Remembering A.S. Kronrod*

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A.S. Kronrod 1921–1986

Aleksandr Semenovich Kronrod was born on October 22, 1921, in Moscow.

Sasha Kronrod discovered mathematics when he was a participant in the now legendary study group for schoolchildren that was affiliated with Moscow State University. His teacher, D.O. Shklyarskiĭ, was a talented young scientist and an outstanding pedagogue. His general method was to encourage students to find solutions to difficult problems on their own. In 1938, Kronrod entered the Faculty of Mechanics and Mathematics at Moscow State University, where he immediately became known to the entire faculty, students, and instructors. They were enthralled by his outstanding

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talent, enormous energy, range of activity, and his sometimes deliberately paradoxical statements — even by his appearance — he was tall and had a beautiful sonorous voice.

While still a freshman, Kronrod did his first independent work. Professor A.O. Gel'fond, who at that time was Chair of Mathemaical Analysis and supervised a student circle, proposed a traditional problem in pre-World War II mathematics (although the problem was not traditional for Alexander Osipovich himself). It was concerned with the description of the possible structure of the set of points of discontinuity of a function that is differentiable at the points of continuity. In 1939, Kronrod's first scientific article, in which this problem was solved, appeared in the journal *Izvestiya Akademii Nauk*.

The normal course of studies for Kronrod's generation was interrupted by the war. Kronrod petitioned to be sent to the front but was rejected; students at the graduate level were exempt from conscription. In subsequent years, they were sent to military academies. In the early days of the war they were mobilized to build trenches around Moscow. On his return, he renewed his application for enlistment, was accepted, and was sent to the front.

His military career was not easy. During the winter offensive of the Soviet army near Moscow, his bravery resulted not only in his receiving his first military decoration, but also his first severe injury. After he was wounded a second time in 1943, his return to the army became out of the question. He preserved his ability to study mathematics, but not to fight. The last injury made him an invalid; its effects were felt throughout his life.

While still in the hospital, Kronrod returned to a problem proposed to him by M.A. Kreynes. The problem was to answer the following question. Let the permutation $i \to k_i$ defined on the set $\mathbb{N} = \{i\} = \{1, 2, 3, \ldots\}$ of natural numbers be such that it changes the sum of some infinite series, $\sum a_i \neq \sum a_{k_i}$. Does there exist a (conditionally) convergent series $\sum b_i$ which the above permutation transforms into a divergent one?

Kronrod greatly extended the scope of the problem. He managed to prove that, with respect to their action on (conditionally convergent) series, permutations fall into several categories. There are permutations mapping some convergent series into divergent ones — Kronrod called these "left". Permutations transforming some divergent series into convergent ones he called "right". Obviously, the inverse of a left permutation will always be a right permutation. The intersection of the sets of right and left permutations form "two-sided" permutations. They can transform a convergent series into a divergent one as well as a divergent series into a convergent one. Permutations which are neither left nor right Kronrod called "neutral". These permutations cannot change the convergence of series and, as it turns out, cannot change the sum of even one series. The latter follows from the fact that the set of permutations that can change the sum of a series (Kronrod called them "essential") happens to be a subset of the set of two-sided permutations.

The final part of the work contained a set of effective criteria which permit deciding to which class a permutation belongs (left, right, two-sided, neutral, essential) and an extension of the main results to series with complex terms.

This extraordinarily fine work, published in 1946 in the journal *Matematicheskiĭ Sbornik*, served as his graduation thesis. It earned him the prize of the Moscow Mathematical Society for young scientists. (We note that, while it may not have been the first time a student has been given this award, it was indeed a rare occasion. Also, A.S. Kronrod was the only person ever to be awarded this prestigious prize twice.)

In the autumn of 1944, Kronrod resumed his 4th-year studies at the Faculty of Mechanics and Mathematics. In February of the following year, an extraordinary event occurred: after a long absence, the academician N.N. Luzin returned to lecture at the Faculty. He announced a course "The theory of functions of two real variables" and at the same time started a seminar closely related to the course.

In those days, Nikolay Nikolaevich Luzin was perceived by the students as an almost mythological figure. Most of the leading scientists of the older and middle generations were his students. The famous "Luzitania" (a group of Luzin's pupils) was surrounded by legends. Since he had been absent as a lecturer during the previous years, a natural gap developed in the sequence of his students. It appears that A.S. Kronrod and G.M. Adel'son-Vel'skiĭ were his last students. Although Adel'son-Vel'skiĭ may have had other mentors (I.M. Gel'fand and, in computational mathematics, the slightly older Kronrod), for Kronrod, Luzin was the only mentor. He always was proud of this, and liked to show a copy of the French edition of Luzin's famous dissertation "The Integral and the Trigonometric Series", which had been presented to him by the author. In addition, he fondly remembered Luzin introducing Kronrod as his student to Jacques Hadamard.

