[50] Homework 4. *Proof Techniques*

Each problem is worth 10 points

- [10] Show that $\sqrt[3]{3}$ is irrational.
- [10] Prove that for positive integer n, $n^3 + 2n$ is divisible by 3.
- [10] Using mathematical induction prove that

$$\sum_{i=1}^{n} i2^i = 2^{n+1}(n-1) + 2.$$

- [10] Define the nth *Harmonic number* as

$$H_n = \sum_{i=1}^{n} \frac{1}{i}.$$

  Prove by induction on $n \geq 0$ that

$$H_{2^n} \geq 1 + \frac{n}{2}.$$

- [10] Derive an explicit formula for the following recurrence for $n \geq 1$

$$a_n = \frac{n}{2} a_{n-1}$$

  with $a_0 = 1$.