Alexander Ostrowski 1893–1986

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Purdue University

Collected Mathematical Papers

Volume 1

- Determinants
- Linear Algebra
- Algebraic Equations

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- Determinants
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Volume 2

- Multivariate Algebra
- Formal Algebra

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Volume 6

- Conformal Mapping
- Numerical Analysis
- Miscellaneous

Ostrowski’s mother

Ostrowski, the cadett

Vol.1. Determinants, Linear Algebra, Algebraic Equations

Determinants and Linear Algebra

- (1937) Matrices with dominant diagonal

$$\boldsymbol{A} = [a_{ij}], \quad d_i := |a_{ii}| - \sum_{j \neq i} |a_{ij}| > 0, \ \text{all } i$$

Hadamard (1899): $\det \boldsymbol{A} \neq 0$

Ostrowski: $|\det \boldsymbol{A}| \geq \prod_i d_i$

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- (1937) M-matrices

$$\boldsymbol{A} = [a_{ij}], \quad a_{ii} > 0, \quad a_{ij} \leq 0 \ (i \neq j)$$

$$a_{11} > 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \ldots, \det \boldsymbol{A} > 0$$

Vol.1 (continued)

Thm. If $\boldsymbol{A}$ is an M-matrix, then $\boldsymbol{A}^{-1} \geq \mathbf{0}$

- (1955) General theory of vector and matrix norms
- (1958–59) Rayleigh quotient iteration for computing real eigenvalues of a matrix $\boldsymbol{A}$

  for $k = 0, 1, 2, \ldots$ do

  $$\left[\begin{array}{l} \rho_0 = 0, \quad \boldsymbol{x}_{-1} \text{ arbitrary} \\ (\boldsymbol{A} - \rho_k \boldsymbol{I})\boldsymbol{x}_k = \boldsymbol{x}_{k-1}, \quad \rho_{k+1} = \boldsymbol{x}_k^T \boldsymbol{A} \boldsymbol{x}_k / \boldsymbol{x}_k^T \boldsymbol{x}_k \end{array}\right.$$

  Convergence $\rho_k \longrightarrow \lambda$ to an eigenvalue λ of $\boldsymbol{A}$ is cubic if $\boldsymbol{A}^T = \boldsymbol{A}$ and quadratic otherwise (in general). Generalized Rayleigh quotient iteration (involving bilinear forms) always converges cubically.
- (1963–64) Positive matrices (Frobenius theory)

David Hilbert

Algebraic Equations

- (1933) Critique and correction of Gauss's first and fourth proof of the Fundamental Theorem of Algebra
- (1939) Continuity of the roots of an algebraic equation as a function of the coefficients: quantitative statement

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- (1939) Continuity of the roots of an algebraic equation as a function of the coefficients: quantitative statement

Theorem Let x_ν, y_ν be the zeros of

$$p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \quad a_0 a_n \neq 0$$

resp.

$$q(z) = b_0 z^n + b_1 z^{n-1} + \cdots + b_n, \quad b_0 b_n \neq 0.$$

If

$$b_\nu - a_\nu = \varepsilon_\nu a_\nu, \quad |\varepsilon_\nu| \leq \varepsilon, \quad 16 n \varepsilon^{1/n} \leq 1,$$

then, with x_ν and y_ν suitably ordered,

$$\left| \frac{x_\nu - y_\nu}{x_\nu} \right| \leq 15 n \varepsilon^{1/n}.$$

Algebraic Equations (continued)

- (1940) Long memoir on Graeffe’s method
- (1928–65) Sign rules (Descartes, Budan-Fourier, Runge)
- (1979!) A little mathematical jewel

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Theorem Let p and q be polynomials of degrees m and n, respectively. Define

$$M_f = \max_{|z|=1} |f(z)|.$$

Then

$$\gamma M_p M_q \le M_{pq} \le M_p M_q, \quad \gamma = \sin^m \frac{\pi}{8m} \sin^n \frac{\pi}{8n}.$$

Ostrowski, the skater

Vol. 2. Multivariate Algebra, Formal Algebra

Multivariate Algebra

- (1918–20) Existence of a finite basis $\mathcal{B}$ for a system $\mathcal{S}$ of polynomials in several variables. *Example*: necessary and sufficient conditions for

$$\mathcal{S} = \{x_1^{\alpha_{1,i}} x_2^{\alpha_{2,i}} \cdots x_n^{\alpha_{n,i}} : \ \alpha_{\nu,i} \in \mathbb{N}_0, \ \ i = 1, 2, 3, \dots\}$$

