1. • We first prove $A \subseteq (A - B) \cup (A \cap B)$. For every $x \in A$, there are two cases:
   (a) $x \in B$. Therefore we have $x \in A$ and $x \in B$, which is $x \in (A \cap B)$. Thus $x \in (A - B) \cup (A \cap B)$.
   (b) $x \notin B$. Therefore we have $x \in A$ and $x \notin B$, which is $x \in (A - B)$. Thus $x \in (A - B) \cup (A \cap B)$.

   • Then we prove $(A - B) \cup (A \cap B) \subseteq A$. For every $x \in (A - B) \cup (A \cap B)$, we know x belongs to $(A - B)$ or $(A \cap B)$ or both. Let's consider two cases (note that they are not mutually exclusive):
   (a) $x \in (A - B)$. By definition of difference, we know $x \in A$.
   (b) $x \in (A \cap B)$. By definition of intersection, we know $x \in A$.

■

2. (a) $x \equiv y \mod 11$ is reflexive, symmetric and transitive, but not antisymmetric.
   - reflexive: $\forall x(x \equiv x \mod 11)$ since $\forall x(x - x = 0 \cdot 11)$.
   - symmetric: $\forall x \forall y$ such that $x \equiv y \mod 11$, we know $\exists k(x - y = k \cdot 11)$. Therefore, we have $y - x = -k \cdot 11$, which means $y \equiv x \mod 11$.
   - not antisymmetric: $1 \equiv 12 \mod 11$ and $12 \equiv 12 \mod 11$, but $1 \neq 12$.
   - transitive: $\forall x \forall y \forall z$ such that $x \equiv y \mod 11$ and $y \equiv z \mod 11$, we know $\exists k_1(x - y = k_1 \cdot 11)$ and $\exists k_2(y - z = k_2 \cdot 11)$. Then we have $x - z = (k_1 + k_2) \cdot 11$, and hence $x \equiv z \mod 11$.

   So it is an equivalence relation, the equivalence classes are:

$$
\begin{aligned}
[0] &= \{11k : k \in \mathbb{Z}\} \\
[1] &= \{11k + 1 : k \in \mathbb{Z}\} \\
[2] &= \{11k + 2 : k \in \mathbb{Z}\} \\
[3] &= \{11k + 3 : k \in \mathbb{Z}\} \\
[4] &= \{11k + 4 : k \in \mathbb{Z}\} \\
[5] &= \{11k + 5 : k \in \mathbb{Z}\} \\
[6] &= \{11k + 6 : k \in \mathbb{Z}\} \\
[7] &= \{11k + 7 : k \in \mathbb{Z}\} \\
[8] &= \{11k + 8 : k \in \mathbb{Z}\} \\
[9] &= \{11k + 9 : k \in \mathbb{Z}\} \\
[10] &= \{11k + 10 : k \in \mathbb{Z}\}
\end{aligned}
$$

   It is not a partial order relation.

   (b) $xy \geq 3$ is symmetric, but not reflexive, antisymmetric or transitive.
   - not reflexive: $0 \cdot 0 \ngeq 3$.
   - symmetric: $\forall x \forall y(xy = yx)$, so $\forall x \forall y(xy \geq 3 \to yx \geq 3)$.
   - not antisymmetric: $2 \cdot 3 \geq 3 \wedge 3 \cdot 2 \geq 3$ but $2 \neq 3$.
   - not transitive: Let $x = 1$, $y = 3$ and $z = 1$, so we have $xy \geq 3$ and $yz \geq 3$, but $xz \ngeq 3$.

   It is neither a equivalence relation nor a partial order relation.

(c) $x = y^2$ is antisymmetric, but not reflexive, symmetric or transitive.

- not reflexive: $4 \neq 4^2$.
- not symmetric: $4 = 2^2$, but $2 \neq 4^2$.
- antisymmetric: if $x = y^2$ and $y = x^2$, then $y = y^4$, which gives us two possible values of y, 0 and 1. We can verify that when $y = 1$, $x = 1$ and when $y = 0$, $x = 0$. So $x = y$.
- not transitive: $16 = 4^2$ and $4 = 2^2$, but $16 \neq 2^2$.

It is neither a equivalence relation nor a partial order relation.

■

3. (a) $f(x) = x^5$ is a bijection, as we can construct its inverse $f^{-1}(y) = y^{1/5}$.

   (b) $f(x) = \cos^2(x)$ is not a bijection, since it's not injective: $f(0) = f(\pi)$ but $x \neq \pi$.

   (c) $f(x) = \frac{x+1}{x+5}$ is a bijection, because we can construct its inverse $f^{-1}(y) = \frac{5y-1}{1-y}$.

■

4. (a) $g^{-1}(\{0\}) = \{x : 0 \leq x < 1\}$

   (b) $g^{-1}(\{x : 0 < x < 1\}) = \varnothing$

■

5. (a)

$$\begin{aligned}\sum_{i=1}^{1000} 3^i &= \sum_{i=1}^{999} 3^{i+1} \\ &= 3 \times \frac{3^{1000} - 1}{3 - 1} \\ &= \frac{3^{1001} - 3}{2}\end{aligned}$$

   (b)

$$\begin{aligned}\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j) &= (1+1) + (1+2) + (1+3) \\ &\quad + (2+1) + (2+2) + (2+3) \\ &= 21\end{aligned}$$

   (c)

$$\begin{aligned}\sum_{j=0}^{100}(3^j - 2^j) &= \sum_{j=0}^{100} 3^j - \sum_{j=0}^{100} 2^j \\ &= \frac{3^{101} - 1}{2} - \frac{2^{101} - 1}{1} \\ &= \frac{3^{101} + 1}{2} - 2^{101}\end{aligned}$$

■