Solution of Homework 3

**** Note that the approximate functions shown below are *not* unique and are provided for the sake of illustration not perfection.**

Prob. 1 Using Stirling's formula:

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2n\pi} \Rightarrow \sum_{k=1}^{n} \ln k = \ln n! \approx n(\ln n - 1) + 0.5 \ln(2n\pi)$$

Algorithm

I(1) = 0
for k = 2 to 500
  I(k) = I(k-1) + ln(k)

n	exact	approximate	n	exact	approximate	n	exact	approximate
1	0	0	170	706.5731	706.572773	340	1645.67517	1645.675123
10	15.10441257	15.09628322	180	758.2481	758.247851	350	1704.12475	1704.12471
20	42.33561646	42.33165135	190	810.4775	810.477225	360	1762.85967	1762.859643
30	74.65823635	74.65565988	200	863.232	863.231772	370	1821.87203	1821.872003
40	110.3206397	110.3187576	210	916.4855	916.485275	380	1881.15432	1881.154301
50	148.477767	148.4763015	220	970.2142	970.214014	390	1940.69945	1940.699441
60	188.6281734	188.6269858	230	1024.397	1024.39642	400	2000.5007	2000.500691
70	230.4390436	230.4380543	240	1079.013	1079.0128	410	2060.55166	2060.551654
80	273.6731243	273.6722838	250	1134.045	1134.0451	420	2120.84624	2120.846246
90	318.1526396	318.1519149	260	1189.477	1189.4767	430	2181.37866	2181.378672
100	363.7393756	363.7387434	270	1245.292	1245.29228	440	2242.1434	2242.143407
110	410.3227765	410.3222202	280	1301.478	1301.47762	450	2303.13516	2303.135176
120	457.812388	457.8118947	290	1358.02	1358.01955	460	2364.34892	2364.348938
130	506.1328253	506.1323855	300	1414.906	1414.90577	470	2425.77985	2425.779873
140	555.2202941	555.2199001	310	1472.125	1472.12484	480	2487.42334	2487.423363
150	605.0201058	605.0197515	320	1529.666	1529.66602	490	2549.27495	2549.274984
160	655.4848567	655.4845371	330	1587.519	1587.51926	500	2611.33046	2611.330493

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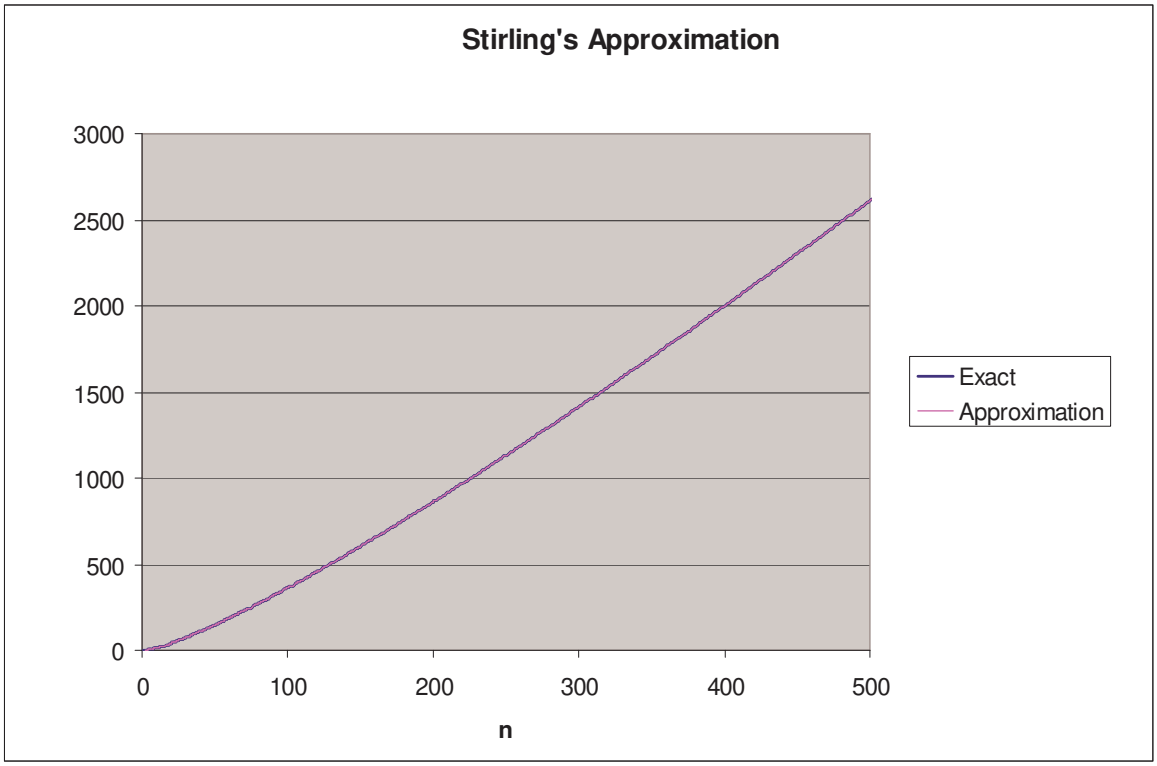


