## [40] Homework 2. Language of Mathematics

Each problem is worth 10 points

[10] Prove

$$A \cap B = \bar{A} \cup \bar{B}$$

without using the de Morgan law and Venn's diagrams.

[10] What is the image of  $f(\mathbf{R})$ , where **R** is the set of all reals:

•  $f(x) = x^4$ 

- $f(x) = \sin(x)$
- [10] Is  $f(x) = x^2 + 1$  a bijection of  $\mathbf{R} \to \mathbf{R}$ ?

Compute also  $f^{-1}(\{y: 0 \le y \le 1\})$ , if exists, where  $f^{-1}(Y)$  denotes an inverse image, that is, the set of all x such that  $f(x) \in Y$ .

[10] What are the values of the following:

$$\sum_{i=5}^{99} 5 \cdot 2^{i-4},$$
$$\sum_{j=3}^{100} (2^{j+1} - 2^j),$$
$$\prod_{k=1}^{100} (-1)^k.$$