Leonhard Euler
His Life, The Man, and His Work

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Purdue University

April 30, 2007

Outline

Basel 1707–1727

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St. Petersburg 1727–1741

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The Man

Basel 1707–1727

Auspicious beginnings

Parish residence and church in Riehen

The old university of Basel and Johann Bernoulli

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- 1724: masters degree; first public lecture (in Latin); advanced mathematics course from Johann Bernoulli; "privatissima"

From Euler's autobiography (1767)

"… In 1720 I was admitted to the university as a public student, where I soon found the opportunity to become acquainted with the famous professor Johann Bernoulli, who made it a special pleasure for himself to help me along in the mathematical sciences. Private lessons, however, he categorically ruled out because of his busy schedule. However, he gave me a far more beneficial advice, which consisted in myself taking a look at some of the more difficult mathematical books and work through them with great diligence, and should I encounter some objections or difficulties, he offered me free access to him every Saturday afternoon, and he was gracious enough to comment on the collected difficulties, which was done with such a desired advantage that, when he resolved one of my objections, ten others at once disappeared, which certainly is the best method of making auspicious progress in the mathematical sciences."

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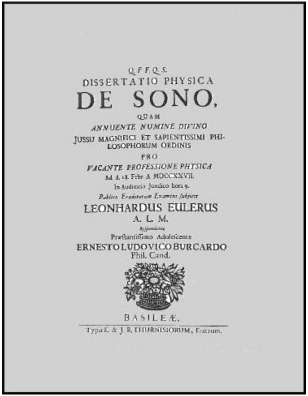
- left Basel (for good) in April of 1727 to assume a junior appointment at the Academy of St. Petersburg



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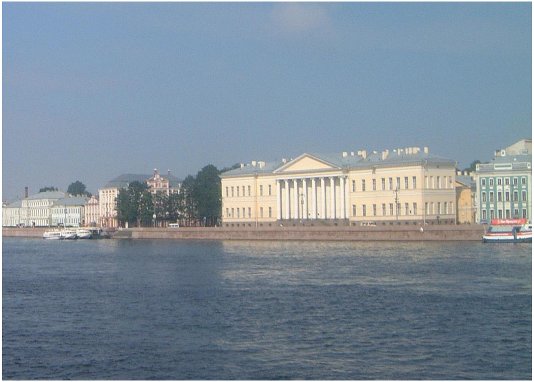
DISSERTATIO PHYSICA
DE SONO,
QUAM
ANNUENTE NUMINE DIVINO
JUSSU MAGNIFICI ET SAPIENTISSIMI PHILOSOPHORUM ORDINIS
PRO
VACANTE PROFESSIONE PHYSICA
Ad d. 18. Febr. A. MDCCXXVII.
In Auditorio Juridico hora 9.
Publico Eruditorum Examini subjicit
LEONHARDUS EULERUS
A. L. M.
Respondente
Præstantissimo Adolescente
ERNESTO LUDOVICO BURCARDO
Phil. Cand.

BASILEÆ,
Typis E. & J. R. THURNISIORUM, Fratrum,

Physical dissertation on sound

St. Petersburg 1727–1741

Meteoric rise to world fame and academic advancement



The Academy in St. Petersburg and Peter I

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- 1741: departure from St. Petersburg following political unrest after the death (1740) of the Empress Anna Ivanovna (a niece of Peter I)

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- kinematics and dynamics of a mass point
- in free motion (vol. I)
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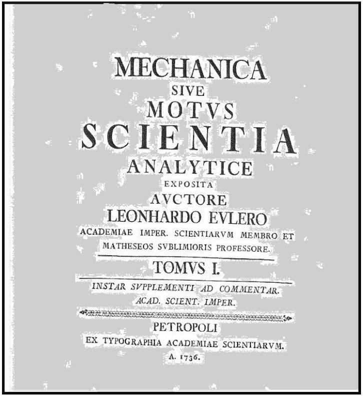
Naval science (1749, written 1740–41)

- principles of hydrostatics
- stability theory
- naval engineering and navigation (vol. II)


