“Things to Know...”

Existential Quantifier: There exists an x such that $P(x)$ is true.

$$(\exists x)P(x)$$

Universal Quantifier: For all x, $P(x)$ is true.

$$(\forall x)P(x)$$

Binomial Coefficient: The number of m-combinations of a set of n distinct objects

$$\binom{n}{m} = \frac{1}{m!(n \Leftrightarrow m)!}$$

Product:

$$\prod_{i \leq j \leq k} f(j) = f(i) \cdot f(i+1) \cdot \ldots \cdot f(k)$$

Sum:

$$\sum_{i \leq j \leq k} f(j) = f(i) + f(i+1) + \ldots + f(k)$$

Logarithm Definition

$$For\ x > 0, \quad \log\ x = y \Leftrightarrow\ 2^y = x$$

Properties:

L1: Log is a strictly increasing function:

$$(\forall x, y)(x > y) \Rightarrow (\log\ x > \log\ y)$$

L2: Log is a one-to-one function:

$$(\forall x, y)(\log\ x = \log\ y) \Rightarrow (x = y)$$

L3: $\log\ 1 = 0$

L4:

$$(\forall a) \log_2 2^a = a$$

L5:

$$(\forall x, y) \log(x * y) = \log\ x + \log\ y$$

L6:

$$(\forall x, a) \log x^a = a * \log\ x$$

L7:

$$(\forall x, y) x^{\log\ y} = y^{\log\ x}$$

L8:

$$\log_b N = \frac{\log_c N}{\log_c b}$$

Summation Properties:

L1: Let f be any function, then

$$\sum_{i=1}^{b} cf(i) = c\sum_{i=a}^{b} f(i)$$

L3:

$$\sum_{i=a}^{b} [f(i) + g(i)] = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

L4:

$$\sum_{i=a}^{b} f(i) = \sum_{i=a}^{b} f(a + b \Leftrightarrow i), \quad a < b$$

L5:

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

Proof by induction:

Base Step: for $n = 1$,

$$\sum_{i=1}^{1} i = \frac{1(1+1)}{2} = 1$$

Induction Hypothesis: Assume that for some $k > 1$,

$$(\forall n \leq k) \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Inductive Step:

$$\sum_{i=1}^{n+1} i = \left(\sum_{i=1}^{n} i\right) + (n+1) = \frac{n(n+1)}{2} + n + 1 = \frac{(n+1)(n+2)}{2} \ .$$

L6:

$$\sum_{i=0}^{n} 2^i = 2^{n+1} \Leftrightarrow 1$$

L7:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad n \leq 1$$

Proof by Induction:

Base Step: for $n = 1$,

$$\sum_{i=1}^{1} i^2 = \frac{1(1+1)(2+1)}{6} = 1$$

Inductive Step: Assume for some $k > 0$ that

$$(\forall n \leq k) \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(n+2)}{6}$$

then

$$\sum_{1 \leq i \leq n+1} i^2 = \sum_{1 \leq i \leq n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2(n+1_{+}1))}{6} \ .$$