Digital Cash

November 29, 2016

Instructions for today's class…

- Please write out the diagrams and any math/text that you need to understand what's going on in each protocol. You'll need your diagrams and notes to answer the questions and perform the activities.
- Copy down the "Principles of Digital Cash" and what they mean. You'll need them, too.

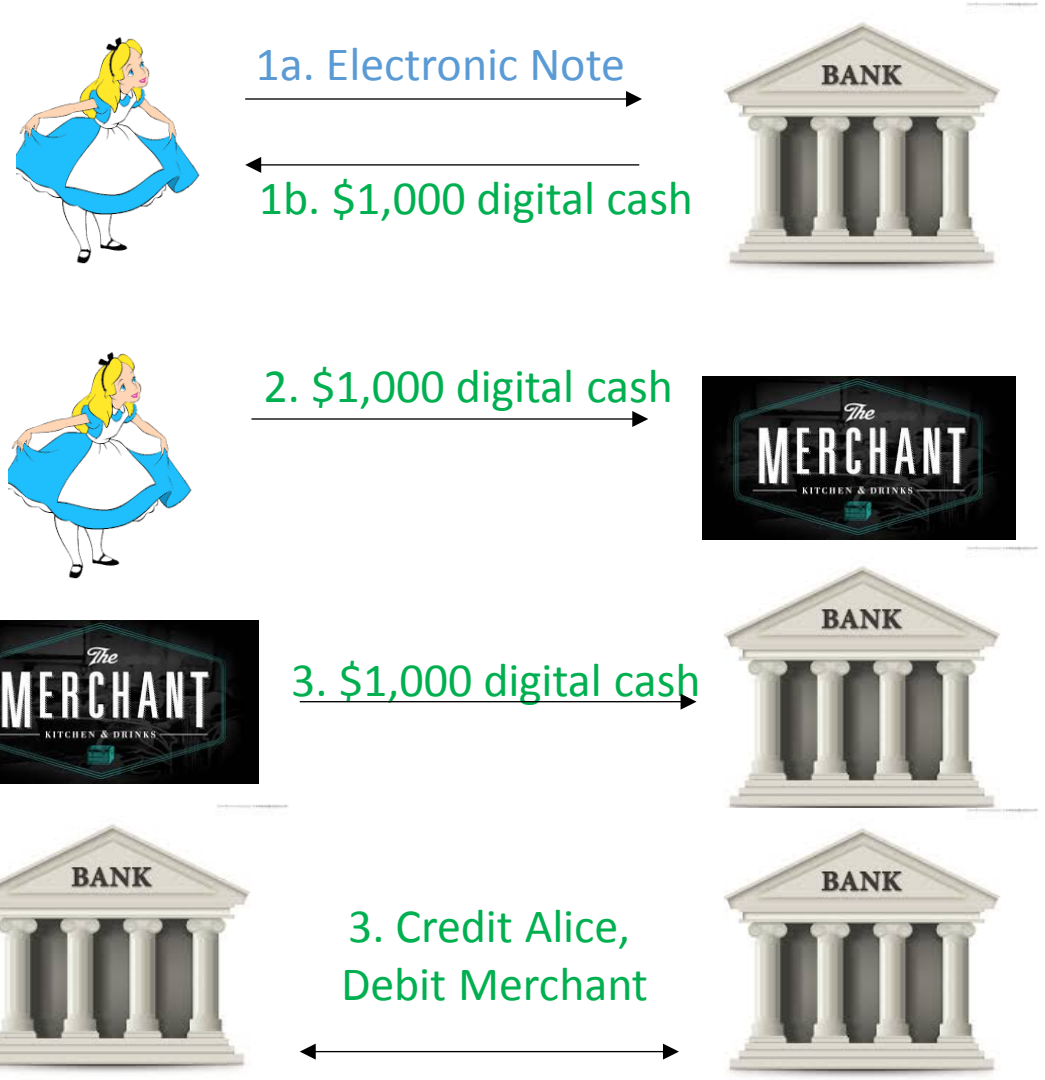
Properties of Digital Cash

1. Not a check, credit card or a debit card (they leave audit trails).
2. Anonymous and untraceable.
3. Can be sent through computer networks.
4. Can be used offline (not connected to a bank).
5. Transferable.
6. Divisible (can make change)
7. Can be stolen
8. Can be spent only once (is secure)
9. Might be used to pay for small things (tolls, food).