MECHANICA
SIVE
MOTVS
SCIENTIA
ANALYTICE
EXPOSITA
AVCTORE
LEONHARDO EVLERO
ACADEMIAE IMPER. SCIENTIARVM MEMBRO ET
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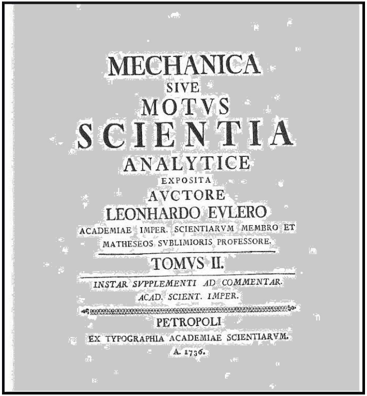
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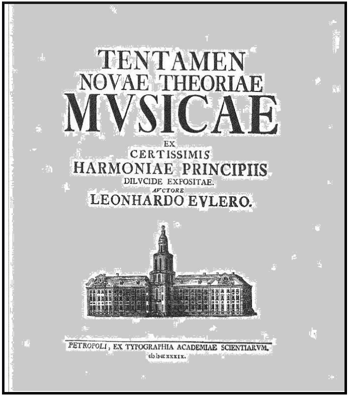
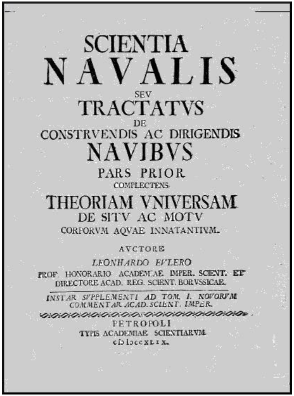
Mechanics I,II


TENTAMEN
NOVAE THEORIAE
MVSICAE
EX
CERTISSIMIS
HARMONIAE PRINCIPIIS
DILVCIDE EXPOSITAE.
AVCTORE
LEONHARDO EVLERO.
PETROPOLI, EX TYPOGRAPHIA ACADEMIAE SCIENTIARVM.
clↄlↄccxxxix.

Tentamen


SCIENTIA
NAVALIS
SEV
TRACTATVS
DE
CONSTRVENDIS AC DIRIGENDIS
NAVIBVS
PARS PRIOR
COMPLECTENS
THEORIAM VNIVERSAM
DE SITV AC MOTV
CORPORVM AQVAE INNATANTIVM.
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TYPIS ACADEMIAE SCIENTIARVM
CIƆIƆCCXLIX.

Naval science I,II


PROSPECT DER RHEINBRÜCKE ZU BASEL
VON SEITEN DER KLEINEN STADT.
Em. Büchel del. 1761.
VUE DU PONT DU RHIN DE BASLE
DU CÔTÉ DE LA PETITE VILLE.
D. Herrliberger exc. Cum Priv.

Basel 1761

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Later in 1750, he was able to prove

$$\zeta(2n) = \frac{2^{2n-1}}{(2n)!}\,|B_{2n}|\pi^{2n}$$

Euler ca. 1737

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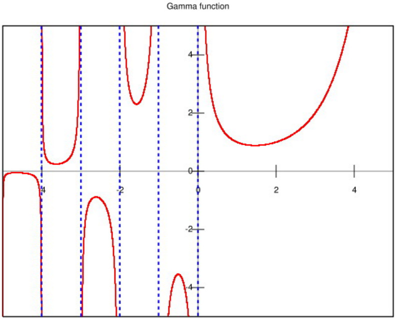
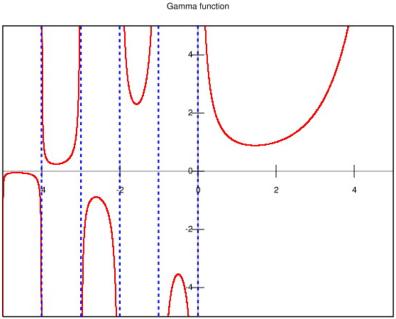
$$\left(\prod_{p \in \mathcal{P}} \frac{p^s - 1}{p^s} \right) \zeta(s) = 1 \qquad \square$$