Luzin's strongest quality had always been his ability to present pupils with problems of great general mathematical importance which, when worked on independently by strong and persistent young students, could lead to the beginning of new directions.

The problem presented to Adel'son-Vel'skiĭ and Kronrod was as follows. Prove the analyticity of a monogenic function by methods of the theory of functions of a real variable without invoking the Cauchy integral and the theory of functions of a complex variable. Specifically, prove that every function $N(x+iy) = u(x,y)+iv(x,y)$, where $u(x,y)$ and $v(x,y)$ satisfy the Cauchy-Riemann conditions, can be developed into a convergent power series. This problem was solved by Adel'son-Vel'skiĭ and Kronrod, and even generalized. They considered arbitrary equations

$$\frac{\partial u}{\partial x} = A(x,y)\frac{\partial v}{\partial y}; \qquad \frac{\partial u}{\partial y} = -B(x,y)\frac{\partial y}{\partial x}$$

with positive functions $A(x,y)$ and $B(x,y)$, and established a relationship between the smoothness of solutions and the smoothness of the coefficients A and B. (In the case of the Cauchy-Riemann equations, the coefficients are identically equal to 1.) The study of level-curves of functions of two variables, $u(x,y)$ and $v(x,y)$, played an essential role in their research, as well as establishing the maximum principle for these functions.

This work became the starting point for studying level-curves of arbitrary (continuous) functions of two variables; this was done in a subsequent series of papers by A.S. Kronrod and G.M. Adel'son-Vel'skiĭ.

However, Kronrod did not stop here. It was not in his character to deal only with a particular problem; we will speak below about Kronrods's maximalism (in life as well as in science). Dealing with functions of two variables, Kronrod discovered that, while the theory of continuous functions of one (real) variable achieved some degree of completeness at that time, a theory of functions of two (and more) variables simply did not exist. Only the most elementary facts from the theory of functions of one variable were extended, and they did not contain anything "essentially two-dimensional". If the theory does not exist, it has to be created. In the course of the next few years all of Kronrod's attention was devoted to exploring this vast problem area.

Over four years, Kronrod developed an orderly theory, containing properties of functions of two real variables and their connections with the concept of variation; it paved the way for studying functions of many variables.

From the beginning, he avoided using definitions that depend on the choice of a given orthogonal coordinate system (e.g., Tonelli variation), and he introduced concepts that are invariant with respect to orthogonal mappings. Variations of functions of two variables are fundamental concepts for his theory. A.S. Kronrod showed that in the transition from the one-variable theory to the two-variable theory, features depending on variation dichotomize, so that for functions of two variables it is natural to introduce two variations. One of them he called *planar*, the other *linear*. The boundedness of the planar variation guarantees the existence almost everywhere of an asymptotic total differential. For a smooth function, this variation turns out to be equal to the integral of the absolute value of its gradient, extended over the domain of definition.

The linear variation was basically a new object. Kronrod introduced the concept of a monotone function of two variables, a natural generalization of the corresponding concept for a function of a single variable. He proved that the boundedness of the linear variation permits the function to be represented as a difference of two monotone functions. For the linear variation itself, he gave a number of equivalent definitions, of which one is of particular interest. It turns out that with a continuous function of two variables one can associate a one-dimensional tree, the elements of which are the components of the level-sets of the function. On them, with the help of the original function, a metric can be defined and, on the tree, a function. The linear variation then turns out to be equal to the usual variation of the function defined on a one-dimensional tree. The boundedness of both the planar and linear variation guarantees the existence almost everywhere of the usual total differential.

Kronrod considered continuous functions, but the concepts he introduced can easily be carried over to the case of discontinuous functions. He also outlined a program for investigating functions of many variables, which was later carried out by his students.

At that time, an active group of students congregated around Kronrod. (More about Kronrod's pedagogical activity is discussed below.) Among them were A.G. Vitushkin, who developed a theory for variations of functions and sets of many variables and A.Ya. Dubovitskiĭ, who studied in detail the set of critical points for functions of many variables and smooth mappings. In particular, he reproved A. Sard's theorem, at the time not known in Moscow, and also obtained a series of more refined theorems on the structure of the critical points.

From the modern perspective, half a century later, it is not A.S. Kronrod's results themselves that are of the greatest interest. (They represent an important but closed phase of development.) The main value lies in the apparatus he created for obtaining the results. For example, Kronrod's one-dimensional tree was used by V.I. Arnol'd to solve Hilbert's 13th problem.

Especially popular nowadays is the following theorem of Kronrod: Let $G \subset \mathbb{R}^n$ be a domain and $f : G \to \mathbb{R}$ a smooth function, $E_t = \{x \in G | f(x) = t\}$ the level-sets of the function f, and ds the $(n-1)$-dimensional surface element on E_t. Then

$$\operatorname{meas} G = \int_{\min f}^{\max f} \int_{E_t} \left(\frac{ds}{|\nabla f|} \right) dt.$$

This theorem, for example, lies at the basis of many modern proofs in the theory of partial differential equations.