Does Bitcoin have all of these properties?

Digital Cash – Protocol 0

1. Bank gives Alice a note for $1000 (like a money order or cashier's check) and subtracts $1000 from Alice's bank account.
2. Alice spends the note with a merchant.
3. Merchant deposits the note in his bank account.
4. Merchant's bank clears note with Alice's bank.



Digital Cash – Problems with Protocol 0

Problems:

1a) The note is electronic. It can be easily copied, so Alice could spend it twice.

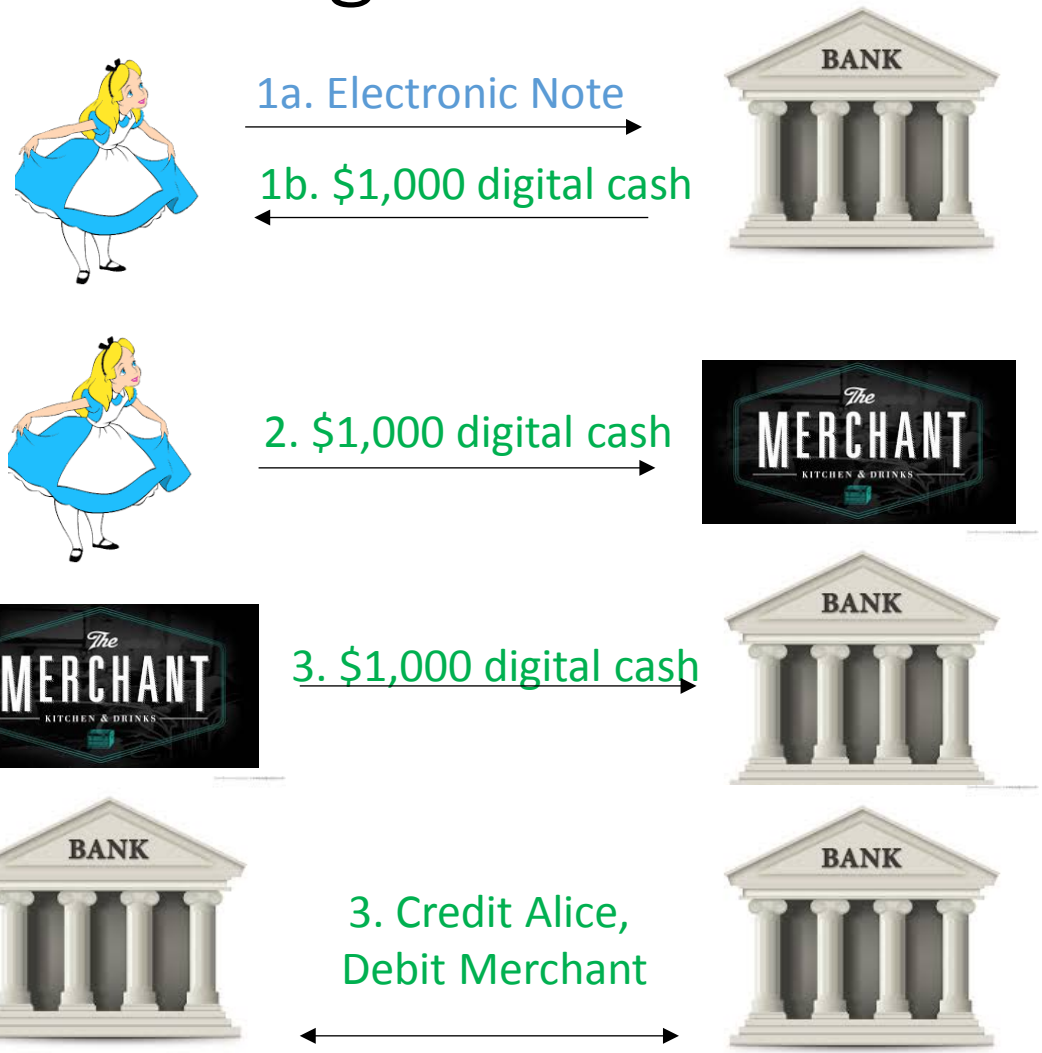
1b) …so could the merchant.