Gamma function
4
2
0
-2
-4
-4
-2
0
2
4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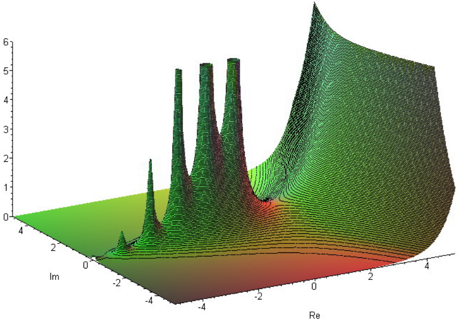
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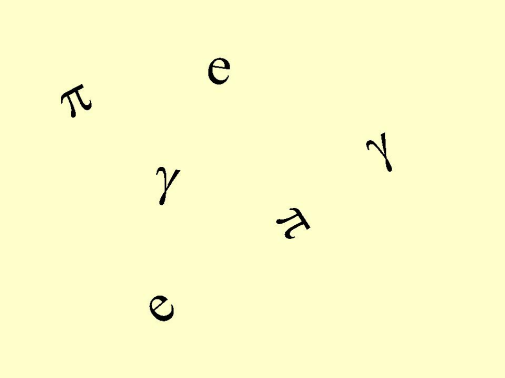
There holds $\Gamma(x+1) = x\Gamma(x)$, $\Gamma(1) = 1$, so that $\Gamma(n+1) = n!$.


e
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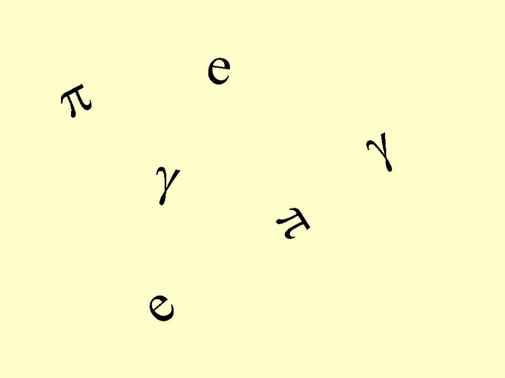

Auguste de Morgan (1806–1871): “[π] comes on many occasions through the window and through the door, sometimes even down the chimney.”

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From Euler's autobiography (1767)

" . . . in 1740, when His still gloriously reigning Royal Majesty came to power in Prussia, I received a most gracious call to Berlin, which, after the illustrious Empress Anne had died and it began to look rather dismal in the regency that followed, I accepted without much hesitation . . . "

Berlin 1741–1766

The emergence of epochal treatises

The Berlin Academy and Frederick II

Chronology

Chronology

- 1746: Berlin Academy opens its doors, with Maupertuis its president and Euler the director of the Mathematics Class; elected foreign member of the Royal Society of London

Chronology

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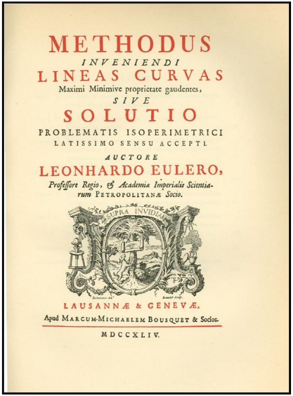
Artillery (1745)

- vastly expanded and annotated German translation of Robins's *New principles of gunnery* (1742)

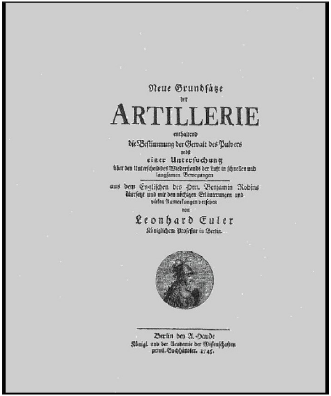


METHODUS
INVENIENDI
LINEAS CURVAS
Maximi Minimive proprietate gaudentes,
SIVE
SOLUTIO
PROBLEMATIS ISOPERIMETRICI
LATISSIMO SENSU ACCEPTI.
AUCTORE
LEONHARDO EULERO,
Professore Regio, & Academiæ Imperialis Scientiarum PETROPOLITANÆ *Socio.*

LAUSANNÆ & GENEVÆ,
Apud MARCUM-MICHAELEM BOUSQUET & Socios.
MDCCXLIV.

