[30] Homework 3. *Programming Assignment: Numerical Evaluation of Certain Sums*

The goal of this assignment is to find good numerical approximations to certain sums. We need first a few definitions. For nonnegative integers $0 \le k \le n$ define

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where $n! = 1 \cdot 2 \cdots n$.

[15] **Stirling's Approximation**:

Tabulate and plot

$$I_n = \sum_{k=1}^{n} \ln k$$

for a range of n (e.g., $1 \le n \le 500$). Based on this numerical computation find a *good* a approximation formula for

$$\ln n! = \sum_{k=1}^{n} \ln k$$

for large n up to a $\log n$ term, if possible. That is, your answer may look like this

$$\log n! \approx^{?} n^2 \log n + n + 3 \log n.$$

Plot your approximation on the same graph as I_n.

[15] **An Interesting Sum**:

Tabulate and plot

$$S_n = \sum_{k=1}^{n} \binom{n}{k} 2^{-n} \log_2 \binom{n}{k}$$

for $1 \le n \le 500$. Based on this numerical computation find a *good* approximation of S_n for large n, as in the previous case, and plot it together with S_n.