MR661060 (83g:41031) 41A55 (01A55 01A60 65D32)

Gautschi, Walter

A survey of Gauss-Christoffel quadrature formulae.

E. B. Christoffel (Aachen/Monschau, 1979), pp. 72–147, *Birkhäuser, Basel-Boston, Mass.*, 1981.

This very substantial work (the bibliography contains about 450 items) presents in an excellent way the history and theory of the Gauss quadrature method and its various generalizations. It is a must for everybody who is interested in quadrature theory. Key words of the content: Multiple nodes; Prescribed nodes; Kronrod procedures; Gauss-type formulas for Chebyshev systems; Nonpositive weight functions; Cauchy principal values; Approximation of the remainder via function theory, functional analysis, approximation theory; Peano kernels; Convergence; Numerical computation of formulas; Tables.

{For the entire collection see MR0661053 (83e:01002)}

Reviewed by *Helmut Brass*

MR875242 (88f:30001a) 30-03 (01A60 30C50)

de Branges, Louis (1-PURD)

The story of the verification of the Bieberbach conjecture.

The Bieberbach conjecture (West Lafayette, Ind., 1985), 199–203, *Math. Surveys Monogr.*, 21, *Amer. Math. Soc.*, *Providence, RI*, 1986.

MR875243 (88f:30001b) 30-03 (01A60 30C50)

Gautschi, Walter (1-PURD-C)

Reminiscences of my involvement in de Branges's proof of the Bieberbach conjecture.

The Bieberbach conjecture (West Lafayette, Ind., 1985), 205–211, *Math. Surveys Monogr.*, 21, *Amer. Math. Soc.*, *Providence, RI*, 1986.

MR875244 (88f:30001c) 30-03 (01A60 30C50)

Askey, Richard (1-WI)

My reaction to de Branges's proof of the Bieberbach conjecture.

The Bieberbach conjecture (West Lafayette, Ind., 1985), 213–215, *Math. Surveys Monogr.*, 21, *Amer. Math. Soc.*, *Providence, RI*, 1986.

In March 1985, an international conference at Purdue University was held for the Symposium on the occasion of the proof of the Bieberbach Conjecture. The Mathematical Surveys and Monographs, Number 21, 1986, published by the American Mathematical Society, contains fifteen mathematical papers dealing with related analysis problems. In the last few pages of the monograph there are also three accounts: (1) The story of the verification of the Bieberbach conjecture, by de Branges, (2) Reminiscenses of my involvement in de Branges' proof of the Bieberbach conjecture, by Gautschi, and (3) My reaction to de Brange's proof of the Bieberbach conjecture, by Askey. The comments below perhaps may suffice as one review for all three of these quite interesting personal reports.

Each of these short articles, coming from three of the several principal members of the drama that unfolded, can be read as stories of human interest by even those without mathematical specialization in complex analysis. The reviewer has no intention to give here a listing of the highlights that develop in the tale. To do so perhaps might diminish the fascination of the first reading by those who have yet to see these articles.

The media, particularly the press, has already alerted the general public to the historical fact that the Bieberbach conjecture and its generalizations finally have been verified. The media picked up the unusual situation that numerous outstanding mathematicians the world over had made courageous attempts without success over a period of sixty-eight years to find a proof for the conjecture. Only in special cases were they successful and even then the proofs were generally not easily found.

Paper (1) is the story from the pen of the talented mathematician de Branges at Purdue University

who had worked successfully at hard problems in other fields of mathematics until a way became clear to him in a new direction that eventually led to his proof of the famous conjecture. Actually he proved a stronger result known as the Milin conjecture which implies the Robertson conjecture which in turn implies the Bieberbach conjecture. His paper deals with the events of the months before and after his discovery. It is an accurate account of the author's somewhat tempestuous time until the difficult task was finished. It is written with admirably calm restraint. He acknowledges the work of those who preceded him and the able expert assistance of others in Leningrad. In particular, colleague Gautschi, author of (2), with his accurate computer system at Purdue, verified the needed positive character of integrals involving Jacobi polynomials of degree high enough to substantiate the proof for at least thirty coefficients appearing in the conjecture. This was a most encouraging development. Also Askey, author of (3), at the University of Wisconsin, a longtime expert on inequalities and special functions, was contacted by Gautschi for the prospect of a formal proof of the positivity required of the special polynomials (of arbitrary degree). Askey soon recalled, apparently much to his surprise, that the missing link had appeared in a joint paper by G. Gasper and himself. He tells how there are now five proofs of the desired polynomial inequality, four due to Gasper and one found by T. Koornwinder. How fate and apparent chance play their parts too in the scheme of things, however elusive!

