[40] **Homework 2.** *Language of Mathematics*

Each problem is worth 10 points

[10] Prove

$$A \cap B = \overline{\bar{A} \cup \bar{B}}$$

without using the de Morgan law and Venn's diagrams.

[10] What is the image of $f(\mathbf{R})$, where $\mathbf{R}$ is the set of all reals:

- $f(x) = x^4$
- $f(x) = x^2 - 4x$
- $f(x) = 2^{x^2}$
- $f(x) = \sin(x)$

[10] Is $f(x) = x^2 + 1$ a bijection of $\mathbf{R} \to \mathbf{R}$?

Compute also $f^{-1}(\{y : \ 0 \le y \le 1\})$, if exists, where $f^{-1}(Y)$ denotes an inverse image, that is, the set of all x such that $f(x) \in Y$.

[10] What are the values of the following:

$$\sum_{i=5}^{99} 5 \cdot 2^{i-4},$$

$$\sum_{j=3}^{100} (2^{j+1} - 2^j),$$

$$\prod_{k=1}^{100} (-1)^k.$$