

Congruences for fun and profit

On Saturday, October 3, 2009, a group of people celebrated the one-hundredth anniversary of an event that happened on a Saturday in early October, 1909.

One of the people wondered whether the event happened on October 3, 1909. He asked whether that date was a Saturday. I said immediately that it was a Sunday.

He didn't believe that I could have found that day of the week without using a calendar or a computer. I bet him that October 3, 1909, was a Sunday. He accepted the bet and lost.

Here is how I did it using congruences.

First, we can code the days of the week as Sunday = 0, Monday = 1, etc., Saturday = 6.

We know October 3, 2009, is a Saturday = 6.

October 3, 1909, was exactly 100 years earlier, so its day of the week was 6 minus (the number of days in 100 years) modulo 7.

Each year has 52 weeks plus 1 day, unless it is a Leap Year, in which case it has 52 weeks plus 2 days. There are 25 leap years in the 100 years between 1909 and 2009. Thus, the number of days in 100 years is $\equiv 100 + 25 = 125 \pmod{7}$.

Now 49 and 98 are multiples of 7, so

$$125 \equiv 125 - 98 = 27 \equiv 6 \pmod{7}.$$

Therefore, the day of the week for October 3, 1909, is

$$6 - 125 \equiv 6 - 6 = 0 \pmod{7},$$

that is, October 3, 1909, was a Sunday.