“Things to Know...”

**Existential Quantifier:** There exists an $x$ such that $P(x)$ is true.

$$\exists x P(x)$$

**Universal Quantifier:** For all $x$, $P(x)$ is true.

$$\forall x P(x)$$

**Binomial Coefficient:** The number of $m$-combinations of a set of $n$ distinct objects

$$\binom{n}{m} = \frac{1}{m!(n \leftrightarrow m)!}$$

**Product:**

$$\prod_{i \leq j \leq k} f(j) = f(i) \cdot f(i + 1) \cdot \ldots \cdot f(k)$$

**Sum:**

$$\sum_{i \leq j \leq k} f(j) = f(i) + f(i + 1) + \ldots + f(k)$$

**Logarithm Definition**

For $x > 0$, $\log x = y \iff 2^y = x$

**Properties:**

**L1:** Log is a strictly increasing function:

$$(\forall x, y)(x > y) \Rightarrow (\log x > \log y)$$

**L2:** Log is a one-to-one function:

$$(\forall x, y)(\log x = \log y) \Rightarrow (x = y)$$

**L3:** $\log 1 = 0$

**L4:**

$$(\forall a) \log_2 2^a = a$$

**L5:**

$$(\forall x, y) \log(x \ast y) = \log x + \log y$$

**L6:**

$$(\forall x, a) \log x^a = a \ast \log x$$

**L7:**

$$(\forall x, y)x^{\log y} = y^{\log x}$$

**L8:**

$$\log_b N = \frac{\log_e N}{\log_e b}$$

**Summation Properties:**
L1: Let \( f \) be any function, then
\[
\sum_{i=1}^{b} cf(i) = c \sum_{i=a}^{b} f(i)
\]

L3:
\[
\sum_{i=a}^{b} [f(i) + g(i)] = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)
\]

L4:
\[
\sum_{i=a}^{b} f(i) = \sum_{i=a}^{b} f(a + b \leftrightarrow i), \; a < b
\]

L5:
\[
\sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}
\]

Proof by Induction:
Base Step: for \( n = 1 \),
\[
\sum_{i=1}^{1} i = \frac{1(1+1)}{2} = 1
\]

Induction Hypothesis: Assume that for some \( k > 1 \),
\[
(\forall n \leq k) \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\]

Inductive Step:
\[
\sum_{i=1}^{n+1} i = \left( \sum_{i=1}^{n} i \right) + (n+1) = \frac{n(n+1)}{2} + n + 1 = \frac{(n+1)(n+2)}{2}.
\]

L6:
\[
\sum_{i=0}^{n} 2^i = 2^{n+1} \iff 1
\]

L7:
\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \; n \leq 1
\]

Proof by Induction:
Base Step: for \( n = 1 \),
\[
\sum_{i=1}^{1} i^2 = \frac{1(1+1)(2+1)}{6} = 1
\]

Inductive Step: Assume for some \( k > 0 \) that
\[
(\forall n \leq k) \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(n+2)}{6}
\]

then
\[
\sum_{1 \leq i \leq n+1} i^2 = \sum_{1 \leq i \leq n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2(n+1)+1)}{6}.
\]