[50] Homework 4. Proof Techniques

Each problem is worth 10 points

- [10] Show that  $\sqrt[3]{3}$  is irrational.
- [10] Prove that for positive integer n,  $n^3 + 2n$  is divisible by 3.
- [10] Using mathematical induction prove that

$$\sum_{i=1}^{n} i2^{i} = 2^{n+1}(n-1) + 2.$$

[10] Define the *n*th Harmonic number as

$$H_n = \sum_{i=1}^n \frac{1}{i}.$$

Prove by induction on  $n \geq 0$  that

$$H_{2^n} \ge 1 + \frac{n}{2}$$

[10] Derive an explicit formula for the following recurrence for  $n\geq 1$ 

$$a_n = \frac{n}{2}a_{n-1}$$

with  $a_0 = 1$ .