Topological space(closed sets, closures)

Sunday, January 3, 2021 4:16 PM

Def (Closed set, closures): • Q is closed if $\mathbb{T} \setminus Q$ is open.

• $Cl\, Q$ is the smallest closed set containing $Q \subseteq \mathbb{T}$.

In Example 1 $\{3,5,7\}$ is closed because complement $\{0,1\}$ is open

$Cl\{0\} = \{0,3,7\}$

Example 2: All sets in the power set are both open and closed

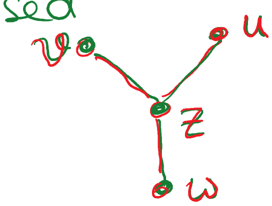
$\mathbb{T} = \{u, v, w\}$, $2^{\mathbb{T}} = \{\dots\dots\}$, take $\{v,w\}$, $\mathbb{T} \setminus \{v,w\} = \{u\}$

open, closed

Example 3: $\{u, z, (u,z)\}$ is closed because $\{w, v, (w,z), (v,z)\}$ is open

$\{u, (u,z)\}$ is neither open nor closed

$CL(\{u, (u,z)\}) = \{u, z, (u,z)\}$



Topological space[Interior, connected, subspace)

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Def. Interior $\text{Int } A$ is the union $\bigcup_i u_i$ where $u_i \subseteq A$ is open

$\text{bnd } A = A \setminus \text{Int } A$

Example 1: $\text{Int}\{3,5,7\} = \{5\}$, $\text{bnd}\{3,5,7\} = \{3,7\}$

Def. (Connected): $(\mathbb{T}, T)$ is disconnected if $\exists\ \underbrace{U, V}_{\text{disjoint}} \in T$ s.t. $\mathbb{T} = U \,\dot\cup\, V$

$(\mathbb{T}, T)$ is connected if it is not disconnected

Example 1 is connected, subspace $\{0,1,5\}$ is disconnected

Example 3 is connected, subspace $\{(u,z),(v,z),(w,z)\}$ disconnected

Def. (Subspace topology): $\mathbb{U} \subseteq \mathbb{T}$, topology induced by $U = \{P \cap \mathbb{U} : P \in T\}$

Example 1: $\mathbb{U} = \{0,1,5\}$ $U = \{\{\emptyset\},\{1\},\{5\},\{1,5\},\{0,1\},\{0,1,5\}\}$

Topological space (Cover, compactness)

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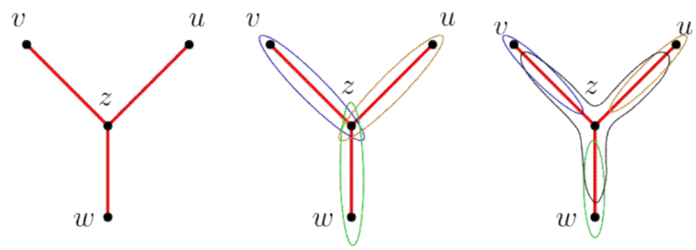
Def. (Cover, compactness): Open (closed) cover C for $(\mathbb{T}, T)$ is a collection of open (closed) sets s.t. $\mathbb{T} = \bigcup_{c \in C} c$.

- $\mathbb{T}$ is compact if every open C has $C' \subseteq C$ where $\mathbb{T} = \bigcup_{c \in C'} c$ and C' finite



(a) (b) (c)

Fig (b): a closed cover: $C = \{\{v, z, (v,z)\}, \{u, z, (u,z)\}, \{w, z, (w,z)\}\}$

Fig (c): an open cover: $C = \{\{u, (u,z)\}, \{v, (v,z)\}, \{w, (w,z)\}, \{z, (u,z), (v,z), (w,z)\}\}$