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- (1922, 1975–77) Various questions of irreducibility

Formal Algebra

- (1913!) Algebra of finite fields (37-page memoir in Ukrainian)

Formal Algebra (continued)

- (1913–17) Theory of valuation on a field. (Generalizing the concept of absolute value in the field of real numbers to general fields.) “Perfect” fields and extensions thereof

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- (1934) Arithmetic theory of fields
- (1936) Structure of polynomial rings

Formal Algebra (continued)

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- (1934) Arithmetic theory of fields
- (1936) Structure of polynomial rings

- (1954) Evaluation of polynomials

$$p_n(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n$$

by Horner’s rule

$$p_0 = a_0, \;\; p_\nu = x p_{\nu-1} + a_\nu, \quad \nu = 1, 2, \ldots, n$$

Complexity: n additions, n multiplications. Optimal for addition, optimal for multiplication when $n \leq 4$.

Formal Algebra (continued)

The year 1954 is generally considered "the year of birth of algebraic complexity theory" (P. Bürgisser and M. Clausen, 1996).

- (1961) Metric properties of block matrices

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \cdots & \boldsymbol{A}_{1n} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \cdots & \boldsymbol{A}_{2n} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{A}_{n1} & \boldsymbol{A}_{n2} & \cdots & \boldsymbol{A}_{nn} \end{bmatrix}, \quad \boldsymbol{A}_{\nu\mu} \in \mathbb{R}^{\nu\times\mu},\ \det \boldsymbol{A}_{\nu\nu} \neq 0$$

  Example: Is Hadamard's theorem still valid if $|\cdot|$ is replaced by $\|\cdot\|$? *Answer*: yes, if the "associated matrix"

$$\begin{bmatrix} \|\boldsymbol{A}_{11}\|^* & -\|\boldsymbol{A}_{12}\| & \cdots & -\|\boldsymbol{A}_{1n}\| \\ -\|\boldsymbol{A}_{21}\| & \|\boldsymbol{A}_{22}\|^* & \cdots & -\|\boldsymbol{A}_{2n}\| \\ \vdots & \vdots & & \vdots \\ -\|\boldsymbol{A}_{n1}\| & -\|\boldsymbol{A}_{n2}\| & \cdots & \|\boldsymbol{A}_{nn}\|^* \end{bmatrix}$$

  is an M-matrix, where

$$\|\boldsymbol{B}\|^* = \min_{\|\boldsymbol{x}\|=1} \|\boldsymbol{B}\boldsymbol{x}\|, \quad \|\boldsymbol{B}\| = \max_{\|\boldsymbol{x}\|=1} \|\boldsymbol{B}\boldsymbol{x}\|$$

Formal Algebra (continued)

- (1961) Convergence of "block iterative methods" for solving $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A}$ is a block matrix

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- (1977) Kronecker's elimination theory in the general setting of polynomial ideals

Ostrowski in his 40s

Vol. 3. Number Theory, Geometry, Topology, Convergence

Number Theory

- (1919) Polynomials with coefficients in a finite arithmetic field taking on integer values for integer arguments. Existence of a “regular” basis

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- (1919) Arithmetic theory of algebraic numbers
- (1921–27, 1964–82) Diophantine equations and approximations

Geometry

- (1955) A study of evolutes and evolvents of a plane curve (under weak differentiability assumptions). Application to ovals

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- (1933) Existence criterion for a common zero of two real functions continuous inside and on the boundary of a disk

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Topology

- (1933) Existence criterion for a common zero of two real functions continuous inside and on the boundary of a disk
- (1935) Topology of oriented line elements

Ostrowski in his 50s

Convergence

- (1930) Infinite products

$$\prod_{\nu=0}^{\infty}(1+x_\nu)=\Phi(x)\ \text{ where } x_0=x,\ x_{\nu+1}=\varphi(x_\nu),\ \nu=0,1,2,\ldots$$

Example: Euler's product $\varphi(x)=x^2$, $\Phi(x)=(1-x)^{-1}$.

Problem: find *all* products which converge in a neighborhood of $x=0$, and for which φ is rational and Φ algebraic.

Convergence

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Example: Euler's product $\varphi(x) = x^2$, $\Phi(x) = (1-x)^{-1}$.

Problem: find *all* products which converge in a neighborhood of $x = 0$, and for which φ is rational and Φ algebraic.

