QuickSort

1. Divide: Split $A[p \ldots r]$ into $A[p \ldots q]$ and $A[q+1 \ldots r]$
   Recurse: Sort $A[p \ldots q]$ and $A[q+1 \ldots r]$.
   Combine: Automatic

```
Procedure QuickSort(p,r)
  if p<r then  q := Partition(p,r);
               Quicksort(p,q);
               Quicksort(q+1,r);
  endif
```

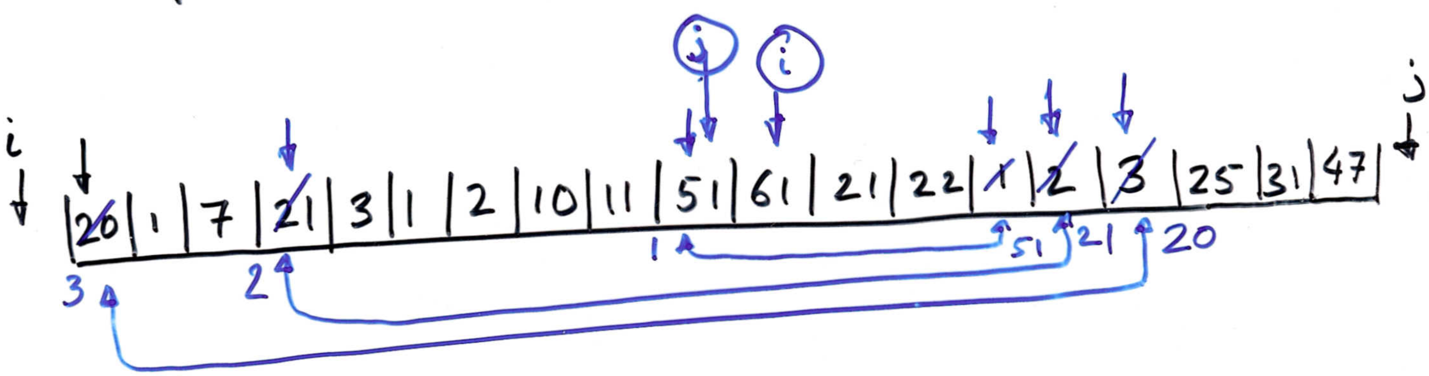
2. The Partition

Objective: split A into a left and right part so that all numbers in the left part are $\leq$ to all numbers in the right part.

Ex. | 20 | 1 | 7 | 21 | 3 | 1 | 2 | 10 | 11 | 51 | 61 | 21 | 22 | 1 | 2 | 3 | 25 | 31 | 47 |

function Partition (p, r)

```
    x := A[p]; i := p-1; j := r+1;
    loop repeat j := j-1 until A[j] ≤ x;
         repeat i := i+1 until A[i] ≥ x;
         if i < j then A[i] ⟷ A[j]
                  else Partition := j; exit
         endif
    forever
```

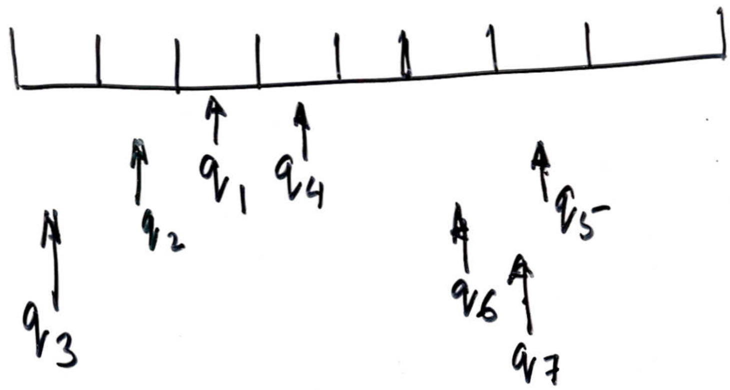


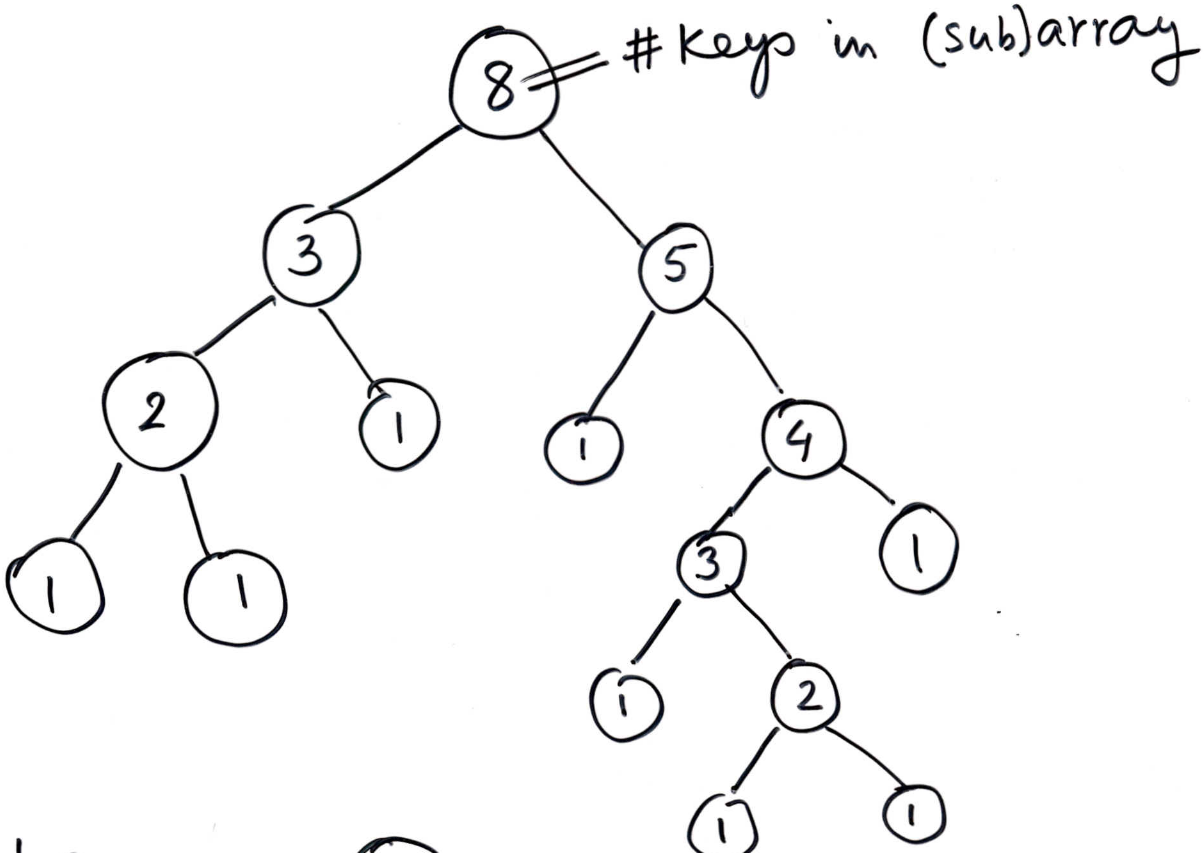
- If $A[p]$ is smallest then Partition $= p$; recursion continues with arrays of lengths 1 and $r-p$