2) Alice could be traced if the bank remembered the serial number of the note.

We solve these problems one-by-one in the following protocols, but...

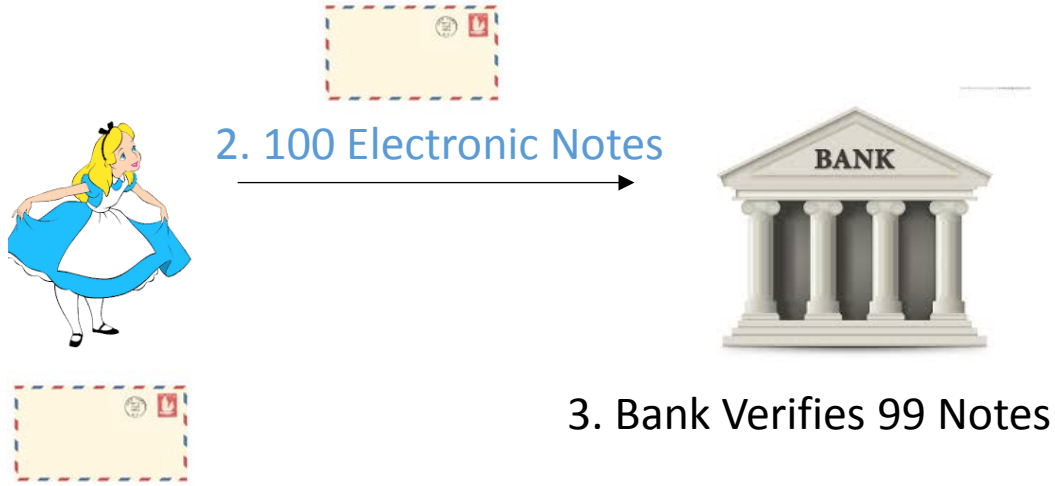
Individual Activity (5 minutes). What could you add/change to make cash usable only once? Would it violate other Principles of Digital Cash?

1. Bank gives Alice a note for $1000 (like a money order or cashier's check) and subtracts $1000 from Alice's bank account.
2. Alice spends the note with a merchant.
3. Merchant deposits the note in his bank account.
4. Merchant's bank clears note with Alice's bank.



Digital Cash – Protocol 1

1. Alice prepares 100 anonymous money orders for $1000 each.
2. Alice blinds (enciphers) each one and gives all to her bank.
3. Bank asks Alice to open (decipher) 99 envelopes, and verifies that each one is a money order for $1000. (If not so, Alice goes to jail.)



1. Alice enciphers 100 “identical” money orders

Digital Cash – Protocol 1 (Cont.)

4. Bank signs the last one, returns it to Alice and deducts $1000 from her bank account.
5. Alice deciphers the money order (now signed by the bank) and spends it with a merchant.
6. Merchant verifies bank's signature (can be done without communication with bank) to make sure it is valid.