Kronrod's work on the theory of functions of two variables constituted the contents of his Masters thesis which he defended in 1949 at the Moscow State University. His official advisors were M.V. Keldysh, A.N. Kolmogorov, and D.E. Men'shov. For this work he was immediately awarded the Doctoral degree in physical-mathematical sciences, by-passing the Masters degree.

The next large problem to attract Kronrod's attention was the following: Let S be a given surface with bounded Lebesgue area, parametrically embedded in $\mathbb{R}^3$. Is it true that S has an asymptotic tangent plane almost everywhere (in the sense of the measure generated on S by Lebesgue area)? This remained an unsolved problem for a long time. Kronrod found a positive answer but did not publish the solution. He did so because he had decided to break with pure mathematics. The decision was firm and forever.

To understand what happened, we must go back a few years. In 1945, during his fourth-year university studies, Kronrod started working for the computer department of the Kurchatov Atomic Energy Institute. Initially, the reason was financial: he was married, and in 1943 a son was born. In particular, there was a need for accommodation. Working for the Institute offered a solution. But Kronrod was not the kind of person who could take his work lightly. Faced with computational mathematics, he went into it with great seriousness. He found that this was an interesting area, quite unlike pure mathematics, in his opinion. He always stressed that computational methods must be kept apart from theorems that are proved about computational mathematics. For example, he used to say that, when applying finite difference methods to solve differential equations, the finite-difference scheme must be set up starting from the physical problem and not from the differential equation. And one should never be interested in whether the solution of the finite-difference equations converges to the solution of the differential equation, because if the scheme that was set up is physically correct and there is no convergence to the solution of the differential equation, then so much the worse for the differential equation. As a rule, one should not do a theoretical estimation of the error. Such an estimation requires the description of a set of functions containing the solution. *A priori*, this set, as well as the distribution of solutions in it, is unknown. Today, all of this seems trivial, but in those days it sounded paradoxical. Kronrod devised a series of algorithms for the fast solution of various problems (e.g., independently of some other authors, he discovered the sweep method[2]).

Thus, Kronrod discovered for himself a new area of activity. Probably this was not enough for such a resolute break with traditional mathematics, in spite of all the maximalism which, as has already been said, was one of the foremost traits in his character.

At that time, besides electric desk calculators — "mercedeses" — tabulators and sorting machines working with punched cards were the computational devices in use. During this period, a fortunate relationship began to develop between Kronrod and Nikolaĭ Ivanovich Bessonov, a talented relay engineer. From some tabulators and supplementary relay machines for multiplying numbers, which he had developed, Bessonov constructed the machine "Combine", on which one could solve more complex computational problems. Kronrod and Bessonov at this point conceived the idea of a universal program-controlled digital computer. Apparently, the logical aspect of the problem was dealt with by Kronrod, and the design aspect, undoubtedly, by Bessonov. It must be remembered that at that time (the beginning of the second half of the forties) there was still no knowledge in the Soviet Union of American electronic computers. The project of such a computer — RVM (R for "relay", in contrast to the E now in use for "electronic")[3] — was accepted to go into production.

If this computer had been built quickly, it would have become the first digital high-speed computer. Among other things, with respect to speed of computation, it would have surpassed the contemporary American EVMs, owing to the profound ideas incorporated into its design; in particular, it used the "cascade method" (a kind of parallelism, a topical modern problem) and the Shannon counter, which was then largely unknown in the Soviet Union. All of this would have opened new perspectives and revolutionized computational methods.

By the end of the 1940s it was recognized that it was necessary to create, side by side with the I.V. Kurchatov Institute, yet another "atomic" institute, the guidance of which was entrusted to A.I. Alikhanov. On the recommendation of I.V. Kurchatov and L.D. Landau, Alikhanov invited Kronrod to his institute in 1949 and entrusted him with the direction of the Mathematical Department, later named the Institute for Theoretical and Experimental Physics (ITEF). Here, it is appropriate to mention yet another aspect of A.S. Kronrod's nature. He was a born organizer. Being in charge of a department, he was given the opportunity to organize its work as efficiently as possible. Computational mathematics, the computer, the opportunity to organize work in this area, and the recognition of its usefulness — all of this took precedence over his call to pure mathematics; besides, he was to a large extent a pragmatist.

Upon transferring to ITEF, Kronrod invited Bessonov to join the staff. The RVM was being built, but the project was moving at an agonizingly slow pace. The machine

[2] The "sweep method" (МЕТОД ПРОГОНКИ) is an algorithm for solving linear second-order two-point boundary value problems or tridiagonal linear systems arising in the finite-difference solution of them. – W.G.

