

[50] **Homework 4. Proof Techniques**

Each problem is worth 10 points

[10] Show that  $\sqrt[3]{3}$  is irrational.

[10] Show that 3 divides  $n^3 + 2n$  whenever  $n$  is a nonnegative integer.

[10] Let  $A$  be a set of cardinality  $n$ . Let  $P(A)$  be the power set, that is, the set of *all* subsets of  $A$ . Prove by induction that cardinality of  $P(A)$  is  $2^n$ , that is,

$$|P(A)| = 2^n.$$

[10] Prove using induction on  $k$  that for any natural  $n$

$$\sum_{i=1}^n i^k \leq \frac{n^k(n+1)}{2}.$$

[10] Derive an explicit formula for the following recurrence for  $n \geq 1$

$$a_n = \frac{n}{2}a_{n-1}$$

with  $a_0 = 1$ .