2. 100 Electronic Notes

4. 1 Signed Electronic Note

3. Bank Verifies 99 Notes

1. Alice enciphers 100 "identical" money orders

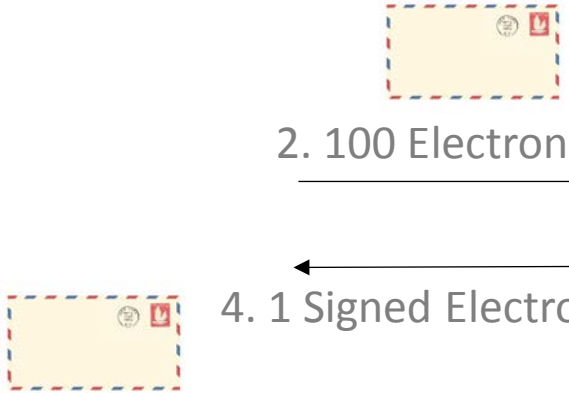
5b. Alice Spends Note

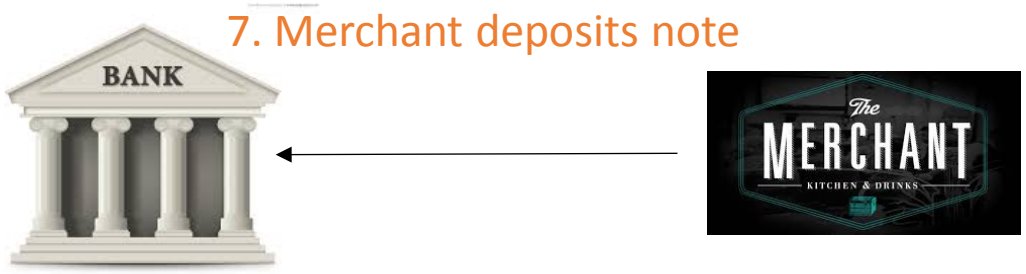
6. Merchant verifies bank signature

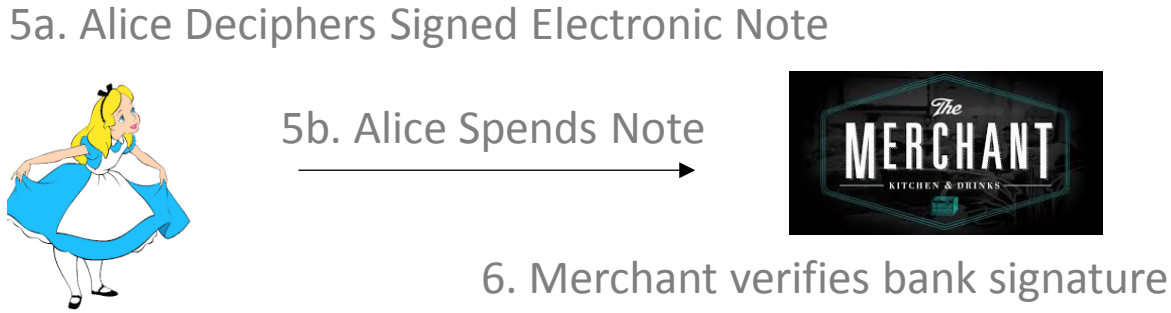
5a. Alice Deciphers Signed Electronic Note

Digital Cash – Protocol 1 (Cont.)

7. Merchant takes money order to his bank.
8. Merchant's bank verifies the signature and adds $1000 to merchant's bank account.

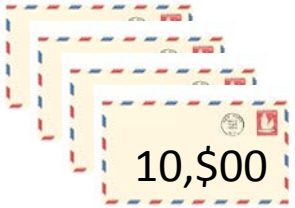






Digital Cash – Protocol 1 - Graphically

1 &2. Alice creates and enciphers 100 “identical” money orders



3a. Open 99 Electronic Notes

3b. 99 open Electronic Notes

3c. Bank Verifies 99 Notes

4. 1 Signed, Encrypted Electronic Note

5a. Alice Deciphers Signed Electronic Note

5b. Alice Spends Note

6. Merchant verifies bank signature

7. Merchant deposits note

8. Merchant’s bank verifies signature, Debits merchant’s account.

Digital Cash – Problems with Protocol 1

Problem:

- This protocol makes anonymous cash, but cash that can still be spent twice.

Do any of the solutions to this problem that were discussed following Protocol 0 conflict with the changes made in Protocol 1?

How does Alice's bank “sign” her enciphered money order?

