Number Systems

Sec. 2.5

A) The Decimal System

- Used in everyday life to represent numbers
- based on 10 digits $\{0, 1, 2, ---, 9\}$
- Consider, for example, the number 9734 (Nine thousands, 7 hundreds, thirty four):

$$9734 = (9 \times 1000) + (7 \times 100) + (3 \times 10) + 4$$
$$= 9 \times 10^3 + 7 \times 10^2 + 3 \times 10 + 4 \times 10^0$$

* The base (radix) is 10 → each digit in the number is multiplied by 10 raised to a power corresponding to that digit's position.

- Same thing holds for decimal fractions but -ve powers of 10 are used.

$$0.485 = 4 \times 10^{-1} + 8 \times 10^{-2} + 5 \times 10^{-3}$$

- In general: $X = \ldots d_2 d_1 d_0 \bullet d_{-1} d_{-2} d_{-3}$

$$= \left(\sum_i d_i \times 10^i \right)_{10}$$

← notation to indicate deci

B) Other Number Systems $(\cdot)_r$

System	Radix	digits
Decimal	10	0, 1, 2, .., 9
Binary	2	0, 1
Octal	8	0, 1, 2, --, 7
Hexadecimal	16	0, 1, 2, --, 9, A, B, C, D, E, F

Ex. 1 Convert $(735)_8$ to decimal.

Sol$^{\underline{n}}$ $(735)_8 = 7 \times 8^2 + 3 \times 8 + 5 \times 8^0 = (477)_{10}$

Ex. 2 Convert $(A3.F2)_{16}$ to decimal.

Sol$^{\underline{n}}$ $(A3.F2)_{16} = 10 \times 16 + 3 \times 16^0 + 15 \times 16^{-1} + 2 \times 16^{-}$

$= (163.9453125)_{10}$

Ex. 3 Convert $(10110.011)_2$ to decimal.

Sol$^{\underline{n}}$ $(10110.011)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2 + 0 \times 2^0.$

$0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$

$= (22.375)_{10}$

C) Converting from Decimal to Binary:

$$(\cdot)_{10} \longrightarrow (\cdot)_2$$

- The binary system is used to represent numbe in the computer since it uses only 2 digits (0, 1 called bits) and therefore simpler to represent using 2 voltage levels (High, low). Besides it requires simpler circuits to manipul binary numbers.

- To convert a number from decimal to binary, the integer part is handled in a different way than the fraction part

$$X = (\underbrace{\ldots d_4 d_3 d_2 d_1 d_0}\, . \,\underbrace{d_{-1} d_{-2} d_{-3} \ldots}\,)_{1c}$$

repeated division by 2 and record the remainders in order of appearance from the decimal point and moving left

repeated multiplicat by 2 and recor the integer part from decimal pc and moving right

$(30)_{10} = \overset{4}{1}\overset{3}{1}\overset{2}{1}\overset{1}{1}\overset{0}{0}$

$$\begin{array}{r|l} 30 & \\ 15 & 0 \\ 7 & 1 \\ 3 & 1 \\ 1 & 1 \\ 0 & 1 \end{array}$$

$= 2^4+2^3+2^2+2^1$

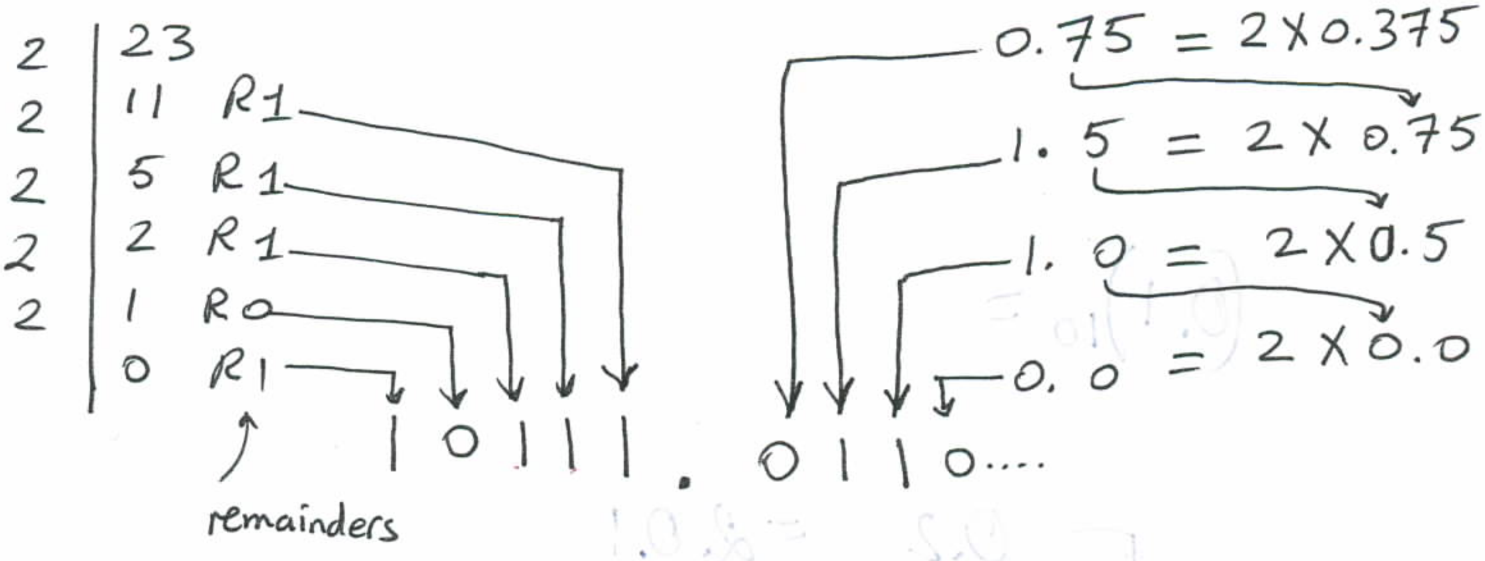
$= 16+8+4+2$

$= 30$

Ex. 4

Convert $(23.375)_{10}$ to binary.

