[40] **Homework 5**: *Big O, Ω.*

[10] Select the best "big Oh" notation for each expression. Justify by showing the constants c and n_0. Note that $f(n) = O(g(n))$ if there are constants $c > 0$ and $n_0 > 0$ so that for all $n \geq n_0$ we have $|f(n)| \leq c \cdot g(n)$.

1. $100n + \log n$.
2. $(5n+1)^3 + 1000n^2$.
3. $n\sqrt{n^3} + \log^5 n$.
4. $n^3 + n + \sqrt{n} + \sqrt{\log n}$.

[10] Show the following:

$$\begin{aligned} 6n^2 \log n - 2n &= \Theta(n^2 \log n) \\ \frac{6n^2 + n}{n \log^6 n + 1} &= \Theta\left(\frac{n}{\log^6 n}\right) \\ \sum_{i=1}^{n} i^2 &= \Theta(n^3) \end{aligned}$$

[10] Is $(\log n)^3 = O(\log n^3)$? Justify your answer?

[10] We say that $f(n) \prec g(n)$ if $g(n)$ grows faster than $f(n)$ (e.g., $\log n \prec n$).

Order the following functions by by $\prec$ from the lowest to the highest:

$$\left(\frac{5}{3}\right)^{2n}, \;\; 10^8, \;\; \sqrt{n^3}\log^2 n, \;\; 2^{\log_2 n}, \;\; \log^4 \sqrt{n}, \;\; 2^{3\log_2 n}, \;\; 2^n.$$

Justify your answer.