- Let's say the bank uses RSA with keys n, e, d.
- Let the money order be M.
- Alice chooses a random k in $1 < k < n$.
- Alice “blinds" (enciphers) the money order by computing $t = Mk^e \ mod \ n$.
- Bank signs t as:

$$t^d \equiv (Mk^e)^d \equiv M^D k^{ed} \equiv M^d k \ (mod \ n)$$

How does Alice's bank “sign” her enciphered money order (cont.)?

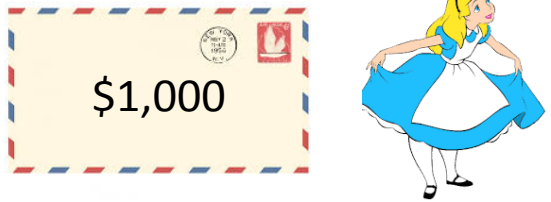
- Alice “unblinds” (deciphers) the signed money order t^d by computing $k^{-1}\ (mod\ n)$ (using extended Euclid) and multiplying:

$$s = t^d k^{-1}\ (mod\ n) \equiv (Mk^e)^d k^{-1} \equiv M^d k^{ed-1} \equiv M^d (mod\ n)$$

- To “open" (decipher) the 99 t, Alice tells the bank M and k for each. The bank verifies that each $t = Mk^e (mod\ n)$

How Alice's Bank signs (multiple messages) - Graphically

1.



$t_i = M_i k_i^e \ (mod\ n)$

…for i=0 to 99 if 100 messages

2.

3. Decipher all i notes except x

(AES Bank of "ned")

5. Bank verifies that open messages are all correct.

6. Bank signs the remaining message: t_x^d

4. All i notes except x

7. Bank signed message: t_x^d

8.

Alice deciphers: $t_x^d = M^d$

Digital Cash – Protocol 2

1. Alice prepares 100 anonymous money orders for $1000 each. On each one she writes a random 20-digit integer.
2. Alice enciphers each one and gives all to her bank.
3. Bank asks Alice to open (decipher) 99 envelopes, and verifies that each one is a money order for $1000 and that all 99 20-digit integers differ (If not so, Alice goes to jail.)
4. Bank signs the last money order, returns it to Alice and deducts $1000 from her bank account.

Digital Cash – Protocol 2

5. Alice deciphers the money order (now signed by the bank) and spends it with a merchant.
6. Merchant verifies bank's signature to make sure it is valid.
7. Merchant takes money order to his bank.
8. Merchant's bank verifies the signature, and checks in a database to make sure that a money order with the same 20-digit integer has not been previously spent.
   - If this has not happened, then it adds $1000 to the merchant's bank account and records the 20-digit number in the database used by all banks.
   - But if the number is already in the database, then the bank doesn't accept the money order.

Digital Cash – Protocol 2 - Graphically

rand
$1,000

1 . Alice creates and enciphers 100 “identical” money orders, and adds a random 20-digit number (rand)

rand x
01,$00

2. 100 Electronic Notes →

3a. Alice, open 99 Notes ←

4. 1 Signed Electronic Note, account deduction

rand x,
bank
01,$00

3b. Bank Verifies 99 Notes

5a. Alice Deciphers Signed Electronic Note

5b. Alice Spends Note →

rand x,
bank
$1,000

6. Merchant verifies bank signature

7. Merchant deposits note →

8. Merchant’s bank verifies signature, checks Alice’s random number against a trusted database, Debits merchant’s account.

Digital Cash – Protocol 2

- Now if Alice tries to spend the money order twice, or if the merchant tries to deposit it twice (in two different banks, say), the second bank will know and not accept it.

- The next protocol tries to identify the cheater (Alice, Merchant, Eve).

How would you add/change Protocol 2 to be able to identify the cheater, but retain as much anonymity for Alice and the Merchant as possible?

