

[30] **Homework 1.** *Basic Logic*

Each problem is worth 10 points

[10] Make truth tables for the following statements:

1. $\neg(p \rightarrow q) \rightarrow (q \rightarrow p)$
2. $(p \rightarrow \neg q) \wedge (\neg p \rightarrow q)$

[10] Using *logical equivalences* discussed in class prove that

$$(p \wedge q) \rightarrow (p \vee q)$$

is a tautology, that is, prove that

$$(p \wedge q) \rightarrow (p \vee q) \equiv T.$$

(**Hint:** Observe first that $p \rightarrow q \equiv \neg p \vee q$.)

[10] Determine the truth value of the following statements when x and y are real numbers:

- $\exists x \forall y \ (x = y^2)$;
- $\exists x \forall y \ (xy = 0)$;
- $\forall x \neq 0 \exists y \ xy = 1$;
- $\exists x \exists y \ (x + y \neq y + x)$.