“Things to Know...”

**Existential Quantifier:** There exists an \( x \) such that \( P(x) \) is true.

\[
(\exists x) P(x)
\]

**Universal Quantifier:** For all \( x \), \( P(x) \) is true.

\[
(\forall x) P(x)
\]

**Binomial Coefficient:** The number of \( m \)-combinations of a set of \( n \) distinct objects

\[
\binom{n}{m} = \frac{n!}{m!(n-m)!}
\]

**Product:**

\[
\prod_{i \leq j \leq k} f(j) = f(i) \cdot f(i+1) \cdot \ldots \cdot f(k)
\]

**Sum:**

\[
\sum_{i \leq j \leq k} f(j) = f(i) + f(i+1) + \ldots + f(k)
\]

**Logarithm Definition**

For \( x > 0 \), \( \log x = y \) \( \iff \) \( 2^y = x \)

**Properties:**

**L1:** Log is a strictly increasing function:

\[
(\forall x, y)(x > y) \Rightarrow (\log x > \log y)
\]

**L2:** Log is a one-to-one function:

\[
(\forall x, y)(\log x = \log y) \Rightarrow (x = y)
\]

**L3:** \( \log 1 = 0 \)

**L4:**

\[
(\forall a) \log_2 2^a = a
\]

**L5:**

\[
(\forall x, y) \log(x * y) = \log x + \log y
\]

**L6:**

\[
(\forall x, a) \log x^a = a * \log x
\]

**L7:**

\[
(\forall x, y) x^{\log y} = y^{\log x}
\]

**L8:**

\[
\log_b N = \frac{\log_e N}{\log_e b}
\]

**summation Properties:**
L1: Let $f$ be any function, then
\[ \sum_{i=a}^{b} cf(i) = c \sum_{i=a}^{b} f(i) \]

L2:
\[ \sum_{i=a}^{b} [f(i) + g(i)] = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i) \]

L3:
\[ \sum_{i=a}^{b} f(i) = \sum_{i=a}^{b} f(a + b - i), \ a < b \]

L4:
\[ \sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2} \]

Proof by induction:
Base Step: for $n = 1$,
\[ \sum_{i=1}^{1} i = \frac{1(1+1)}{2} = 1 \]

Induction Hypothesis: Assume that for some $k > 0$,
\[ (\forall n \leq k) \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]

Inductive Step:
\[ \sum_{i=1}^{n+1} i = \left( \sum_{i=1}^{n} i \right) + (n+1) = \frac{n(n+1)}{2} + n + 1 = \frac{(n+1)(n+2)}{2}. \]

L5:
\[ \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \]

L6:
\[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \ n \leq 1 \]

Proof by Induction:
Base Step: for $n = 1$,
\[ \sum_{i=1}^{1} i^2 = \frac{1(1+1)(2+1)}{6} = 1 \]

Inductive Step: Assume for some $k > 0$ that
\[ (\forall n \leq k) \sum_{i=1}^{n} i^2 = \frac{n(n+1)(n+2)}{6} \]
then
\[ \sum_{1 \leq i \leq n+1} i^2 = \sum_{1 \leq i \leq n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2(n+1)+1)}{6}. \]