Secret Splitting (used in the next protocol)

1. Let the secret be s.
2. Choose a random r of the same length as s.
3. One party gets r ; the other gets $r \oplus s$.
4. Together the two parties can compute: $r \oplus (r \oplus s) = s$

$$secret\ (s)$$

$$r \longrightarrow \boxed{r \oplus (r \oplus s)} \longleftarrow r \oplus s$$

Bit Commitment(used in the next protocol)

1. Alice wants to commit to a “bit" (it can be a string) now so that she can't change it later, but only Alice knows it for now.
2. Alice commits to b by generating two random strings R_1, R_2.
3. She creates a message (R_1, R_2, b) and computes a hash (SHA. say) of it.
4. She reveals (or tells Bob) the hash value and R_1.
5. When the time comes to reveal b, Alice shows the message (R_1, R_2, b).
6. Bob checks that R_1 is the same as it was earlier and verifies the hash value.

Why is R_2 needed?

- Because if b were a short string (one bit, literally, say), then Bob could guess it from the hash value.

Bit Commitment - Graphically

1. Alice generates a bit, b.
2. Alice commits to b by generating r_1, r_2.
3. Alice hashes the message (r_1, r_2, b).

4. Alice sends Bob r_1 and the hash of (r_1, r_2, b). →

(When Alice wants to reveal B…)

5. Alice sends Bob (r_1, r_2, b). →

6. Bob verifies r_1 and that the hash is the same as the one Alice originally sent.

Digital Cash – Protocol 4

1. Alice prepares 100 anonymous money orders for $1000 each. On each one she writes a random 20-digit integer and 100 pairs of identity bit strings: $(I_{1L}; I_{1R}), \ldots, (I_{100L}; I_{100R})$
2. Each part is a bit-committed packet that Alice can be asked to open, and whose proper opening can easily be checked. Any pair, $(I_{59L}; I_{59R})$, say, reveals Alice's identity by secret splitting, that is,

$$I_{59L} \oplus I_{59R} = \text{Alice's Identity}$$

3. Alice enciphers each money order and gives all 100 to her bank.

Digital Cash – Protocol 4

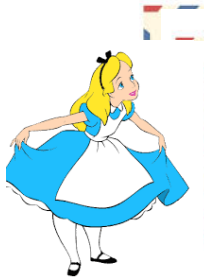
3. Bank asks Alice to open (decipher) 99 envelopes, and verifies the contents. If it finds an error, Alice goes to jail.
4. Bank signs the last money order, returns it to Alice and deducts $1000 from her bank account.
5. Alice deciphers the money order (now signed by the bank) and spends it with a merchant.
6. Merchant verifies bank's signature to make sure it is valid.
7. Merchant asks Alice to randomly reveal either the left or right half of each of the 100 identity strings. (Merchant chooses the random choices of left or right half.)
8. Alice complies.

Digital Cash – Protocol 4

9. Merchant takes money order to his bank.
10. Bank verifies the signature and checks the database for the 20-digit number. If it is not found therein, the bank credits the merchant with $1000 and records the money order in the database.
11. If the 20-digit integer is already in the database, the bank does not accept the money order. It compares the 100 identity strings on the money order with those in the database.
    - If they agree, the bank knows that the merchant copied the money order.
    - If they differ, a second merchant deposited the money order earlier and it was Alice who copied. Some of the 100 identity strings will have both halves revealed, so Alice can be identified. Alice goes to jail.

Digital Cash – Protocol 4 - Graphically

rand, IdL, IdR
$1,000



rand x, IdxL, IdxR
01,$00

1 . Alice creates and enciphers 100 “identical” money orders, and adds a random 20-digit number (rand), and a pair of secret split ID numbers

2. 100 Electronic Notes →

3a. Alice, open 99 Notes →

4. 1 Signed Electronic Note, account deduction

rand x, IdxL, IdxR
01,$00

3b. Bank Verifies 99 Notes

5a. Alice Deciphers Signed Electronic Note

5b. Alice Spends Note →

rand x, IdxL, IdxR
bank
$1,000

6. Merchant verifies bank signature

7. Merchant deposits note →

8. Merchant’s bank verifies signature, , checks Alice’s random number against a trusted database, Debits merchant’s account.

