[40] Homework 1. *Basic Logic*

Each problem is worth 10 points

[10] Make truth tables for the following statements:

1. $p \vee (\overline{r \vee q})$;
2. $(p \wedge \neg q) \rightarrow r$.

[10] Using *logical equivalences* discussed in class prove that

$$(p \wedge q) \rightarrow (p \vee q)$$

is a tautology, that is, prove that

$$(p \wedge q) \rightarrow (p \vee q) \equiv T.$$

[10] Let

$$P(x, y) : \ x + y \geq 5 \text{ where } x, y \text{ positive integers.}$$

Tell whether the following statements are true or false:

- $\forall_x \ \forall_y \ P(x, y)$
- $\forall_x \exists_y \ P(x, y)$.

[10] Which of the following is equivalent to $\overline{\forall_x \exists_y P(x, y)} \equiv \neg \forall_x \exists_y P(x, y)$:

(a) $\exists_x \overline{\forall_y P(x, y)}$;
(b) $\forall_x \overline{\exists_y P(x, y)}$;
(c) $\exists_x \forall_y \overline{P(x, y)}$;
(d) $\exists_x \exists_y \overline{P(x, y)}$.