Calculus of variations

Euler 1753


Neue Grundsätze
der
ARTILLERIE
enthaltend
die Bestimmung der Gewalt des Pulvers
nebst
einer Untersuchung
über den Unterscheid des Wiederstands der Luft in schnellen und
langsamen Bewegungen

aus dem Englischen des Hrn. Benjamin Robins
übersetzt und mit den nöthigen Erläuterungen und
vielen Anmerkungen versehen
von
Leonhard Euler
Königlichem Professor in Berlin.

Berlin bey A. Haude
Königl. und der Academie der Wissenschaften
privil. Buchhändler. 1745.

Artillery

Introduction to the analysis of the infinite (1748)

- elementary functions
- infinite series and products; continued fractions
- analytic geometry: algebraic curves and surfaces

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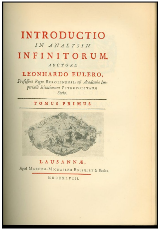
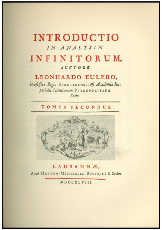
Letters to a German princess (written 1760–1762)

- Euler's philosophical views on science, religion, and ethics


INTRODUCTIO
IN ANALYSIN
INFINITORUM.
AUCTORE
LEONHARDO EULERO,
Professore Regio Berolinensi, & Academiae Imperialis Scientiarum Petropolitanae Socio.
TOMUS PRIMUS.
LAUSANNÆ,
Apud Marcum-Michaelem Bousquet & Socios.
MDCCXLVIII.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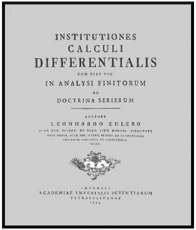
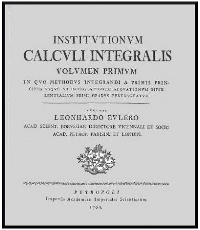
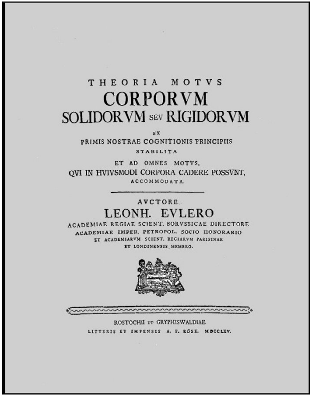
TOMUS SECUNDUS.
LAUSANNÆ,
Apud Marcum-Michaelem Bousquet & Socios.
MDCCXLVIII.

Infinitesimal analysis I,II


INSTITUTIONES
CALCULI
DIFFERENTIALIS
CUM EIUS USU
IN ANALYSI FINITORUM
AC
DOCTRINA SERIERUM
AUCTORE
LEONHARDO EULERO
IMPENSIS
ACADEMIAE IMPERIALIS SCIENTIARUM
PETROPOLITANAE
1755.


INSTITVTIONVM
CALCVLI INTEGRALIS
VOLVMEN PRIMVM
IN QVO METHODVS INTEGRANDI A PRIMIS PRINCIPIIS VSQVE AD INTEGRATIONEM AEQVATIONVM DIFFERENTIALIVM PRIMI GRADVS PERTRACTATVR.
AVCTORE
LEONHARDO EVLERO
ACAD. SCIENT. BORVSSIAE DIRECTORE VICENNALI ET SOCIO
ACAD. PETROP. PARISIN. ET LONDIN.
PETROPOLI
Impensis Academiae Imperialis Scientiarum
1768.