- (1930) "Normal" power series:

$$\sum_{\nu=-\infty}^{\infty} a_\nu z^\nu \text{ with } a_\nu \geq 0,\ a_\nu^2 \geq a_{\nu-1}a_{\nu+1}$$

and all coefficients between two positive ones are also positive.

Theorem: The product of two normal power series is also normal.

Convergence (continued)

- (1955) Convergence and divergence criteria of Ermakov for

$$\int^{\infty} f(x)dx$$

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- (1956) Fixed point iteration

$$x_{\nu+1} = f(x_\nu), \quad \nu = 0, 1, 2, \dots,$$

where $f(x) \subseteq I$ if $x \in I$. Behavior of $\{x_\nu\}$ in case of divergence

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- (1972) Summation of slowly convergent positive or alternating series

Vol. 4. Real Function Theory, Differential Equations, Differential Transformations

Real Function Theory

- (1919–38) Strengthening, or simplifying, the proof of many known results from real analysis

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- (1946) Indefinite integrals of "elementary" functions (Liouville Theory)

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- (1957) Points of attraction and repulsion for fixed point iteration in Euclidean space

Real Function Theory (continued)

- (1958) Univalence of nonlinear transformations in Euclidean space

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- (1969–70) Study of the remainder term in the Euler-Maclaurin formula

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- (1958) Univalence of nonlinear transformations in Euclidean space
- (1966) Theory of Fourier transforms
- (1969–70) Study of the remainder term in the Euler-Maclaurin formula
- (1970) The (frequently cited) Ostrowski–Grüss inequality

$$\left|\int_0^1 f(x)g(x)dx - \int_0^1 f(x)dx \int_0^1 g(x)dx\right| \leq \frac{1}{8} \operatorname*{osc}_{[0,1]} f \max_{[0,1]} |g'|$$

Real Function Theory (continued)

- (1975) Asymptotic expansion of integrals containing a large parameter

Real Function Theory (continued)

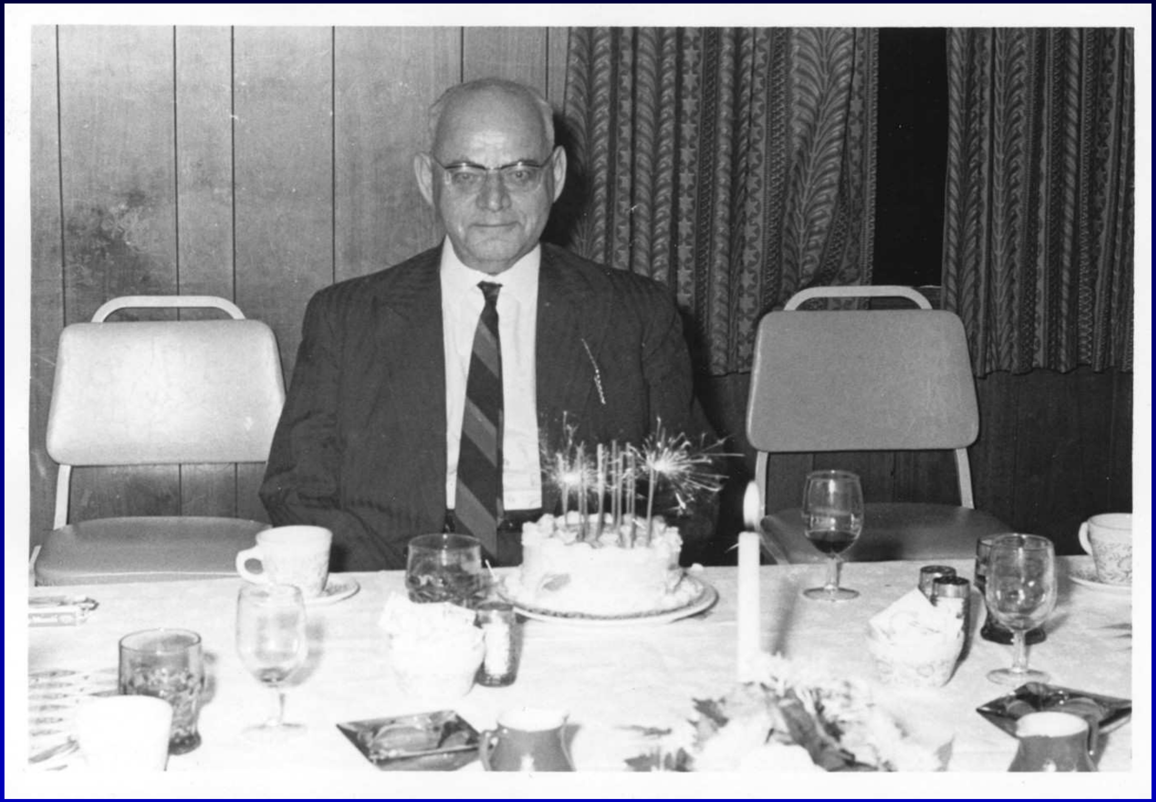
- (1975) Asymptotic expansion of integrals containing a large parameter