Digital Cash – Protocol 4, Alice Spends the note - Graphically

5a. Alice Deciphers Signed Electronic Note

5b. Alice Spends Note

6. Merchant verifies bank signature

7. Give me IdL or IdR for each note.

9. Merchant deposits note

8. Alice complies

rand x, IdxL, IdxR
bank
$1,000

10&11.

- Is rand (Alice's random number) already in a trusted database?
  - No. Debits merchant's account.
  - Yes. Reject
    - Are the IdL and IdR pairs the same as in Step 3?
      - Yes. Merchant copied money
      - No. Alice spent money more than once.

Digital Cash – Remarks on Protocol 4

- The cash is not transferable or divisible.
- Can Alice cheat?
  - No. She can copy her $1000 money order. The first time she spends it is okay. But she gets caught the second time she spends it.
- Can she create a money order with a bad ID string?
  - Yes, but she only has one chance in 100 of being successful.
- Alice can't change the 20-digit number or the identity strings, because then the bank's signature would no longer be valid.
- Can the merchant cheat?
  - No.

Digital Cash – Remarks on Protocol 4 (cont.)

- Can the merchant and Alice collude to spend the e-cash twice?
  - No, because they can't change the 20-digit number signed by the bank, so the bank will not have to pay the $1000 more than once.
- Can Eve copy Alice's money order and spend it first?
  - Yes. It is like cash.
  - Even worse, if Alice didn't know that Eve copied it and spent it, then Alice would be caught when she spent it the first time.
  - Eve could eavesdrop on communication between Alice and the merchant and deposit the money (acting as a merchant) before the merchant deposits it. When the merchant tries to deposit it, he will be found as a cheater.
  - So, Both Alice and the merchant must protect their e-cash as if it were cash. It must be enciphered when it is send across the Net.

The Perfect Crime

1. Alice kidnaps a baby.
2. Alice prepares 10,000 anonymous money orders for $1000 each.
3. Alice blinds them, sends them to authorities, and demands that
   a) a bank signs all 10,000 money orders, and
   b) the results be published in a newspaper.
4. The authorities comply.
5. Alice buys the newspaper, unblinds all the money orders and spends them.
6. Alice frees the baby.
7. Note that there is no physical contact.

Group Activity (15 minutes):

- In groups of 2 or 3, what changes would you make to Protocol 4 in order to make sure that Alice can’t get away with the “The Perfect Crime”?
- Be prepared to discuss your answer in terms of Protocol 4 and in terms of the Principles of Digital Cash.

END

Digital Cash – Protocol 3

1. Alice prepares 100 anonymous money orders for $1000 each. On each one she writes a random 20-digit integer.
2. Alice enciphers each one and gives all to her bank.
3. Bank asks Alice to open (decipher) 99 envelopes, and verifies that each one is a money order for $1000 and that all 99 20-digit integers differ. (If not so, Alice goes to jail.)
4. Bank signs the last money order, returns it to Alice and deducts $1000 from her bank account.

Digital Cash – Protocol 3

5. Alice deciphers the money order (now signed by the bank) and spends it with a merchant.
6. Merchant verifies bank's signature to make sure it is valid.
7. Merchant asks Alice to write a random identity string on the money order.
8. Alice does so.
9. Merchant takes money order to his bank.

Digital Cash – Protocol 3

10. Merchant's bank verifies the signature, and checks in a database to make sure that a money order with the same 20-digit integer has not previously deposited.
    - If it hasn't, the bank credits $1000 to the merchant's account and records the 20-digit integer and the random identity string in the database.
11. If the 20-digit integer is already in the database, the bank refuses the money order.
    - Then it compares the identity string on the money order with the one in the database.
        - If they are the same identity string, the bank knows it was the merchant who copied the money order (and he goes to jail).
        - If they differ, the bank knows that the person who bought the money order from a bank copied it.

- But the bank doesn't know in the latter case that the person was Alice. The next protocol repairs this flaw.