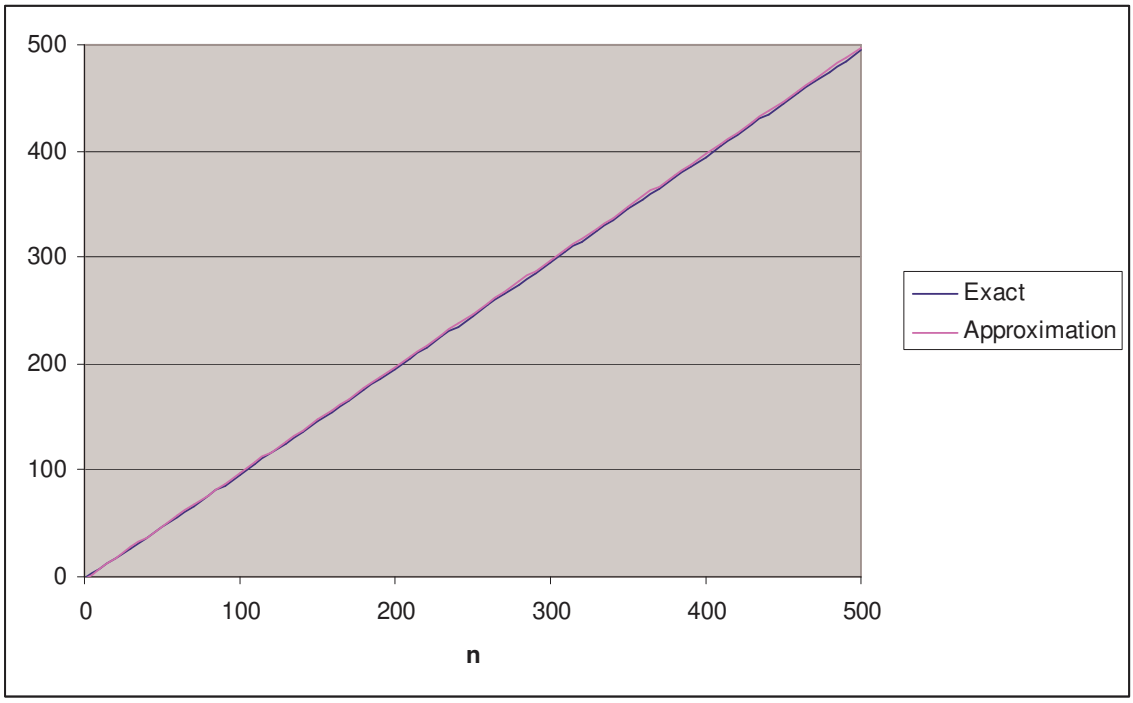
Solution of Homework 3

Prob. 2

$$S_n = \sum_{k=1}^{n} \binom{n}{k} 2^{-n} \log_2 \binom{n}{k} \approx n - 3$$

Algorithm

for n = 1 to 500
    S(n) = 0
    for k = 1 to n
        S(n) = S(n) + C(n,k) $\log_2$(C(n,k))
    S(n) = 2^{-n} S(n)

n	exact	approximate	n	exact	approximate	n	exact	approximate
1	0	-2	170	165.2482132	167	340	334.7482	337
10	7.293571037	7	180	175.2069816	177	350	344.7273	347
20	16.79227734	17	190	185.16798	187	360	354.707	357
30	26.49960272	27	200	195.1309794	197	370	364.6872	367
40	36.2920196	37	210	205.0957844	207	380	374.668	377
50	46.13102647	47	220	215.0622271	217	390	384.6492	387
60	55.99949369	57	230	225.0301617	227	400	394.631	397
70	65.88828818	67	240	234.9994612	237	410	404.6132	407
80	75.79195964	77	250	244.9700142	247	420	414.5958	417
90	85.70699305	87	260	254.9417223	257	430	424.5788	427
100	95.63098859	97	270	264.9144983	267	440	434.5622	437
110	105.5622347	107	280	274.8882645	277	450	444.546	447
120	115.4994676	117	290	284.8629513	287	460	454.5302	457
130	125.4417277	127	300	294.8384964	297	470	464.5146	467
140	135.3882691	137	310	304.8148435	307	480	474.4995	477
150	145.3385005	147	320	314.7919415	317	490	484.4846	487
160	155.2919451	157	330	324.7697444	327	500	494.47	497

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