Differential and integral calculus



THEORIA MOTVS
CORPORVM
SOLIDORVM SEV RIGIDORVM
EX
PRIMIS NOSTRAE COGNITIONIS PRINCIPIIS
STABILITA
ET AD OMNES MOTVS,
QVI IN HVIVSMODI CORPORA CADERE POSSVNT,
ACCOMMODATA.

AVCTORE
LEONH. EVLERO
ACADEMIAE REGIAE SCIENT. BORVSSICAE DIRECTORE
ACADEMIAE IMPER. PETROPOL. SOCIO HONORARIO
ET ACADEMIARVM SCIENT. REGIARVM PARISINAE
ET LONDINENSIS MEMBRO.

ROSTOCHII ET GRYPHISWALDIAE
LITTERIS ET IMPENSIS A. F. RÖSE. MDCCLXV.

DIOPTRICAE
PARS PRIMA
CONTINENS
LIBRVM PRIMVM,
DE
EXPLICATIONE
PRINCIPIORVM,
EX QVIBVS
CONSTRVCTIO TAM TELESCOPIORVM
QVAM
MICROSCOPIORVM
EST PETENDA.

AVCTORE
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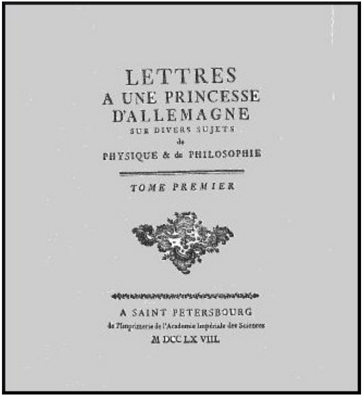
PETROPOLI
Impensis Academiae Imperialis Scientiarum
1769.

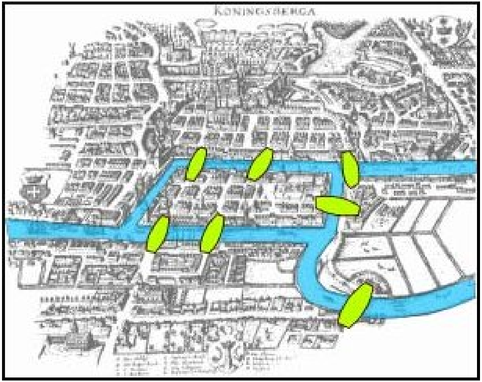
Second mechanics and Optics

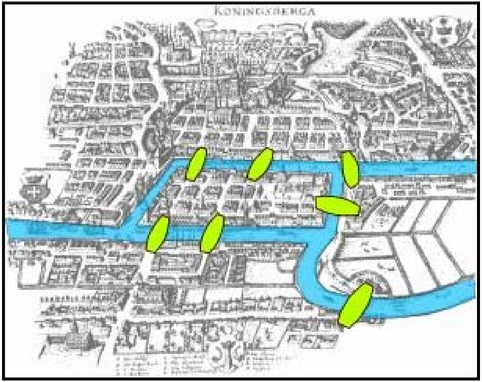

LETTRES
A UNE PRINCESSE
D'ALLEMAGNE
SUR DIVERS SUJETS
de
PHYSIQUE & de PHILOSOPHIE

TOME PREMIER

A SAINT PETERSBOURG
de l'Imprimerie de l'Academie Impériale des Sciences
M DCC LX VIII.

Letters


KONINGSBERGA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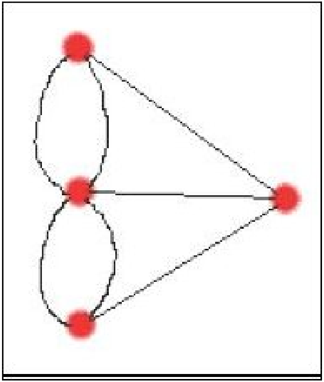

Selecta Euleriana

Selecta Euleriana

Selectio 5 The Königsberg bridge problem (1741)

