[40] **Homework 5**: *Big O, Ω.*

[10] Select the best "big Oh" notation for each expression. Justify by showing the constants c and n_0. Note that $f(n) = O(g(n))$ if there are constants $c > 0$ and $n_0 > 0$ so that for all $n \geq n_0$ we have $|f(n)| \leq c \cdot g(n)$.

1. $100n + 1$.
2. $(10n + 1)^4$.
3. $3n^3 - 5n^2 - 500$.
4. $n^2 + n + \sqrt{n} + \log^2 n$.

[10] Show the following:

$$\begin{aligned} 6n^2 - 2n &= \Theta(n^2) \\ \frac{6n^2}{\log^3 n + 1} &= O(n^3) \\ 3n^3 + 44n^2 &= \Omega(n^2) \end{aligned}$$

[10] Is $(\log n)^3 = O(\log n^3)$? Justify your answer?

[10] We say that $f(n) \prec g(n)$ if $g(n)$ grows faster than $f(n)$ (e.g., $\log n \prec n$).

Order the following functions by by $\prec$ from the lowest to the highest:

$$\left(\frac{3}{2}\right)^n, \quad 100, \; n^3 \log^2 n, \; 2^{\log_2 n}, \; \log^4 n, \; 2^{3\log_2 n}, \; 2^n, n!, n^n.$$

Justify your answer.