Leonhard Euler: 300 years old

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Purdue University

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The three stations of Euler's life

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Basel 1707-1727

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Auspicious beginnings

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- left Basel (for good) in April of 1727 to assume a junior appointment at the Academy of St. Petersburg

St. Petersburg 1727–1741

Meteoric rise to world fame and academic advancement

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- 1741: departure from St. Petersburg following political unrest after the death (1740) of the Empress Anna Ivanovna (a niece of Peter I); accepted an invitation of Frederick II to help set up an Academy in Berlin.

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- kinematics and dynamics of a mass point
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Naval science (1749, written 1740–41)

- principles of hydrostatics
- stability theory
- naval engineering and navigation (vol. II)

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Later in 1750, he was able to prove (rigorously)

$$\zeta(2n) = \frac{2^{2n-1}}{(2n)!}\,|B_{2n}|\pi^{2n}$$

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$$\left(\prod_{p \in \mathcal{P}} \frac{p^s - 1}{p^s}\right) \zeta(s) = 1 \qquad \square$$

Berlin 1741–1766

The emergence of epochal treatises

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Artillery (1745)

- vastly expanded and annotated German translation of Robins's *New principles of gunnery* (1742)

Introduction to the analysis of the infinite (1748)

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Letters to a German princess (written 1760–1762)

- Euler's philosophical views on science, religion, and ethics

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Königsberg bridge graph: $n = 4$

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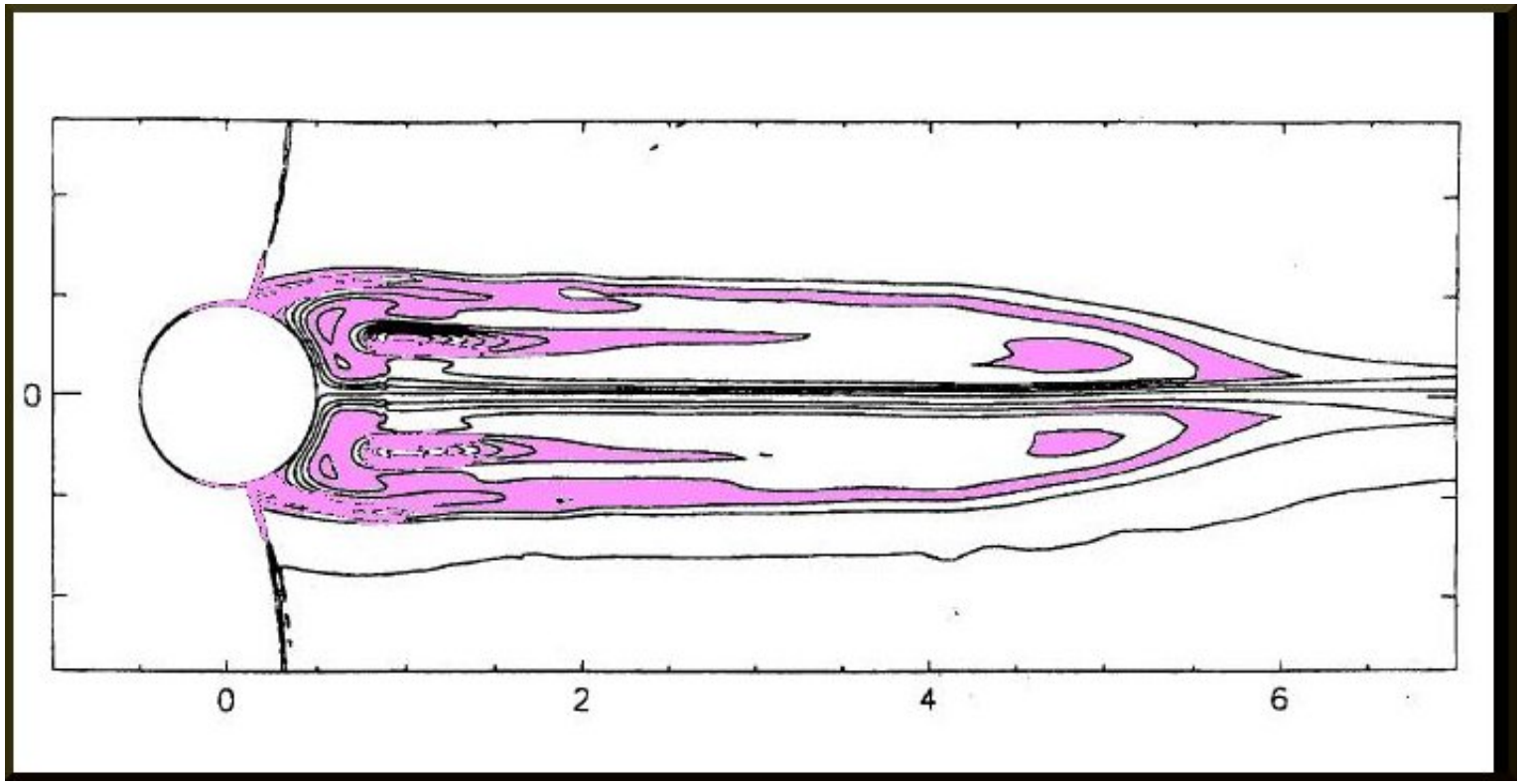

Transonic Euler flow at Mach .85 about a cylinder

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St. Petersburg 1766–1783

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- 1776: Euler remarries
- 1783: On September 18, Euler dies of a stroke

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Second theory of ships (1773)

- construction and maneuvering of ships
- written for people (e.g., sailors) with no, or little, mathematical knowledge

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Selectio 6 Euler's disk

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Selection 7 Gear transmission; Euler's tooth profile

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The Man

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Craftsmanship

- superb expositor
- his goal: ultimate clarity and simplicity
- yet fearless and aggressive in his quest for discovery

Epilogue

LEONHARD EULER
1707–1783
mathematician, physicist, engineer, astronomer and philosopher, spent his youth in Riehen. He was a great scholar and a kind man.