On different lives of information concept

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Outline

1. Intuitive notion of information
   - Different formalizations
   - Entropic and Logic conceptions

2. Remarks on some formal notions
   - Shannon’s information theory
   - A non-probabilistic entropy
   - Kolmogorov/Chaitin approach

3. A structural approach
   - Structural information
   - The case of words

4. Conclusions
   - Information in Physics and Biology
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Different formalizations

- Information is an intuitive concept which has a wide ‘semantic halo’ so that several formalizations are possible.

- Intuitively, information means *minimum amount of ‘data’ which are required to ‘determine’ an ‘object’ into a given ‘class’.*

- Several approaches have been proposed in order to formalize and quantify the notion of information. Any definition of information requires a suitable specification of the terms ‘data’, ‘determine’, and ‘class of objects’ used in the intuitive definition.

- These approaches, called *technical, semantic, pragmatic, descriptive, algorithmic, logic, structural,* etc., are conceptually very different.
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In the different approaches there exist some analogies, even though often only formal, between the considered quantities.

Some formalizations of the concept of information, even though meaningful and interesting, lack a solid mathematical frame in which one can evaluate the actual implications of these concepts or find deep theorems.

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Entropy and complexity

There exist two main conceptions about the notion of information

A. **Entropic**. It is based on a global ‘measure of ignorance’ about the state of a system. This measure is called ‘entropy’ in analogy to the physical entropy. Any determination of the state of a system yields an (average) information proportional to the entropy.

B. **Logic**. It is essentially based on ‘formal logic’. In this case information is related to the ‘complexity’, *static* or *dynamic*, required to compute or generate an object of a given class.

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Shannon’s entropy

- The source $S$ is described by an ergodic Markov chain.

- The entropy $H(S)$ can be interpreted as the average amount of uncertainty in making a prevision on the letter which will be emitted from the source.

- Equivalently, the entropy of $S$ measures the average amount of information that one receives from the knowledge of the letter emitted from the source.
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Aldo de Luca, Settimo Termini, A definition of a non-probabilistic entropy in the setting of fuzzy sets. *Information and Control* 20 (1972) 301-331

- The ‘entropy’ can be considered as a measure of the global ‘distance’ of a non-Boolean universe (described by fuzzy sets) from a Boolean one.

- Several entropy measures can be introduced. They have to satisfy some basic axioms. The choice of a particular measure depends on the ‘context’, i.e., the class of considered problems.
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- ‘information’ is defined in terms of the algorithmic static complexity (*program complexity*)

- The algorithmic approach is based on the theory of recursive functions which is a very solid and well developed mathematical theory.

- There is the possibility of defining ‘random’ objects as those for which the program complexity is approximately equal to the size of the object. Random objects pass all conceivable statistical tests.

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Structural information

Analysis of a complex object, or structure, in terms of simpler components of the same type. One can ask the following general question:

*what information does one have about a complex structure from the knowledge of smaller substructures?*

For instance, what information has one about a word, a tree, or a graph by knowing a certain set of subwords, subtrees, or subgraphs?

*What is the minimal size of these substructures capable of determining uniquely the structure itself?*
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This approach is for some aspects similar to the Kolmogorov approach.

The main difference concerns the ‘set of data’. In fact, in this structural approach the data about the given structure belong to a class of a certain kind, namely substructures.
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The case of words

In the case of a word the substructures are subwords.

- A word $w$ is any sequence of letters belonging to a given alphabet $A$, $w = a_1 \cdots a_n$.
- A subword, or factor, of $w$ is any block of consecutive letters of $w$.
- A factor $u$ of $w$ is called right (resp. left) special if there exist two distinct letters $x, y$ such that $ux$ and $uy$ (resp. $xu, yu$) are factors of $w$.
- A factor is bispecial if it is right and left special.
- A factor $xsy$ of $w$ with $s$ bispecial and $x, y$ letters is called a proper box.
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The maximal box theorem

Example

Let $w = abccbabcab$. One has

- The set of right special factors is $\{\varepsilon, b, c, bc, abc\}$.
- The set of left special factors is $\{\varepsilon, a, b, c, ab\}$.
- The maximal proper boxes are

  $abc, cba, bcc, ccb, bca$.

- The initial box is $abcc$ and the terminal box is $cab$.

Theorem (A. Carpi, A. de Luca, 2001)

(Maximal Box theorem). Any word is uniquely determined by the initial box, the terminal box, and the set of maximal proper boxes.
Proposition

Any word $w$ is uniquely determined by the set of its factors up to the length $G_w + 2$, where $G_w$ is the maximal length of a repeated factor of $w$.

If $n$ is the length of a word $w$ over a $d$-letter alphabet, then the following holds:

- $\left\lfloor \log_d n \right\rfloor \leq G_w$

- For almost all words of length $n$
  
  $$G_w \leq \left\lceil 2\log_d n \right\rceil + \log_d(\log_d n)$$

- The average value $\langle G_w \rangle_n$ of $G_w$ over all words of length $n$ has the following upper bound
  
  $$\langle G_w \rangle_n \leq \left\lceil 2\log_d n \right\rceil - 1/2.$$
Remarks

- The preceding approach is a ‘structural’ approach in a non-probabilistic frame.
- In the case of words the underlying mathematical theory is the ‘Algebraic Combinatorics on words’.
- There is some similarity with the algorithmic approach. However, differently from Kolmogorov theory, it is not an asymptotic theory.
- There exist very efficient algorithms for ‘sequencing’ a text and, conversely, for ‘recovering’ the initial text.
- The formalism can be generalized to more general combinatorial structures such as trees and 2-D arrays.
- Important applications in molecular Biology for the problem of ‘sequence assembly’.
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Information does not seem to be a Physical entity like mass, energy, force, etc.

Several different formalizations of the intuitive notion exist.

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- In Biology these exist sophisticated mechanisms for the information processing, which are often *coding processes*. For instance, DNA, RNA coding mechanism by means of which a sequence of bases in a four letter alphabet is transformed in a protein which is a word in a 20 letter alphabet.

- Also the *brain* and especially the *cortical areas* have specialized systems for processing and coding information. In fact, despite its physical limitations, the brain possesses sophisticated mechanisms to process information which increase considerably its efficiency.
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The *Life* is the only known case, in the great variety of phenomena of physical word, in which there exist some natural coding mechanisms such as the genetic code. The natural origin of these mechanisms is very surprising since the codified objects are very different from the uncoded objects (for instance, *genes* and *proteins*).

Biology seems to show that the notion of information cannot be independent from the ‘semantic’ and ‘pragmatic’ aspects of the information which are strongly related with its utilization, i.e., with the characteristic features (mechanisms of information processing) of the receiver.