[40] **Homework 5**: *Big O, Ω.*

[12] Select the best "big Oh" notation for each expression. Justify by showing the constants c and n_0. Note that $f(n) = O(g(n))$ if there are constants $c > 0$ and $n_0 > 0$ so that for all $n \geq n_0$ we have $|f(n)| \leq c \cdot g(n)$.

1. $100n^2 + n$.
2. $(15n + \log n)^3$.
3. $3n^5 - 5n^2 - 100$.
4. $n^2 \log n + n + \sqrt{n} + \log n$.

[6] Show the following:

$$\begin{aligned} 5n^2 - n\log n &= \Theta(n^2) \\ \frac{n^2}{n\log^3 n + 1} &= O(n^3) \end{aligned}$$

[8] Justify that $n \log n + \sqrt{n}$ is **not** $O(n)$.

[14] We say that $f(n) \prec g(n)$ if $g(n)$ grows faster than $f(n)$ (e.g., $\log n \prec n$).

Order the following functions by by $\prec$ from the lowest to the highest:

$$\left(\frac{3}{2}\right)^n, \quad 100, \; n^2 \log n, \; 2^{\log_2 n}, \; \log^2 n, \; 2^{2\log_2 n}, \; 2^n.$$

Justify your answer.