Towards Algorithmic Synthesis of Synchronization for Shared-Memory Concurrent Programs

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Problem Definition

Given a shared-memory concurrent program P, composed of unsynchronized processes $P_1, \ldots, P_k$, and a (branching-time) temporal logic specification ϕ_{spec} such that $P \not\models \phi_{spec}$, automatically generate synchronized processes $P_1^s, \ldots, P_k^s$ such that $P^s \models \phi_{spec}$.

Example

P :

$x : \{0, 1, 2\}$ with $x = 1$

$P_1 \parallel P_2$

P_1 :

```
L1:   while (x < 2)
L2:       x := x + 1;
L3:   while true;
```

P_2 :

```
T1:   while (x > 0)
T2:       x := x - 1;
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```

$$\phi_{spec} : AF(L_3 \wedge T_3 \wedge (x = 0 \vee x = 2))$$

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$x : \{0, 1, 2\}$ `with` $x = 1$

$P_1 \;\|\; P_2$

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Q&A

- Why shared-memory concurrent programs?
  - Ubiquitous
  - Hard to write
  - Harder to verify
- Why temporal logic specifications? Why branching-time?
  - Terminating, ongoing programs
  - Safety, liveness
  - Expressivity
- Why synthesize only the synchronization?
  - Trickiest part
  - Simple specifications, tractable

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A Vocabulary L

- L-symbols
  - Variable symbols (x)
  - Function, constant symbols (1, 2, $+$, $-$)
  - Predicate, proposition symbols ($\mathtt{L}_1$, $\mathtt{T}_3$, $>$, $<$)
  - *Sort* symbols (`bool`, `int0to2`)
  - Distinguished equality predicate symbol $=$
- (Sorted) L-terms and L-atoms:
  - L-term (t): v or $f(t_1, \ldots, t_m)$ (x, 1, $x + 1$)
  - L-atom(G): $B(t_1, \ldots, t_m)$ or $t_1 = t_2$ ($x > 1$, $\mathtt{L}_1$, $x = 1$)
  - Sorts mapped to nonempty domains ($\mathtt{bool} \rightarrow \{\mathtt{T}, \mathtt{F}\}$, $\mathtt{int0to2} \rightarrow \{0, 1, 2\}$)
  - Symbols, terms, atoms mapped to values in appropriate domains.

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(Finite-state) Concurrent Programs over **L**

- Syntax:
  - P ::= vardeclaration process || process
  - vardeclaration ::= v_1 : $domain_1, \ldots, v_n$: $domain_n$
  - process ::= localvardeclaration body
  - localvardeclaration ::= vardeclaration
  - body ::= labeledstmt ; body | labeledstmt
  - labeledstmt ::= LOC: atomicstmt
  - atomicstmt ::= assignment | conditiontest | goto | CCR
  - assignment ::= ⟨ v:=t ⟩
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CTL-like Specifications over **L** (**L**CTL)

- Syntax: $\phi ::= \mathbf{L}\text{-}atom \mid \neg\phi \mid \phi \vee \phi \mid \mathsf{EX}\,\phi \mid \mathsf{EX}_i\,\phi \mid \mathsf{A}[\phi\,\mathsf{U}\,\phi] \mid \mathsf{E}[\phi\,\mathsf{U}\,\phi]$
- Semantics:
  Defined from semantics of **L**-atoms, propositional and temporal operators

Revisiting Example

P :
$x : \{0, 1, 2\}$ `with` $x = 1$
$P_1 \parallel P_2$

P_1 :

```
L1:  ⟨ if (x < 2) L2, L4 ⟩;
L2:      ⟨ x := x + 1 ⟩;
L3:      ⟨ goto L1 ⟩;
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$$AF(L_4 \wedge T_4 \wedge (x = 0 \vee x = 2))$$

