[30] Homework 1. *Finite Sums and the Euler-Maclaurin Summation Formula*

Due by: September 6 by the end of the class.

In the class you will learn the Euler-Maclaurin Summation formula

$$\sum_{k=m+1}^{n-1} f(k) + \frac{f(m)+f(n)}{2} = \int_m^n f(x)dx + \int_m^n \left(x - \lfloor x \rfloor - \frac{1}{2}\right) f'(x)dx$$

for any integers $m \leq n-2$ and differentiable function $f(x)$.

[15] Stirling's Approximation:

[10] Tabulate (and/or plot)

$$\sum_{k=1}^{n} \log k$$

for a range of n (e.g., for n in the range $1 \leq n \leq 500$). Based on this numerical computation find a *good* approximation for

$$\log n! = \sum_{k=1}^{n} \log k$$

for large n up to a *constant* term (e.g., your computations may indicate that $\log n! \approx n\sqrt{n} + 3.2n + 4$; note that this is **not** the correct answer!)

[5] Use the Euler-Maclaurin formula to compute $\log n!$ up to a constant term.

[15] A Very Simple Sum:

[10] Tabulate (and/or plot)

$$S_n = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$

for a range of n. Based on this numerical computation find a *good* approximation for S_n.

[5] What answer gives you the Euler-Maclaurin formula? Justify your response.

Hints. You may want to know that:

$$\begin{aligned}
\int_1^\infty \left(x - \lfloor x \rfloor - \frac{1}{2}\right) \frac{dx}{x} &\approx -0.080894281, \\
\int_1^\infty \left(x - \lfloor x \rfloor - \frac{1}{2}\right) \frac{dx}{x^2} &\approx -0.077215333, \\
\int_1^\infty \left(x - \lfloor x \rfloor - \frac{1}{2}\right) \frac{dx}{x^3} &\approx -.07246695009, \\
\int_1^\infty \left(x - \lfloor x \rfloor - \frac{1}{2}\right) \frac{2x+1}{x^2(x+1)^2} &\approx -.056852656.
\end{aligned}$$