

[50] **Homework 4. Proof Techniques**

Each problem is worth 10 points

[10] Show that $\sqrt[3]{3}$ is irrational.

[10] Prove that for positive integer n , $n^3 + 2n$ is divisible by 3.

[10] Using mathematical induction prove that

$$\sum_{i=1}^n i2^i = 2^{n+1}(n-1) + 2.$$

[10] Define the n th *Harmonic number* as

$$H_n = \sum_{i=1}^n \frac{1}{i}.$$

Prove by induction on $n \geq 0$ that

$$H_{2^n} \geq 1 + \frac{n}{2}.$$

[10] Derive an explicit formula for the following recurrence for $n \geq 1$

$$a_n = \frac{n}{2}a_{n-1}$$

with $a_0 = 1$.