- If $A[p]$ is K-smallest, for $K \geq 2$, then partition $= K-1$, so we recurse with lengths $K-1$ and $(r-p+1)-(K-1) = r-p-K+2$

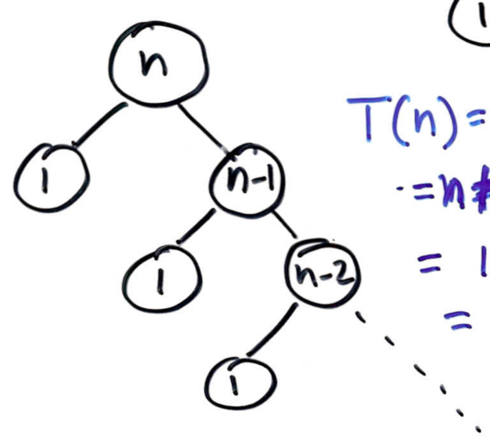
- Time complexity $= O(\quad)$

3. Worst and Best Case



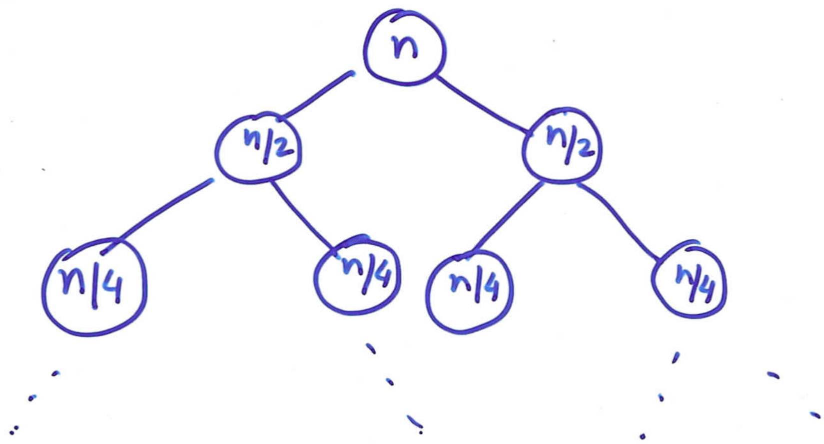


Worst case



$$T(n) = T(n-1) + T(1) + n$$
$$= \cancel{n} (n-1) + (n-2) \cdots + 2 + n$$
$$= 1 + 2 + \cdots + (n) + (n-1)$$
$$= \binom{n+1}{2} + (n-1) = \Theta(n^2)$$

Best Case.



$$T(n) = 2T(n/2) + n \quad , \quad T(1) = 1$$
$$= \Theta(n \log n)$$

Randomized QuickSort.

function Random-Partition (p, r)
    $i :=$ Random (p, r) ; $A[p] \longleftrightarrow A[i]$;
    Random-Partition $:=$ Partition (p, r).

Average Analysis

$$T(n) = \frac{1}{n}\left(\underbrace{T(1) + T(n-1)}_{= O(n^2)} + \sum_{q=1}^{n-1}\left(T(q) + T(n-q)\right)\right) + \Theta(n)$$

$$= \frac{1}{n}\sum_{q=1}^{n-1}\left(T(q) + T(n-q)\right) + \Theta(n) = \frac{2}{n}\sum_{k=1}^{n-1} T(k) + \Theta(n)$$

TKD, QS5

Assume inductively $T(n) \le an\log n + b$

$$T(n) \le \frac{2}{n} \sum_{k=1}^{n-1} (ak\log k + b) + \Theta(n)$$

$$= \frac{2a}{n} \sum_{k=1}^{n-1} k\log k + \frac{2b(n-1)}{n} + \Theta(n)$$

$$\le an\lg n - \frac{an}{4} + 2b + \Theta(n)$$

$$= an\log n + b + \left(\Theta(n) + b - \frac{an}{4}\right) \le an\log n + b$$

$$\left(\sum_{k=1}^{n-1} k\lg k \le \frac{1}{2} n^2 \log n - \frac{n^2}{8}\right)$$

$$\Theta(n) + b - \frac{an}{4} \le 0$$

or, $\frac{an}{4} \geq cn + b$

or, $a \geq 4c + 4b$