Synthesis Algorithm Sketch

1. Formulate an **L**CTL formula ϕ_P for semantics of P.
2. Construct a tableau T_ϕ for $\phi : \phi_P \wedge \phi_{spec}$.
   If T_ϕ is empty, declare specification as unsatisfiable.
3. If T_ϕ is non-empty, extract a model M for ϕ from it.
4. Decompose M to obtain CCRs to synchronize each process.
5. Compile CCRs into lower level synchronization primitives ([EmeSam2011])

Formulation of ϕ_P

P :
$x : \{0, 1, 2\}$ with $x = 1$
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P$_1$:

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```

P$_2$:

```
T1:  ⟨ if (x > 0) T2, T4 ⟩;
T2:       ⟨ x := x - 1 ⟩;
T3:       ⟨ goto T1 ⟩;
T4:  ⟨ goto T4 ⟩;
```

- At any step, only one process moves
  - $AG(L_1 \Rightarrow AX_2(L_1)) \wedge \ldots$

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- A process is always in exactly one of its locations
  - $AG(L_1 \lor L_2 \lor L_3 \lor L_4)$
  - $AG(L_1 \Rightarrow \neg(L_2 \lor L_3 \lor L_4))$
  - ...

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```

- Some process always moves.
  - AG(EX *true*)

Formulation of ϕ_P

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- Initial condition
  - $L_1 \wedge T_1 \wedge x = 1$

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T2:      ⟨ x := x - 1 ⟩;
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```

- Local process execution
  - $AG((L_1 \wedge x < 2) \Rightarrow AX_1(L_2))$
  - $AG((L_1 \wedge x \geq 2) \Rightarrow AX_1(L_4))$
  - $AG(\bigwedge_{c \in \{0,1,2\}}(L_2 \wedge x = c) \Rightarrow AX_1(L_3 \wedge x = c + 1))$
  - $AG(L_3 \Rightarrow AX_1(L_1))$
  - ...

Construction of Tableau T_ϕ

- Tableau basics:
  - Finite, rooted, directed AND/OR graph
  - Nodes labeled with formulas
  - OR-node formulas valid iff formulas in some AND-node succ. valid
  - AND-node formulas valid iff formulas in all OR-node succ. valid
  - Assume ability to evaluate **L**-atoms and **L**-terms
- Algorithm Sketch:
  - Let root node of T_ϕ be OR-node, labeled ϕ
  - Repeat until no new nodes can be added:
    Add appropriate AND-node succ. of OR-nodes and OR-node succ. of AND-nodes, merging *equivalent* nodes
  - Prune *inconsistent* nodes to get final T_ϕ

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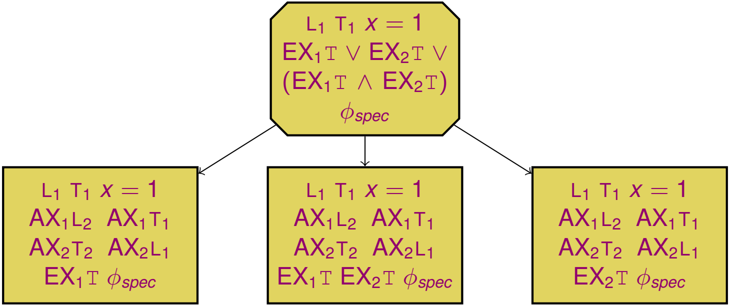

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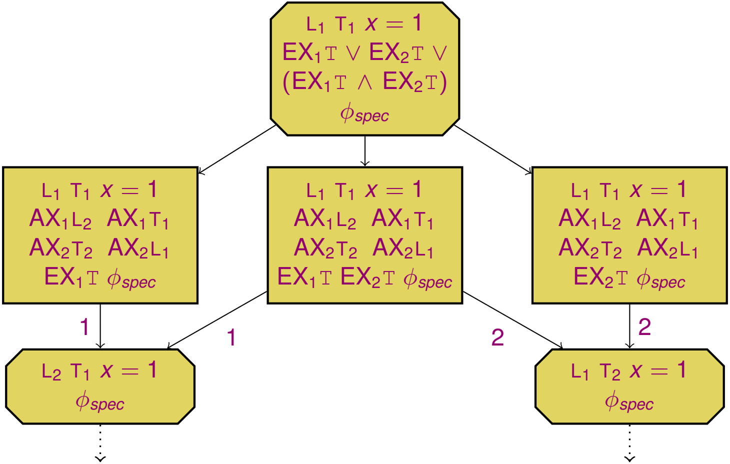