[3] The V stands for "vychislitel'naya" ("computing") and the M for "machine". – W.G.

was cheap, and unfortunately this created an attitude of low interest toward it. Quite competent and well-meaning people gave Kronrod wise advice on how to speed up the construction. For example, one could make contacts out of gold, which would somewhat improve the quality of the machine, and would make it considerably more expensive. This would radically change the attitude toward the machine. Kronrod could only laugh at this kind of advice. His honesty would never allow him to use such tricks. By the time the machine was completed, a project to build the first electronic computer had already been started. Thanks to the many rich ideas incorporated into the design of the RVM, it would have operated at the high speed of the EVM, but, of course, it had no future. On the other hand, if the computer had been built more quickly, even with golden contacts, it would have repaid the expenses.

We are talking about this RVM in such detail in order to underscore one of A.S. Kronrod's leading principles: an idea is nothing, its implementation everything. Even though rich with brilliant ideas, he did not value them. He gracefully gave them away left and right, quite honestly convinced that the authorship belongs to the one who implements them. In this respect, he was quite the opposite of his teacher, Luzin. With regard to the RVM, he resolutely declared Bessonov (definitely a talented person) to be its sole inventor.

Having had a clear and deep insight, Kronrod quickly realized the advantages of electronic computers over relay computers. He actively participated in discussions on building the first EVM. He was a member of many and diverse committees planning to build such a machine at that time. One must say, though, that, his ideas often been ahead of their time, he was often left in the minority in these discussions. For example, he unsuccessfully insisted on hardware support for floating-point numbers. However, our first machines used fixed-point numbers; operations with floating-point numbers were implemented by means of software. This, theoretically, would increase the speed, but in fact brought down the speed to a very low level. Yet, the relay machine still remained his favorite accomplishment, bringing tears when it was dismantled.

During the period 1950–1955, Kronrod's main activity was finding numerical solutions to physical problems. He collaborated much with physicists, in particular theoretical physicists, among whom, with respect to work, he was closest to I.Ya. Pomeranchuk, and, on a purely personal level, L.D. Landau. For his work on problems of importance to the state he was awarded the Stalin Prize and an Order of the Red Banner.

Only in 1955 did a real opportunity arise for A.S. Kronrod to work with an electronic computer. It was the M-2 computer constructed by I.S. Bruk, M.A. Kartsev, and N.Ya. Matyukhin in the laboratory of the Institute of Energy named after Krzhizhanovskiĭ and directed by I.S. Bruk. This laboratory later became the Institute for Electronic Control Machines. The mathematics/machine interface was developed by A.L. Brudno, a great personal and like-minded friend of Kronrod.

At this point, a new period started in the life of A.S. Kronrod. We will speak about this later, but to preserve the chronological order, we will mention yet another aspect of his activity. During the years 1946–1953, Kronrod led a seminar, called the Kronrod circle. At that time, it was probably not less known among young mathematicians than the Luzin seminar. An atmosphere of enthusiasm always surrounded the seminars he led. Its participants were convinced that mathematics was the most important science and that A.S. Kronrod was one of its prophets. At the same time, he was not the master, but simply Sasha, and it so continued to the end of his days. His seminar studied the theory of functions of a real variable, set theory, and set-theoretical topology. Work continued with the same fervor, even after he left pure mathematics. Then and later, he believed that the theory of functions of a real variable offers the best method for encouraging a student's creativity. Here, in his way of thinking, a minimal amount of initial knowledge enables one to derive complex results. Many mathematicians of the older generation participated in this seminar (E.M. Landis, A.Ya. Dubovitskiĭ, E.V. Glivenko, R.A. Minlos, F.A. Berezin, A.A. Milyutin, A.G. Vitushkin, R.L. Dobrushin, N.N. Konstantinov, and many others).

After the university moved to a new building, Kronrod quit as the leader of the seminar. Shortly thereafter, studies resumed, but were devoted to computer principles.

When he started with enthusiasm to program the M-2 machine, Kronrod quickly came to the conclusion that computing is not the main application of computers. The main goal is to teach the computer to think, i.e., what is now called "artificial intelligence" and in those days "heuristic programming".

Kronrod captivated a large group of mathematicians and physicists (G.M. Adel'son-Vel'skiĭ, A.L. Brudno, M.M. Bongard, E.M. Landis, N.N. Konstantinov, and others). Although some of them had arrived at this kind of problems on their own, they unconditionally accepted his leadership. In the room next to the one housing the M-2 machine, the work of the new Kronrod seminar started. At the gatherings there were heated discussions on pattern-recognition problems (this work was led by M.M. Bongard; versions of his program "Kora" are still functioning), transportation problems (the problem was introduced to the seminar and actively worked on by A.L. Brudno), problems of automata theory, and many other problems.