The reviewer purposely omits the details of these three papers. They are needed in the papers to give them true meaning, depth and body. However, this ink-stained writer of reviews at this stage urges kindred souls to read for themselves the story briefly outlined in the three papers under discussion. To this reviewer the account is more than just a story about mathematics. It is a story of the struggle of the human spirit, some of the great driving forces that lure mankind and above all the ever-present intellectual honesty that binds together so many of the great research-driven mathematicians. Are we not all fortunate to catch now and then something of the majestic mystery that lies beyond our search for logical mathematical proofs? The conjecture is replaced by a theorem but could not what seems to be an end be only part of a new beginning?

{For the entire collection see MR0875226 (87k:30002)}

Reviewed by *M. S. Robertson*

MR1339719 (96e:65002) 65-03 (01A60 01A70 65D30)

Gautschi, Walter (1-PURD-C)

The work of Philip Rabinowitz on numerical integration. (English summary)

Numer. Algorithms **9** (1995), *no. 3-4,* 199–222.

The author gives a summary and appreciation of Philip Rabinowitz' contributions to numerical analysis, with emphasis on his multifaceted work on integration (Sections 2–8 in this paper). The brief §1 contains a review of Rabinowitz' activities at the National Bureau of Standards, where he had access to the SEAC machine (in the 1950s), and where his collaboration with P. J. Davis began. §9 (Expository writings) mentions the well-known monograph on numerical integration written jointly with Davis (the three editions appeared in 1967, 1975 and 1984) and the textbook in numerical analysis by R. Ralston and Rabinowitz (1978). The article is accompanied by an ample bibliography of 97 titles.

MR1343520 65D20 (01A70 33C05 39B12)

Gautschi, Walter (1-PURD-C)

Luigi Gatteschi's work on special functions and numerical analysis. (English summary)

Special functions (Torino, 1993).
Ann. Numer. Math. **2** (1995), *no. 1-4*, 3–19.

{This item will not be reviewed individually.}
{For the entire collection see MR1343518 (96c:00037)}

MR1961202 (2004b:65034) 65Fxx (33C45)

Gautschi, Walter (1-PURD-C)

The interplay between classical analysis and (numerical) linear algebra—a tribute to Gene H. Golub. (English summary)

Electron. Trans. Numer. Anal. **13** (2002), 119–147 (*electronic*).

This expository paper, which was written to honor Gene Golub on his 70th birthday, gives a nice overview of the many connections between numerical linear algebra and classical analysis that have been uncovered by Golub and his collaborators over the years.

Topics covered include Gaussian quadrature formulas and their relationship to tridiagonal matrices and orthogonal polynomials, Stieltjes and Lanczos processes, updating and downdating least squares problems, the analysis of iterative methods for solving linear systems, and much more. Have a look at this interesting paper.

Reviewed by *David Scott Watkins*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2376765 (2009d:01015) 01A50 (11-03 40-03 40A05)

Gautschi, Walter (1-PURD-C)

Leonhard Eulers Umgang mit langsam konvergenten Reihen. (German) [Leonhard Euler's handling of slowly convergent series]

Elem. Math. **62** (2007), *no. 4,* 174–183.

This paper deals with Euler's handling of the (very slowly converging) series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (the "Basler problem"); computation of the Euler constant $\gamma = \lim_{n\to\infty}(\sum_{v=1}^{n} \frac{1}{v} - \log n)$; Euler's transformation of series to replace slowly convergent series by other more rapidly converging ones; and the Lambert series $\sum_{v=1}^{\infty} \frac{1}{x^v - 1}$, $x > 1$, especially when $x = 10$.

Reviewed by *S. L. Segal*

MR2281939 (2008b:65004) 65-06 (01A75 65Fxx)

Golub, Gene H.

★**Milestones in matrix computation: selected works of Gene H. Golub, with commentaries.**
Edited by Raymond H. Chan, Chen Greif and Dianne P. O'Leary.
Oxford Science Publications.