Selecta Euleriana

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connected graph

- path
- circuit
- Eulerian path or circuit
- degree of vertex

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connected graph

• path • circuit • Eulerian path or circuit • degree of vertex

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(a) If $n = 0$, the graph has at least one Eulerian circuit;

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Königsberg bridge graph: $n = 4$

Selectio 6 Euler's buckling formula (1744)

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Selectio 7 Euler flow (1757)

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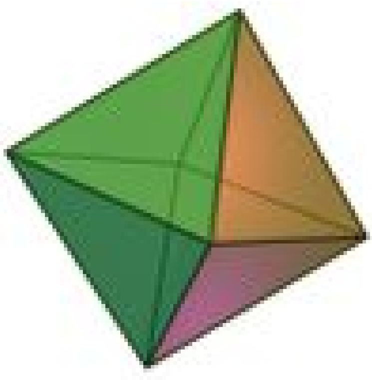
Octahedron



$$V = 6 \qquad E = 12 \qquad F = 8$$

Selectio 8 Euler's polyhedral formula (1753)

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In a three-dimensional convex polyhedron let

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F &= \text{ number of faces}
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$$

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Theorem (Euler)

$$V - E + F = 2$$

86

CONSIDERATIO
QVARVMDAM SERIERVM, QVAE SINGVLARIBVS PROPRIETATIBVS SVNT PRAEDITAE.

AVCTORE
L. EVLERO.

§. 1.

Saepe numero *conſideratio ſerierum*, quae quaſi caſu ſe nobis offerunt, non contemnenda ſuppeditare ſolet artificia, quibus deinceps in vniuerſa ſerierum doctrina ſummo cum fructu vti licet. Cum igitur doctrina *de ſeriebus* ſit maximi momenti in Analyſi, huiusmodi ſpeculationes omnino dignae ſunt habendae, quae omni induſtria euoluantur. Hunc in finem ſequentem ſeriem offerre conſtitui, quae, tum ob ſingulares, quibus praedita deprehenditur proprietate, tum vero propter inſignes vſus, quos nobis exhibet, omni attentione digna videtur. Series autem ita ſe habet:

$$\frac{1-x}{1-a}+\frac{(1-x)(a-x)}{a-a^3}+\frac{(1-x)(a-x)(a^2-x)}{a^3-a^6}+\frac{(1-x)(a-x)(a^2-x)(a^3-x)}{a^6-a^{10}}+\text{etc.}$$

Lex numeratorum ex ſola inſpectione eſt manifeſta, formantur enim ex multiplicatione terminorum huius ſeriei: $1-x$; $a-x$; a^2-x; a^3-x; a^4-x; a^5-x; a^6-x; etc. Denominatores omnes duobus conſtant terminis, qui ſunt poteſtates ipſius a, quarum exponentes ſunt numeri trigonales. Hinc terminus ordine n ſeriei propoſitae erit:

$$\frac{(1-x)(a-x)(a^2-x)(a^3-x)\;.\;.\;.\;.\;.\;(a^{n-1}-x)}{a^{n(n-1):2}-a^{n(n+1):2}}$$

§. 2.

Selectio 9 Euler and q-theory

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the story begins 1734 with a letter of Euler to Daniel Bernoulli, in which he wants to interpolate the logarithm from the data

x	1	10	100	1000	$\cdots$
$\log x$	0	1	2	3	$\cdots$

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today we know that $S(x; a)$ is a q-analogue of the logarithm where $q = 1/a$

St. Petersburg 1766–1783

The glorious final stretch

The Euler house and Catherine II

Euler 1778

Chronology

Chronology

- 1771: Euler loses his (good) left eye following a cataract operation and becomes virtually blind; Euler's wooden house burns down during the great St. Petersburg fire

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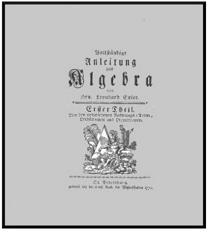
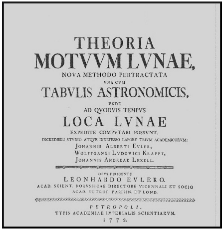
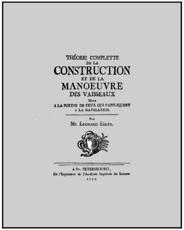
- 1773: Euler's wife Katharina dies
- 1776: Euler remarries
- 1783: On September 18, Euler dies of a stroke


Vollständige
Anleitung
zur
Algebra
von
Hrn. Leonhard Euler.
Erster Theil.
Von den verschiedenen Rechnungs-Arten, Verhältnissen und Proportionen.
St. Petersburg,
gedruckt bey der Kays. Acad. der Wissenschaften 1770.


