Introduction

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- Topological spaces that have the same topology, but different geometry.

- topology encodes connectivity in spaces
- It captures global features in spaces
- So, it can be used to compare data wrt. global features
- Because of global 'features', topology more robust tools against small perturbations due to noise.

Example of Clusters

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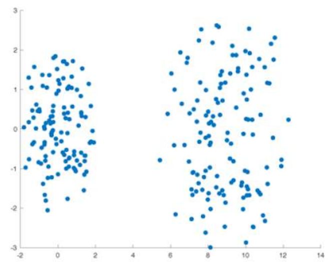
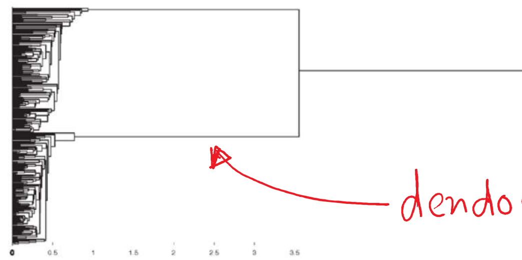
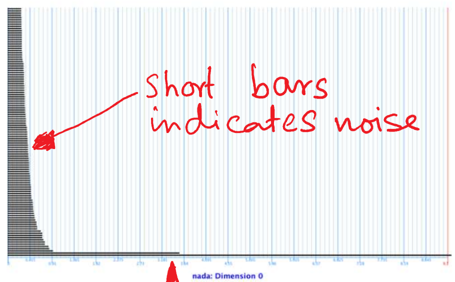
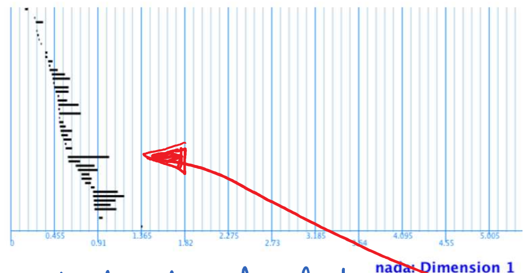




dendogram



Short bars indicates noise



- Two clusters in a point cloud data
  Two long bars in persistence barcode for 0-dim. homology
- No long bar in 1-dim homology barcode

Example: cyclic structures

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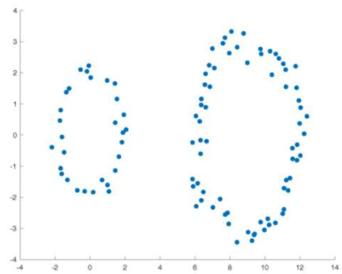

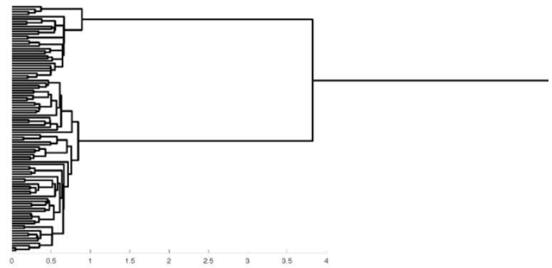

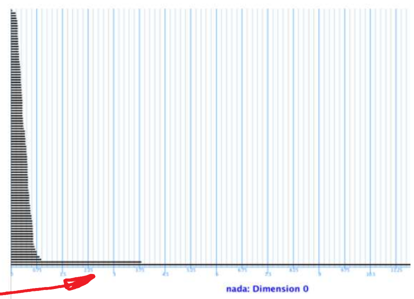

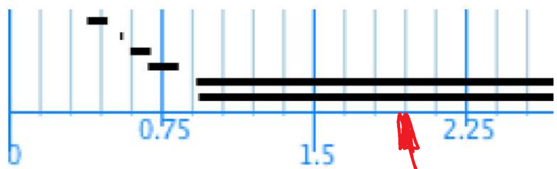

- Two clusters, each one has cyclic structures
- Two long bars in 0-dim.
- Two long bars in 1-dim.

