Homework 1. Basic Logic

1a. $p \vee (\overline{r \vee q}), \overline{r \vee q} = \neg(r \vee q) = \neg r \wedge \neg q$.

Table 1: 1a

p	q	r	$r \vee q$	$\neg(r \vee q)$	$p \vee (\overline{r \vee q})$
T	T	T	T	F	T
T	T	F	T	F	T
T	F	T	T	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	T

1b.

Table 2: 1b

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

2.

$$
\begin{aligned}
&(p \wedge q) \rightarrow (p \vee q) && \\
\equiv&\neg(p \wedge q) \vee (p \vee q) && Table\ 7 \\
\equiv&(\neg p \vee \neg q) \vee (p \vee q) && De\ Morgan's\ laws \\
\equiv&(\neg p \vee p) \vee (\neg q \vee q) && Associative\ laws \\
\equiv&T \vee T && Negation\ laws \\
\equiv&T && Domination\ laws
\end{aligned}
$$

3a. $P(x, y) : x + y = 5$ where x, y are positive integers. $\forall x \forall y P(x, y)$. This statement is false.
Choose $x = 1, y = 1$, then we have,
$1 + 1 \geq 5$
$2 \geq 5$
which is false.
3b. $\forall x \exists y P(x, y)$. This statement is true.

We can translate this as, for all x, there is some y, such that $x + y \geq 5$. Let $x = 1$, which is the min possible value. If we choose $y \geq 4$, then this statement holds as it only needs to be true for some x.

4. Pushing our negation all the way through we have,

$$
\begin{aligned}
&\neg\forall x \exists y P(x, y) \\
\equiv &\exists x \neg\exists y P(x, y) \\
\equiv &\exists x \forall y \neg P(x, y)
\end{aligned}
$$

Hence, it follows that just (c) is equivalent.