[50] **Homework 4. Proof Techniques**

Each problem is worth 10 points

[10] Show that $\sqrt[3]{3}$ is irrational.

[10] Let $A$ be a set of cardinality $n$. Let $P(A)$ be the power set, that is, the set of all subsets of $A$. Prove by induction that cardinality of $P(A)$ is $2^n$, that is, $|P(A)| = 2^n$.

[10] Prove by induction on $n \geq 1$

$$\sum_{i=1}^{n} i \cdot i! = (n + 1)! - 1.$$  

[10] The harmonic number $H_n$ is defined as for $n \geq 1$

$$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$  

Prove by induction that $H_{2^n} \geq 1 + \frac{n}{2}$

whenever $n$ is a nonnegative natural number.

[10] Derive an explicit formula for the following recurrence for $n \geq 2$

$$a_n = \frac{n - 1}{3} a_{n-1}$$

with $a_1 = 1$. 