On different lives of information concept

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Outline

Different formalizations

- Information is an intuitive concept which has a wide ‘semantic halo’ so that several formalizations are possible.
- Intuitively, information means *minimum amount of ‘data’ which are required to ‘determine’ an ‘object’ into a given ‘class’*.
- Several approaches have been proposed in order to formalize and quantify the notion of information. Any definition of information requires a suitable specification of the terms ‘data’, ‘determine’, and ‘class of objects’ used in the intuitive definition.
- These approaches, called *technical, semantic, pragmatic, descriptive, algorithmic, logic, structural, etc.*, are conceptually very different.

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- In the different approaches there exist some analogies, even though often only formal, between the considered quantities.
- Some formalizations of the concept of information, even though meaningful and interesting, lack a solid mathematical frame in which one can evaluate the actual implications of these concepts or find deep theorems.
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Entropy and complexity

There exist two main conceptions about the notion of information

A . **Entropic.** It is based on a global 'measure of ignorance' about the state of a system. This measure is called 'entropy' in analogy to the physical entropy. Any determination of the state of a system yields an (average) information proportional to the entropy.

B . **Logic.** It is essentially based on 'formal logic'. In this case information is related to the 'complexity', *static* or *dynamic*, required to compute or generate an object of a given class.

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Shannon's entropy

- The source S is described by an ergodic Markov chain.
- The entropy $H(S)$ can be interpreted as the *average amount of uncertainty in making a prevision* on the letter which will be emitted from the source.
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- The 'entropy' can be considered as a measure of the global 'distance' of a non-Boolean universe (described by fuzzy sets) from a Boolean one.
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- 'information' is defined in terms of the algorithmic static complexity (*program complexity*)
- The algorithmic approach is based on the theory of recursive functions which is a very solid and well developed mathematical theory.
- There is the possibility of defining 'random' objects as those for which the program complexity is approximately equal to the size of the object. Random objects pass all conceivable statistical tests.
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- The theory is an asymptotic theory. Indeed, the program complexity $K_U(x)$ is defined for a single object only up to an additive constant (depending on the universal machine U).
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Structural information

Analysis of a complex object, or structure, in terms of simpler components of the same type. One can ask the following general question:

what information does one have about a complex structure from the knowledge of smaller substructures?

For instance, what information has one about a word, a tree, or a graph by knowing a certain set of subwords, subtrees, or subgraphs?

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The case of words

In the case of a word the substructures are subwords.

- A word w is any sequence of letters belonging to a given alphabet A, $w = a_1 \cdots a_n$.
- A *subword*, or *factor*, of w is any block of consecutive letters of w.
- A factor u of w is called *right (resp. left) special* if there exist two distinct letters x, y such that ux and uy (resp. xu, yu) are factors of w.
- A factor is *bispecial* if it is right and left special.
- A factor xsy of w with s bispecial and x, y letters is called a *proper box*.
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The maximal box theorem

Example

Let $w = abccbabcab$. One has

- The set of right special factors is $\{\varepsilon, b, c, bc, abc\}$.
- The set of left special factors is $\{\varepsilon, a, b, c, ab\}$.
- The maximal proper boxes are

$$abc, cba, bcc, ccb, bca.$$

- The initial box is $abcc$ and the terminal box is cab.

Theorem (A. Carpi, A. de Luca, 2001)

(Maximal Box theorem). Any word is uniquely determined by the initial box, the terminal box, and the set of maximal proper boxes

Proposition

Any word w is uniquely determined by the set of its factors up to the length $G_w + 2$, where G_w is the maximal length of a repeated factor of w.

If n is the length of a word w over a d-letter alphabet, then the following holds:

- $$\lfloor log_d n \rfloor \leq G_w$$
- For *almost all* words of length n
$$G_w \leq \lceil 2 log_d n \rceil + log_d(log_d n)$$
- The *average value* $\langle G_w \rangle_n$ of G_w over all words of length n has the following upper bound
$$\langle G_w \rangle_n \leq \lceil 2 log_d n \rceil - 1/2.$$

Remarks

- The preceding approach is a ‘structural’ approach in a non-probablistic frame.
- In the case of words the underlying mathematical theory is the ‘Algebraic Combinatorics on words’.
- There is some similarity with the algorithmic approach. However, differently from Kolmogorov theory, it is not an asymptotic theory.
- There exist very efficient algorithms for ‘sequencing’ a text and, conversely, for ‘recovering’ the initial text.
- The formalism can be generalized to more general combinatorial structures such as *trees* and *2-D arrays*.
- Important applications in molecular Biology for the problem of ‘sequence assembly’.

Information and Physical sciences

- At the end of $40's$ it was introduced in the setting of physical sciences a new entity called ‘information’ of great importance in the communication of human beings and machines.
- Information does not seem to be a Physical entity like *mass, energy, force, etc.*
- Several different formalizations of the intuitive notion exist.
- What is the role of Mathematics in an Information science?

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Information and Biology

- In Biology these exist sophisticated mechanisms for the information processing, which are often *coding processes*. For instance, DNA, RNA coding mechanism by means of which a sequence of bases in a four letter alphabet is transformed in a protein which is a word in a 20 letter alphabet.

- Also the *brain* and expecially the *cortical areas* have specialized systems for processing and coding information. In fact, despite its physical limitations, the brain possesses sophisticated mechanisms to process information which increase considerably its efficiency.

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- The *Life* is the only known case, in the great variety of phenomena of physical word, in which there exist some natural coding mechanisms such as the genetic code. The natural origin of these mechanisms is very surprising since the codified objects are very different from the uncoded objects (for instance, *genes* and *proteins*).

- Biology seems to show that the notion of information cannot be independent from the ‘semantic’ and ‘pragmatic’ aspects of the information which are strongly related with its utilization, i.e., with the characteristic features (mechanisms of information processing) of the receiver.