Kronrod skillfully guided the enthusiasm of the seminar participants toward applications. He proposed to choose a standard problem, so that an advance in the solution allowed judgment on the level reached by the authors in the area of heuristic programming. As such a problem, he proposed an intellectual game. The first problem chosen and programmed was the card game "crazy eights". This choice (in spite of the smiles it provoked) was not accidental and not meant to be frivolous. It is a complex game with no established theory. Considering the low capabilities of the computer and its limited memory, the game's simple description of positions was very important. The program was written and played. It worked fine as long as there were enough cards remaining and in conditions of "incomplete information". After the game became open and everything was reduced to an enumeration of all possible strategies, the computer's capacity was too limited to handle the extremely large size of the game's tree. (The game was abandoned, never again to be resumed. It is not clear whether even modern computers have enough capacity for this game.)

In the process of creating the program, general heuristic programming principles were formulated for the first time. They included a length-independent programmed search (*a priori* it is not clear whether this is possible or not), algorithms for organizing information, etc. Since the "crazy eights" game clearly did not qualify as a standard test problem because it was a strictly regional (or national) game, Kronrod proposed as a standard another game — chess. Chess is played throughout the world. In the USA, people had already started to create chess-playing programs. Such programs were already developed on special-purpose machines: in the Mathematics Division of the ITEF a first, and then a second M-20 machine was installed. The chess program was written by a group of mathematicians (Adel'son-Vel'skiĭ, V.I. Arlazarov, A.R. Bitman, and A.V. Uskov) which did not include Kronrod himself. Nevertheless, when a difficulty was encountered regarding the development of a general recursive search scheme, he entered the group and invented an improvement which helped to overcome the difficulties. He assumed the role of an organizer. It was necessary, but not easy, to create appropriate working conditions for the chess group at the institute. Most of his colleagues treated heuristic programming and anything not connected with their immediate needs as mere entertainment.

He organized a chess match between the institute's program and the best (at that time) American program, developed at Stanford University under the guidance of J. McCarthy. Over the telegraph a match of four games was played, ending with a score of 3 to 1 in favor of the institute's program.

However, the Mathematical Department, of course, existed as a service medium for physical problems, and the time has come to say how this work was organized by A.S. Kronrod. This may be instructive, for in all scientific institutes with a need for mathematical service, work is organized differently.

Kronrod believed that a mathematician solving the mathematical aspect of a physical problem should understand this problem, beginning with its formulation, and should understand how the results obtained are going to be used. Moreover, the mathematician must work out the algorithm, usually according to the physical formulation, write the program, and run it. The programming must be done by the mathematician, because only in this way can the optimal variant of the solution be chosen. For this, one needs mathematicians with sufficiently high qualifications, and Kronrod attracted many good graduates from the Faculty of Mechanics and Mathematics to ITEF, also those who specialized in abstract areas. Why precisely people from the Faculty of Mechanics and Mathematics? He liked to quote I.M. Gel'fand: "The objective of the Faculty of Mechanics and Mathematics is to make people capable", meaning that for a mathematician it suffices to formulate the definitions and the rules operating on them.

For a mathematician to be able to program, without expending unnecessary efforts, however, one must provide him maximum ease and liberate him from all tasks not requiring his qualifications. The mathematician would use a language that is close to the common language, write on a form printed on high-quality paper, using a pencil that allowed erasing an unlimited number of times. There was a rich library of standard programs which were easily accessible. A program (or any piece of it) would be sent to the coding center. Coding, checking the code, punching cards, checking the cards — all this did not require the programmer's attention. The next day he would receive two copies of the program without any coding or punching mistakes. The debugging was done in front of the control panel, and there was no time problem. A programmer was given as much access to the control panel as he needed, and he did not need much. Programs were partitioned into small blocks, each of which could be debugged separately and usually ran the very first time. A correction could be introduced into a program by pushing a key on the control panel, just as an editor does now. A woman responsible for card-punching worked next to the programmer and could immediately change the respective card. For this, colored cards were used. The next day, a corrected white card took its place in the deck.

Each program was required to undergo a check by hand computation. A general rule, strictly followed, was that a program which worked and produced reasonable answers is not necessarily correct, even if the result is accurate in special cases.

It turned out that the work of the coding and card-punching groups was extremely important in the course of writing a program. These groups consisted of women, since they were believed to be more accurate in this kind of work; on each form for writing a program which was prepared for Kronrod's department, on the bottom was written "program written by (a male name)", "coded by (a female name)", "coding checked by (a female name)", "punched by (a female name)", "punching checked by (a female name)".