Sol$^{\underline{n}}$



Thus, $(23.375)_{10} = (10111.011)_2$

Ex. 5

Convert $(9.2)_{10}$ to binary.

Sol$^{\underline{n}}$

$$\begin{array}{l|ll} 2 & 9 & \\ 2 & 4 & R1 \\ 2 & 2 & R0 \\ 2 & 1 & R0 \\ & 0 & R1 \end{array}$$

$$\begin{array}{l|l} 9 & \\ 4 & 1 \\ 2 & 0 \\ 1 & 0 \\ & 1 \end{array}$$

$0.4 = 2 \times 0.2$

$0.8 = 2 \times 0.4$

$1.6 = 2 \times 0.8$

$1.2 = 2 \times 0.6$

$0.4 = 2 \times 0.2$

⋮

it repeats afterwards

$\therefore \quad (9.2)_{10} = (1001.\overline{0011})_2$

Ex. 6 (Used to be a job interview question at a major software company).

Consider the following program segment:

```
num = 0.0 ;
for i = 1 to 10 do
    num = num + 0.1 ;
if (num = 1)
    print ("OK");
else
    print ("Not OK");
```

$0.2 = 2 \cdot 0.1$

$0.4 = 2 \cdot 0.2$

$0.8 = 2 \cdot 0.4$

$1.6 = 2 \cdot 0.8$

$1.2 = 2 \cdot 0.6$

$= 2 \cdot 0.$

$(0.1)_{10} = .\overline{00011}$

What should the program print?

Solⁿ The program prints "Not OK". The reason is that the fraction part $(0.1)_{10}$ has no exact binary representation (see previous example) and is subject to truncation error due to finite precision. Adding $(0.1)_{10}$ ten times will therefore result in a value $< (1.0)_{10}$

D) Operations in Binary Number System

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0 \\ +\ 1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ 1 \\ +\ 1 \\ \hline \text{carry}\ 0 \end{array}$$

$$\begin{array}{r} 0 \\ -\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ -\ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ -\ 1 \\ \hline 0 \end{array} \qquad \begin{array}{r} \text{borrow} \rightarrow \overset{10}{\cancel{0}} \\ -\ 1 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 0 \\ \times\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ \times\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0 \\ \times\ 1 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ \times\ 1 \\ \hline 1 \end{array}$$

Ex. 7 Carry out the following operations in binary system:

i)
$$\begin{array}{r} {}^{1\,1} \\ 1011.01 \\ +\ 110.1 \\ \hline 10001.11 \end{array}$$

16 4 1

$$\begin{array}{l} \overset{8\ 2\ \ 2\ 1}{1011} = 11 \\ \underline{6} \\ 17 \end{array}$$

ii) 100101 − 1011

$$\begin{array}{r} \overset{0}{\cancel{1}}\;\overset{\overset{1}{\cancel{10}}}{\cancel{0}}\;\overset{10}{\cancel{0}}\;\overset{0}{\cancel{1}}\;\overset{10}{\cancel{0}}\;1 \\ -\qquad 1\;0\;1\;1 \\ \hline 1\;1\;0\;1\;0 \end{array}$$

$$\begin{array}{r} \overset{\overset{1}{\cancel{10}}}{\cancel{1}}\;\overset{10}{0}\;0\;\overset{10}{\cancel{1}}\;0\;1 \\ 1\;0\;1\;1 \\ \hline 1\;1\;0\;1\;0 \end{array}$$

$$\begin{array}{r} \overset{\overset{1}{10}}{\cancel{1}}\;\overset{10}{0}\;\overset{0}{0}\;\overset{10}{\cancel{1}}\;0\;1 \\ -\qquad 1\;0\;1\;1 \\ \hline 1\;1\;0\;1\;0 \end{array}$$

iii) 11101 X 101

$$\begin{array}{r} 1\;1\;1\;0\;1 \\ \times\qquad 1\;0\;1 \\ \hline 1\;1\;1\;0\;1 \\ 0\;0\;0\;0\;0\; \\ +\quad 1\;1\;1\;0\;1\;\; \\ \hline 1\;0\;0\;1\;0\;0\;0\;1 \end{array}$$

iv) $10010001 \div 101$

$$
\begin{array}{r}
11101 \\
101\;\overline{)\,10010001} \\
\underline{1001} \\
-\;101 \\
\hline
1000 \\
-\;101 \\
\hline
0110 \\
-\;101 \\
\hline
101 \\
101 \\
\hline
0
\end{array}
$$

$$
\begin{array}{r}
1\;\;\;1 \\
1\;\;\;10 \\
0\;10\;10\;10 \\
1000 \\
101 \\
\hline
011
\end{array}
$$

Thus, $10010001 \div 101 = 11101$

$$
\begin{array}{r}
10\;11 \\
11\cancel{1}1 \\
999 \\
\hline
12
\end{array}
$$

$$
\begin{array}{r}
1\;\;1\;\;10 \\
10\;10\;10 \\
1000 \\
101 \\
\hline
011
\end{array}
\qquad\qquad
\begin{array}{r}
1 \\
10\;\;1\;\;10 \\
1000 \\
101 \\
\hline
0110
\end{array}
$$