Oxford University Press, Oxford, 2007. *xii*+565 *pp*. £65.00. *ISBN* 978-0-19-920681-0

Gene Golub has been one of the leading figures in the field of matrix computations for the past half century. This volume, which appeared just in time for Gene's 75th birthday, is a collection of 21 of his most influential papers, divided into five categories. In addition there is an opening section (Part I) that includes a brief biography of Gene by Chen Greif, lists of Gene's publications and major awards, and a list of Gene's Ph.D. students. Each of the other five parts is headed by a commentary written by a leading figure in the field. Part II, which opens with commentary by Anne Greenbaum, is entitled "Iterative Methods for Linear Systems" and focuses on the Chebyshev semi-iterative and preconditioned conjugate gradient methods. Part III, entitled "Solution of Least Squares Problems", includes the papers in which computational techniques for the QR and singular value decompositions are developed and their application to the least squares problem is advocated. The commentary is by Åke Björck. Part IV, entitled "Matrix Factorizations and Applications", is a mixed bag that overlaps substantially with Part III. Topics discussed: singular value decomposition, updating of matrix factorizations, fast Poisson solvers, computation of angles between subspaces. The commentary is by Nick Higham. Part V is on orthogonal polynomials and quadrature. Headed by commentary by Walter Gautschi, this part features three papers that discuss the interplay between eigenvalue problems, orthogonal polynomials, Gaussian quadrature, and classical analysis. Part VI, entitled "Eigenvalue Problems", is another mixed bag. Topics discussed include computation of the Jordan form, block Lanczos methods, and solution of inverse eigenvalue problems for Jacobi matrices. The commentary is by G. W. Stewart. Also included in the book are ten photos of Gene at various stages of his life and one of Gene's license plate. The lists of topics given in this review are not meant to be complete lists of the topics discussed in the book.

Reviewed by *David Scott Watkins*

MR2441235 (2009g:01006) 01A50 (65-03)

Gautschi, Walter (1-PURD-C)

On Euler's attempt to compute logarithms by interpolation: a commentary to his letter of February 16, 1734 to Daniel Bernoulli. (English summary)

J. Comput. Appl. Math. **219** (2008), *no. 2*, 408–415.

Summary: "In the letter to Daniel Bernoulli, Euler reports on his attempt to compute the common logarithm $\log x$ by interpolation at the successive powers of 10. He notes that for $x = 9$ the procedure, though converging fast, yields an incorrect answer. The interpolation procedure is analyzed mathematically, and the discrepancy explained on the basis of modern function theory. It turns out that Euler's procedure converges to a q-analogue $S_q(x)$ of the logarithm, where $q = \frac{1}{10}$. In the case of the logarithm $\log_\omega x$ to base $\omega > 1$ (considered by Euler almost twenty years later), the limit of the analogous procedure (interpolating at the successive powers of ω) is $S_q(x)$ with $q = 1/\omega$. It is shown that by taking $\omega > 1$ sufficiently close to 1 and interpolating at sufficiently many points, the logarithm $\log x$ can indeed be approximated arbitrarily closely, although, if x, $1 < x < 10$, is relatively large, extremely high-precision arithmetic is required to overcome severe numerical cancellation. An alternative procedure for computing $\log x$ by interpolation at points in $[1, 10^\omega]$, $\omega > 0$, accumulating at the lower end point, is shown to converge to the desired limit, but also not without numerical complications."

MR2403056 (2009d:01016) 01A50 (01A70)

Gautschi, Walter (1-PURD-C)

Leonhard Euler: his life, the man, and his works. (English summary)

SIAM Rev. **50** (2008), *no. 1,* 3–33.

Summary: “On the occasion of the 300th anniversary (on April 15, 2007) of Euler’s birth, an attempt is made to bring Euler’s genius to the attention of a broad segment of the educated public. The three stations of his life—Basel, St. Petersburg, and Berlin—are sketched and his principal works are identified in a more or less chronological order. To convey a flavor of his work and its impact on modern science, a few of Euler’s memorable contributions are selected and discussed in more detail. Remarks on Euler’s personality, intellect, and craftsmanship round out the presentation.”

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Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2457088 (2009j:33002) 33C15 (26C10 30C15 33-03 33C10 33C45)

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Luigi Gatteschi's work on asymptotics of special functions and their zeros. (English summary)

Numer. Algorithms **49** (2008), *no. 1-4,* 11–31.

The work of Luigi Gatteschi on special functions is reviewed by the authors. Up to 55 of Gatteschi's papers are cited in this review paper, and a brief account of some of the main results is given. The paper is organized into 5 sections, following methodological and chronological lines: Introduction; Early influence of Szegő, van der Corput and Tricomi; Methods based on differential equations; Uniform expansions; Miscellanea.

From the introduction: "In the realm of special functions and their zeros, Luigi Gatteschi is without any doubt one of the major exponents of, and contributor to, the quantitative theory of asymptotics. His results are not only of a concrete numerical nature, but often attain a degree of sharpness rarely found elsewhere in the literature".

Reviewed by *Javier Segura*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

No review is available for [**196**].