THEORIA
MOTVVM LVNAE,
NOVA METHODO PERTRACTATA
VNA CVM
TABVLIS ASTRONOMICIS,
VNDE
AD QVODVIS TEMPVS
LOCA LVNAE
EXPEDITE COMPVTARI POSSVNT,
INCREDIBILI STVDIO ATQVE INDEFESSO LABORE TRIVM ACADEMICORVM:
JOHANNIS ALBERTI EVLER,
WOLFGANGI LVDOVICI KRAFFT,
JOHANNIS ANDREAE LEXELL.
OPVS DIRIGENTE
LEONHARDO EVLERO.
ACAD. SCIENT. BORVSSICAE DIRECTORE VICENNALI ET SOCIO
ACAD. PETROP. PARISIN. ET LOND.
PETROPOLI,
TYPIS ACADEMIAE IMPERIALIS SCIENTIARVM.
1772.

Algebra and Second lunar theory


THÉORIE COMPLETTE
DE LA
CONSTRUCTION
ET DE LA
MANOEUVRE
DES VAISSEAUX
MISE
À LA PORTÉE DE CEUX QUI S'APPLIQUENT
À LA NAVIGATION.
PAR
MR. LEONARD EULER.
À ST. PETERSBOURG,
De l'Imprimerie de l'Académie Impériale des Sciences.
1773.

Second theory of ships

Major works

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Algebra (1770)

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Second theory of ships (1773)

- construction and maneuvering of ships
- written for people (e.g., sailors) with no, or little, mathematical knowledge

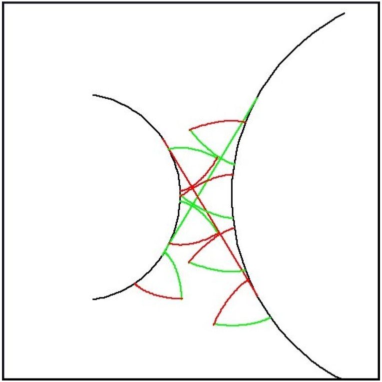
Selecta Euleriana

Selecta Euleriana

Selectio 10 Euler's disk

Selecta Euleriana

Selectio 10 Euler's disk



Selection 11 Gear transmission; Euler's tooth profile

Selection 11 Gear transmission; Euler's tooth profile

The Man

Personality

- modest, inconspicuous, uncomplicated, yet cheerful and sociable
- "honesty, uncompromising rectitude—the acknowledged national virtues of Swiss people—he possessed to a superior degree" (Fuchs)
- free of priority concerns
- generous in acknowledging and furthering other people's work

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Intellect

- phenomenal memory, erudite
- unusual power of mental calculation
- ability to concentrate on mental work under adversity

Personality

- modest, inconspicuous, uncomplicated, yet cheerful and sociable
- "honesty, uncompromising rectitude—the acknowledged national virtues of Swiss people—he possessed to a superior degree" (Fuchs)
- free of priority concerns
- generous in acknowledging and furthering other people's work

Intellect

- phenomenal memory, erudite
- unusual power of mental calculation
- ability to concentrate on mental work under adversity

Craftsmanship

- superb expositor
- his goal: ultimate clarity and simplicity
- yet fearless and agressive in his quest for discovery

Euler memorial plaque in Riehen

Epilogue

Epilogue

LEONHARD EULER
1707–1783
mathematician, physicist, engineer, astronomer and philosopher, spent his youth in Riehen. He was a great scholar and a kind man.