How did Kronrod achieve such accurate work in all these subdivisions? First, he selected good female employees; second, he managed to provide high salaries for them; and finally, he set the salary in accordance with the quality of the work done. For error-free work, he would give a monthly 20% raise, for two mistakes per month that were made by a card-punching checker, this raise was cut in half. For an additional two mistakes per month, there was no raise at all. (Mistakes on colored cards were not counted.) Here, Kronrod was merciless, but in everything not connected with the quality of work, he was very open and accommodating. His colleagues liked and respected him and took their work to heart — and there were few mistakes.

Bessonov, retraining himself quickly in electronics, kept the computers in exemplary working order. There were practically no malfunctions. One must say here that under the guidance of Kronrod, Bessonov constantly introduced improvements to the machines. In 1963, he completely overhauled the system of commands, thereby increasing the capacity of the machine by a factor of two.

Kronrod proceeded from the assumption that a normal computational problem must run quickly. There are, of course, special cases in which lengthy computations are necessary, but this is not the rule but a rather rare exception. The following policy was adopted: if the debugged program ran more than 10 minutes, its author was invited to see the "Senior Council", headed by Kronrod. There, the algorithms were properly analyzed, and usually the computing time was shortened.

All in all, this was similar to a well-organized factory operation. The results were astonishing. On their low-speed machines, the mathematicians of the ITEF surpassed the West in difficult problems. For example, tracking observations in scintillating cameras produced more accurate results in half the time of a similar program at CERN, running on a computer 500 times faster. In a couple of hours during the night it could compute all that an accelerator could do in 24 hours. That is why there was time to repair and maintain the machine, which was obligatory for vacuum tube machines, and also plenty of time for heuristic and other problems that we will discuss below.

In the world of Soviet theoretical physics of that time, a clear tendency was prevalent: the more talented a theoretical physicist is, the less computing is done for him. There was one physicist for whom nothing was ever computed, namely L.D. Landau. Less gifted physicists as a rule demanded a lot of computation, some of them expressing dismay when asked by mathematicians about the source of the equations dealt with, or the utility of the results. We should say here that Kronrod liked to quote Hamming: "Before starting a computation, decide what you will do with the results". The practice in the department was to check with the mathematician every physical problem formulation that demanded a large amount of computation. Sometimes it was discovered that a qualitative result that could be found without computation was sufficient, that the problem was over- or under-determined, that the computational errors invalidated the effect of interest, that the problem's formulation was not correct, etc. Kronrod even put a poster on his door: "Not to be bothered with integral equations of the first kind!" It did not mean at all that he thought integral equations of the first kind could not be solved. For example, the Mathematics Division of the ITEF computed shapes of magnetic poles for several large accelerators. This leads to a Cauchy problem for the Laplace equation, which, as is well known, can be reduced to an integral equation of the first kind. But that was a special case — it was really necessary to do some computing. Incidentally, the work was done by an excellent mathematician, A.M. Il'in.

Returning to A.S. Kronrod, it must be said that he perfectly understood that in some cases equations of the first kind must be solved by virtue of the nature of the problem. At the same time he believed that much more often one does not need the solution of the first-kind equation itself, but some mean value. For this mean value, as a rule, a simple and, importantly, a more correct problem can be formulated.

Two-and-a-half decades have passed. Generations of electronic computers have succeeded one another. Their speed has been increased by many orders of magnitude, and their memory has become practically unlimited. Along with this, the man/machine interface and the type of machine use have changed. For the most part, the machines are no longer used for computing, but for processing and storing information. Nevertheless, much of what was introduced into the practice by Kronrod is still relevant to this day. If a mathematician participates (in the role of computer and programmer) in solving a natural science problem, he must begin by understanding its physical, chemical, biological, economical, etc. formulation of the problem. Collaborating with the physicist, chemist, biologist, economist, he must together with them (or, if need be, instead of them) formulate the mathematical problem, create an algorithm, and write the program, never ignoring the fact that whether or not an algorithm for a serious problem is reasonable can only be discovered in the process of writing the program. At the same time, the mathematician must be provided with maximum assistance to free him from tasks that do not require his qualifications.

At the end of the fifties, Kronrod began to interest himself in questions of economics, in particular price formation. He observed that the basic principles of price formation were wrong. L.V. Kantorovich came to the same conclusion, as did other economists. A USSR Cabinet Ministry commission on the subject was formed, among which the mathematicians included Kantorovich and Kronrod. As a result of this committee's work, new price formation principles were adopted. Their implementation required computing the so-called "Leont'ev matrices" of material expenditure balances across the country. This colossal computational work was directed by Kronrod and carried out first on the RVM, and then on the same two M-20 machines. Later, the work was further developed by a pupil of A.S. Kronrod, the now well-known economist V.D. Belkin[4].

Another problem which interested Kronrod in the 1960s was the computerized differential diagnostics for some diseases. In the Cancer Institute named after Gertsen, a laboratory was created, which was headed by P.E. Kunin, a physicist by training and one of Kronrod's students in heuristic programming. The laboratory conducted research, in particular on the differential diagnostics of lung cancer and central pneumonia. (The results were considered crucial for deciding whether or not surgery is needed.) Kronrod supervised the research. Quite encouraging results were obtained. The sudden death of Kunin cut short this work.

