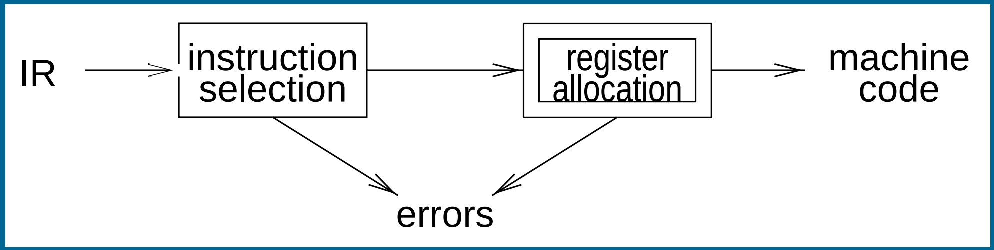
Register allocation



Register allocation:

- have value in a register when used
- limited resources
- changes instruction choices
- can move loads and stores
- optimal allocation is difficult
  $\Rightarrow$ NP-complete for $k \geq 1$ registers

Liveness analysis

Problem:

- IR contains an unbounded number of temporaries
- machine has bounded number of registers

Approach:

- temporaries with disjoint *live* ranges can map to same register
- if not enough registers then *spill* some temporaries
  (i.e., keep them in memory)

The compiler must perform *liveness analysis* for each temporary:

It is *live* if it holds a value that may be needed in future

Control flow analysis

Before performing liveness analysis, need to understand the control flow by building a *control flow graph* (CFG):

- nodes may be individual program statements or basic blocks
- edges represent potential flow of control

Out-edges from node n lead to *successor* nodes, *succ*$[n]$
In-edges to node n come from *predecessor* nodes, *pred*$[n]$
Example:

$$
\begin{array}{rl}
 & a \leftarrow 0 \\
L_1: & b \leftarrow a+1 \\
 & c \leftarrow c+b \\
 & a \leftarrow b \times 2 \\
 & \text{if } a < N \text{ goto } L_1 \\
 & \text{return } c
\end{array}
$$

Liveness analysis

Gathering liveness information is a form of *data flow analysis* operating over the CFG:

- liveness of variables "flows" around the edges of the graph
- assignments *define* a variable, v:
  - $\mathit{def}(v) =$ set of graph nodes that define v
  - $\mathit{def}[n] =$ set of variables defined by n
- occurrences of v in expressions *use* it:
  - $\mathit{use}(v) =$ set of nodes that use v
  - $\mathit{use}[n] =$ set of variables used in n

Liveness: v is *live* on edge e if there is a directed path from e to a *use* of v that does not pass through any $\mathit{def}(v)$

v is *live-in* at node n if live on any of n's in-edges

v is *live-out* at n if live on any of n's out-edges

$v \in \mathit{use}[n] \Rightarrow v$ live-in at n

v live-in at $n \Rightarrow v$ live-out at all $m \in \mathit{pred}[n]$

v live-out at $n, v \notin \mathit{def}[n] \Rightarrow v$ live-in at n

Liveness analysis

Define:
$\mathit{in}[n]$: variables live-in at n
$\mathit{in}[n]$: variables live-out at n

Then:

$$\mathit{out}[n] = \bigcup_{s \in \mathit{succ}(n)} \mathit{in}[s]$$

$$\mathit{succ}[n] = \phi \Rightarrow \mathit{out}[n] = \phi$$

Note:

$$\mathit{in}[n] \supseteq \mathit{use}[n]$$

$$\mathit{in}[n] \supseteq \mathit{out}[n] - \mathit{def}[n]$$

$\mathit{use}[n]$ and $\mathit{def}[n]$ are constant (independent of control flow)

Now, $v \in \mathit{in}[n]$ iff. $v \in \mathit{use}[n]$ or $v \in \mathit{out}[n] - \mathit{def}[n]$

Thus, $\mathit{in}[n] = \mathit{use}[n] \cup (\mathit{out}[n] - \mathit{def}[n])$

Iterative solution for liveness

foreach n

$in[n] \leftarrow \phi$

$out[n] \leftarrow \phi$

repeat

foreach n

$in'[n] \leftarrow in[n];$

$out'[n] \leftarrow out[n];$

$in[n] \leftarrow use[n] \cup (out[n] - def[n])$

$out[n] \leftarrow \bigcup_{s \in succ[n]} in[s]$

until $in'[n] = in[n] \wedge out'[n] = out[n], \forall n$

Notes:

- should order computation of inner loop to follow the "flow"
- liveness flows *backward* along control-flow arcs, from *out* to *in*
- nodes can just as easily be basic blocks to reduce CFG size
- could do one variable at a time, from *uses* back to *defs*, noting liveness along the way

Iterative solution for liveness

Complexity: for input program of size N

- $\leq N$ nodes in CFG
  $\Rightarrow \leq N$ variables
  $\Rightarrow N$ elements per *in/out*
  $\Rightarrow O(N)$ time per set-union
- **for** loop performs constant number of set operations per node
  $\Rightarrow O(N^2)$ time for **for** loop
- each iteration of **repeat** loop can only add to each set
  sets can contain at most every variable
  $\Rightarrow$ sizes of all in and out sets sum to $2N^2$,
  bounding the number of iterations of the **repeat** loop

$\Rightarrow$ worst-case complexity of $O(N^4)$

- ordering can cut **repeat** loop down to 2-3 iterations
  $\Rightarrow O(N)$ or $O(N^2)$ in practice

Iterative solution for liveness

Least fixed points

There is often more than one solution for a given dataflow problem (see example).

Any solution to dataflow equations is a *conservative approximation*:

- v has some later use downstream from n
  $\Rightarrow v \in \mathit{out}(n)$
- but not the converse

Conservatively assuming a variable is live does not break the program; just means more registers may be needed.

Assuming a variable is dead when it is really live *will* break things.

May be many possible solutions but want the “smallest”: the least fixpoint.

The iterative liveness computation computes this least fixpoint.

Another DF analysis example

- Problem: given a program, identify all possible null pointer dereference errors.

```
1. p=&A;
2. i=0;
3. While (i<N) {
4.   sum=sum+*p;
5.    if (i>3)
6.        p=0;
       else
 7.       p++;
8.     i++
    }
```

There is a null pointer dereference error in the code snippet on the left, when the program takes the path 6-8-3-4

- A naïve solution: identify all pointer dereference points, for each deref point, enumerate all backward paths from the point to see if a null assignment (def) can be encountered without encountering another def.
  - Problem: path explosion and loops

- Data flow equation:

$IN[n]=\bigcup_{p \in pred(n)} OUT[p]$

$OUT[n]=(IN[n]\text{- all null defs })\cup(\text{if } n \text{ is a null def then } \{n\} \text{ else } \{\})$

- Full algorithm:

```
Initialize IN[] and OUT[] to {}
changed =1;
While (changed) {
    changed =0
    for (each node n in topological order)
        update IN [n] and OUT[n] according to the above equations.
    changed = new IN[n]/OUT[n] is observed
}
```

- Proof of Termination:

$IN[n]=U_{p\in pred(n)}OUT[p]$
OUT[n]=(IN[n]- all null defs)U (if n is a null def then {n} else {})

The set of all null defs and the set (if n is a null def then {n} else{}) are constants regarding a specific n, lets represent them as Kill and Gen, the equation becomes:

$IN[n]=U_{p\in pred(n)}OUT[p]$
OUT[n]=(IN[n]- Kill)U Gen

Since they are constant, the two equations are monotonic, meaning IN[n] increases if OUT[p] increases, and vice versa

And, the maximal value of IN[n] and OUT[n] is bounded.