- (1976) Generalized Cauchy-Frullani integral

$$\int_0^\infty \frac{f(at)-f(bt)}{t}\,dt = [M(f)-m(f)]\ln\frac{a}{b}, \quad a>0,\ b>0$$

where

$$M(f)=\lim_{x\to\infty}\frac{1}{x}\int_1^x f(t)dt,\ \ m(f)=\lim_{x\downarrow 0}\ x\int_x^1 \frac{f(t)}{t^2}\,dt$$



Ostrowski’s 60th birthday

Differential Equations

- (1919) Göttingen thesis: Dirichlet series and algebraic differential equations

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- (1942) Invertible transformations of line elements

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- (1929) Quasi-analytic functions, the theory of Carleman
- (1933, 1955) Analytic continuation of power series and Dirichlet series

Vol. 6. Conformal Mapping, Numerical Analysis, Miscellanea

Conformal Mapping

- (1929) Constructive proof of the Riemann Mapping Theorem

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Numerical Analysis

- (1937–38) Newton's method for a single equation and a system of two equations: convergence, error estimates, robustness with respect to rounding

The Ostrowskis in their home in Montagnola

Numerical Analysis (continued)

- (1953) Convergence of relaxation methods for linear $n \times n$ systems, optimal relaxation parameters for $n = 2$

Numerical Analysis (continued)

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- (1954) Matrices close to a triangular matrix

$$\boldsymbol{A} = [a_{ij}],\ |a_{ij}| \le m\ (i > j),\ |a_{ij}| \le M\ (i < j),\ 0 < m < M$$

Theorem All eigenvalues of $\boldsymbol{A}$ are contained in the union of disks $\cup_i D_i$, $D_i = \{z \in \mathbb{C} :\ |z - a_{ii}| \le \delta(m, M)\}$, where

$$\delta(m, M) = \frac{Mm^{\frac{1}{n}} - mM^{\frac{1}{n}}}{M^{\frac{1}{n}} - m^{\frac{1}{n}}}\,.$$

The constant $\delta(m, M)$ is best possible.

Numerical Analysis (continued)

- (1956) “Absolute convergence” of iterative methods for solving linear systems

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- (1958) A device of Gauss for speeding up iterative methods

Numerical Analysis (continued)

- (1964) Convergence analysis of Muller’s method for solving nonlinear equations

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- (1969) A descent algorithm for the roots of algebraic equations
- (1971) Newton's method in Banach spaces
- (1972–73) *a posteriori* error bounds in iterative processes

Today's Special

- (1964) A determinant with combinatorial numbers

$$\begin{vmatrix} \binom{x}{k_0} & \binom{x}{k_0-1} & \cdots & \binom{x}{k_0-m} \\ \binom{x}{k_1} & \binom{x}{k_1-1} & \cdots & \binom{x}{k_1-m} \\ \cdots & \cdots & \cdots & \cdots \\ \binom{x}{k_m} & \binom{x}{k_m-1} & \cdots & \binom{x}{k_m-m} \end{vmatrix}$$

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$$= [\Gamma(x+1)]^{m+1} \frac{\prod_{\mu=1}^{m}(x+\mu)^{m+1-\mu}\Delta(k_0,\ldots,k_m)}{\prod_{\mu=0}^{m}[\Gamma(k_\mu+1)\Gamma(x+1+m-k_\mu)]}$$

where

$$\Delta(k_0,\ldots,k_m) = \prod_{\lambda>\mu}(k_\lambda - k_\mu)$$

Ostrowski in his 90s

Vol. 6. Miscellaneous

- Miscellaneous papers on probability distribution functions

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- Lectures

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- Obituaries (G. H. Hardy, 1877–1947; W. Süss, 1895–1958; Werner Gautschi, 1927–1959)

Gentilino


IN MEMORIAM DI
MARGHERITA
OSTROWSKI - SACHS
1897 - 1982
PROF. DR.
A.M. OSTROWSKI
1893 - 1986