During this time, Kronrod organized mathematics courses for high schools and developed teaching methods for them.

After signing a petition in 1968 in support of the prominent dissident and logician Alexander Esenin-Volpin, the son of the famous poet Esenin, Kronrod was summarily fired from his position at ITEF. He later became head of the mathematical laboratory of the Central Scientific Research Institute of Patent Information (CNIIPI). Setting up the mathematical and informational part (and for this, among other things, he needed to create software for Kronrod conditions for the machine "Razdan" located at the CNIIPI and to assemble a coherent group of mathematicians), Kronrod became interested in matters strictly related to patents and discovered that, here also, radical reforms were needed that would stimulate inventions.

Kronrod proposed a number of measures that would help improve the prevailing situation, and entered the high echelons, where he found understanding. The director of the CNIIPI, who was supportive of Kronrod, departed, and the new director wanted to free himself of such a worrisome colleague. A.S. Kronrod left the CNIIPI.

His last employment was at an institute called the Central Geophysical Expedition. Here Kronrod headed a laboratory processing exploration-drilling data. He implemented a series of new computational ideas, but this work, of course, did not match the level of his talent, and so he set new goals for himself.

It must be said that Kronrod's personality attracted many talented people from quite different fields. And while some of them were attracted by his professional competence (e.g., for the prominent oil researcher Lapuk he had to compute the optimal regime for exploiting oil and gas deposits), communicating with others involved quite different interests. You could meet at his home with the actor Evstigneev, the screenwriter Nusinov, and others. Kronrod could be seen with academician I.G. Petrovskiĭ at the Burdel sculpture exhibition, not discussing mathematical problems, but questions of fine art. Among his friends also were prominent physicians: the surgeon Simonyan, the pediatrician Pobedinskaya, the radiologist-oncologist Marmorshteĭn,

[4] During this period, A.S. Kronrod in 1964 also published his work on extended Gaussian quadrature rules, now known as Gauss-Kronrod formulae, for which he is best known among numerical analysts. – W.G.

and others.

Having a keen sense of philanthropy, with a strong desire to immediately help people, he was captivated by the professional stories of physicians, sharing their successes and failures. Gradually he understood that saving the terminally ill is the most important thing that can and must be done. At that time, he became acquainted with a Bulgarian doctor, Bogdanov, developer of a medicine called anabol, based on a Bulgarian sour milk extract. This medicine often caused remission in cancer patients. Incidentally, Bogdanov treated I.N. Vekua and S.A. Lebedev with anabol.

Kronrod started advertising this medicine. The medicine was not easy to obtain, as it was produced in Bulgaria in limited quantities. Kronrod organized the delivery of this medicine for terminally ill patients. But this was not the solution; anabol was rare and expensive. It had to be produced in large quantities and by a simple procedure. Thus, a new medicine appeared, which was sour-clotted milk, based on a Bulgarian milk extract. He gave this medicine the name milil (in honor of Mechnikov Il'ya Il'ich). He developed a simple technology for its production and ways of using the medicine.

Kronrod did not treat patients without a physician. Physicians used milil according to his instructions (there were more and more who came to believe in Kronrod's medicine). The medicine was used in hopeless cases for patients who were doomed to die. Milil became well known and accepted to some degree: A.A. Vishnevskiĭ set aside a ward at his institute to treat patients according to the method of A.S. Kronrod. Kronrod was promised a laboratory for animal experimentation, but this remained a promise, and he did all the experiments on himself.

No longer a novice in medicine, Kronrod replaced mathematics books by medical books, many of which he obtained from physicians he knew. He already had considerable clinical experience. He kept a large card file on the history of patients' diseases. And he had an important advantage over physicians: he could do a correct statistical inference from the thousands of cards in his file. The well-known therapist I.G. Barenblat (the father of the mechanical engineer G.I. Barenblat) was struck, after a conversation with Kronrod, by his medical erudition. And is it surprising? If a very talented person works hard in a medical field, and if he is helped by good specialists, he is likely to become proficient in it, at least as much as an average, or even a good student in a medical school. But he did not have a medical degree, and milil was not an approved medicine. In the medical field, this could not be tolerated. Recall, for example, the story of artificial pneumothorax. On the other hand, Kronrod did not treat patients without physicians, and was not paid for the treatment. In fact, he spent his fortune on the treatment. (At the end, he was so badly dressed that the laboratory assistants offered him a suit as a birthday gift.) In spite of this, a criminal case was opened against Kronrod and, more seriously, his card files were confiscated. The story had a tragicomical end. Either the mother or the wife of the prosecutor who brought the case had cancer. And he needed milil. Naturally, the case was dismissed, and the card file was returned. But for A.S. Kronrod himself, the story turned into a tragedy. He had a stroke, and he completely lost his speech and his abilities to read and write. Recovery was very slow, but he learned how to speak, read, and write once again. He left his position at the Central Geophysical Expedition. He quit working on mathematics. Now he was interested only in medicine. But at this point he suffered a second stroke. The situation was precarious. The physician believed that a final stage of agony had started. But Kronrod was conscious and asked to be put in a very hot tub and to remain there for several hours. One of the prominent neuropathologists remarked later that this was the only correct solution. This time, he survived. But he did not survive the third stroke.