stability against noise

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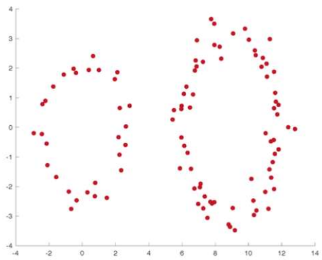

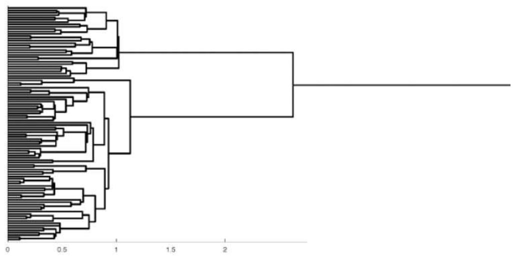

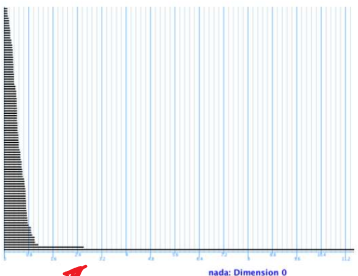

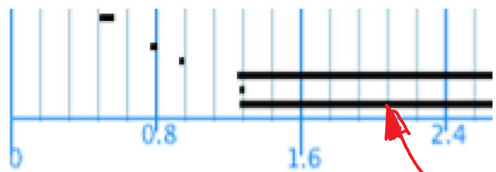

Stable

- Two long bars in 1-dim.

- Two clusters with cyclic structures, but with perturbed points.
- Two long bars in 0-dim.

Noise perturbed the bars slightly, but not lot

Topological space(Definition)

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Def : A point set $\mathbb{T}$ is a topo space endowed with a system of subsets $T \subseteq 2^{\mathbb{T}}$ (T is topology on $\mathbb{T}$)

- $\mathbb{T}, \phi \in T$
- Every $U \subseteq T$, union of sets in U is in T
- Every finite $U \subseteq T$, common intersection $\bigcap U \in T$.

- Every $U \in T$ is an open set
- An open U is a neighborhood of $p \in \mathbb{T}$ if $p \in U$

Example1: $\mathbb{T} = \{0, 1, 3, 5, 7\}$, $T = \{\{\phi\}, \{1\}, \{5\}, \{0,1\}, \{1,5\}, \{0,1,5\}, \{0,1,3,5,7\}\}$

$T = \{\{\phi\}, \{1\}, \{1,5\}, \{0,1,5\}, \{0,1,3,5,7\}\}$ is also a topology

$T = \{\{\phi\}, \{0\}, \{1\}, \{1,5\}, \{0,1,5\}\}$ is not a topology

$\{0\} \cup \{1\} = \{0,1\}$ is not in T

Topological space(Examples)

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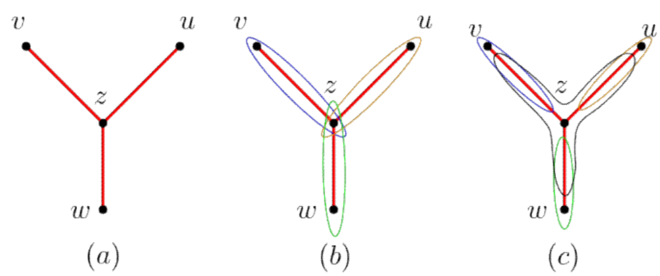
Example 2: $\Pi = \{u, v\}$, $T = 2^{\Pi} = \{\{\emptyset\}, \{u\}, \{v\}, \{u, v\}\}$

- For any set Π, 2^{Π} is a topology called discrete topology

Example 3: $\Pi = \{u, v, w, z, (u,z), (v,z), (w,z)\}$: a graph



$T_1 = \{\{(u,z)\}, \{(v,z)\}, \{(w,z)\}\}$

$T_2 = \{\{u, (u,z)\}, \{v, (v,z)\}, \{w, (w,z)\}, \{z, (u,z), (v,z), (w,z)\}\}$

$2^{T_1 \cup T_2}$ is a topology (Star topology, Alexandrov topology)

(open stars in Fig (c))

(closed stars in Fig (b))