He died on October 6, 1986.

Publications of A.S. Kronrod

1. A. Kronrod, *Sur la structure de l'ensemble des points de discontinuité d'une fonction dérivable en ses points de continuité* (Russian), Bull. Acad. Sci. URSS. Sér. Math. [Izvestia Akad. Nauk SSSR] **1939**, 569–578.

2. G.M. Adel'son-Vel'skiĭ and A.S. Kronrod, *On a direct proof of the analyticity of a monogenic function* (Russian), Doklady Akad. Nauk SSSR (N.S.) **50** (1945), 7–9.

3. G.M. Adel'son-Vel'skiĭ and A.S. Kronrod, *On the level set of continuous functions possessing partial derivatives* (Russian), Doklady Akad. Nauk SSSR (N.S.) **50** (1945), 239–241.

4. G.M. Adel'son-Vel'skiĭ and A.S. Kronrod, *On the maximum principle for an elliptic system* (Russian), Doklady Akad. Nauk SSSR (N.S.) **50** (1945), 559–561.

5. A. Kronrod, *On permutation of terms of numerical series* (Russian), Rec. Math. [Mat. Sbornik] N.S. **18 (60)** (1946), 237–280.

6. A.S. Kronrod and E.M. Landis, *On level sets of a function of several variables* (Russian), Doklady Akad. Nauk SSSR (N.S) **58** (1947), 1269–1272.

7. A.S. Kronrod, *On linear and planar variations of functions of several variables* (Russian), Doklady Akad. Nauk SSSR (N.S.) **66** (1949), 797–800.

8. A.S. Kronrod, *On a line integral* (Russian), Doklady Akad. Nauk SSSR (N.S.) **66** (1949), 1041–1044.

9. A.S. Kronrod, *On surfaces of bounded area* (Russian), Uspehi Matem. Nauk (N.S.) **4** (1949), no. 5 (33), 181–182.

10. A.S. Kronrod, *On functions of two variables*, Uspehi Matem. Nauk (N.S.) **5** (1950), no. 1 (35), 24–134.

11. A.S. Kronrod, *Numerical solution to the equation of the magnetic field in iron with allowance saturation*, Soviet Physics Dokl. **5** (1960), 513–514.

12. A.S. Kronrod, *Integration with control of accuracy*, Soviet Physics Dokl. **9** (1964), 17–19.

13. Aleksandr Semenovich Kronrod, *Nodes and weights of quadrature formulas. Sixteen-place tables.* Authorized translation from the Russian, Consultants Bureau, New York, 1965.

14. V.D. Belkin, A.S. Kronrod, U.A. Nazarov, and V.Y. Pan, *The rational price calculation based on contemporary economic information*, Akad. Nauk SSSR, Ekonomika i Matematičeskie Metodi (**1965**) 1, no. 5, 699–717.

15. V.L. Arlazarov, A.S. Kronrod, and V.A. Kronrod, *On a new type of computers*, Dokl. Akad. Nauk SSSR (1966) **171**, no. 2, 299–301.

16. A.S. Kronrod, V.A. Kronrod, and I.A. Faradžev, *The choice of the step in the computation of derivatives* (Russian), Dokl. Akad. Nauk SSSR **194** (1970), 767–769. English translation in: Reports of the Academy of Sciences of the USSR **194**, New York, 1970.

17. O.N. Golovin, G.M. Žislin, A.S. Kronrod, E.M. Landis, L.A. Ljusternik, and G.E. Šilov, *Aleksandr Grigor'evič Sigalov. Obituary* (Russian), Uspehi Mat. Nauk **25** (1970), no. 5 (155), 227–234.

18. A.S. Kronrod, *The selection of the minimal confidence region* (Russian), Dokl. Akad.Nauk SSSR **205** (1972), 1036.

19. A.S. Kronrod, *A nonmajorizable prescription for the choice of a confidence region for a given level of reliability* (Russian), Dokl. Akad. Nauk SSSR **208** (1973), 1026.

20. A.S. Kronrod, *A nonmajorizable prescription for the selection of a confidence region of a certain form of target function* (Russian), Dokl. Akad. Nauk SSSR **210** (1973), 18–19.