Extraction of Model M from T_ϕ

- Model-fragment:
  - Rooted DAG of AND-nodes in T_ϕ
  - All eventualities in root node fulfilled
- Identify model fragments rooted at each AND-node of T_ϕ
- If multiple model fragments, pick one of *minimal size*
- Join model fragments to get model

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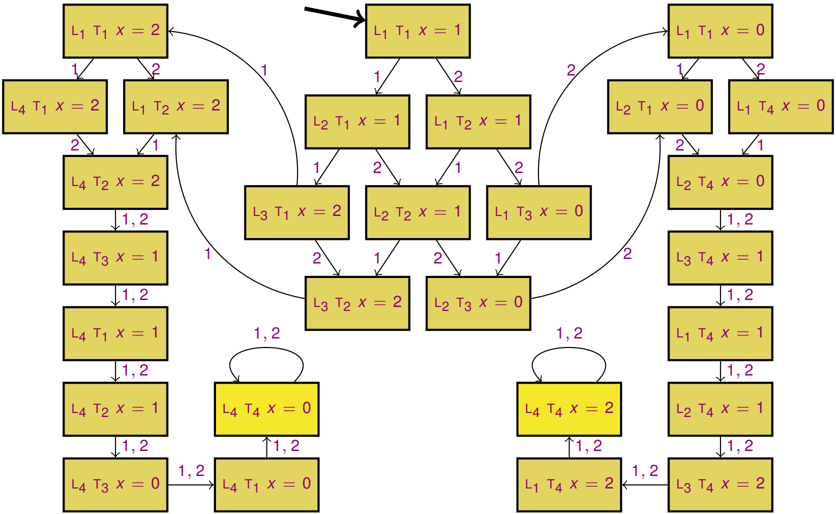
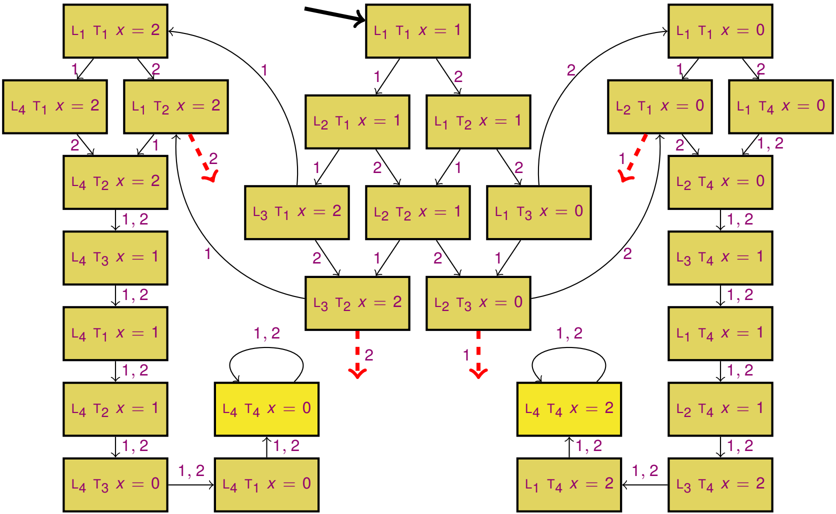


Obtaining P_1^s, P_2^s from M



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Assumptions, Correctness, Complexity

[Assumptions]:

- P_1, P_2 are finite-state programs over **L**
- All program variables are shared variables, initialized to specific values
- **L**-atoms and **L**-terms can be evaluated

[Soundness]:
Given unsynchronized processes P_1, P_2 and an **L**CTL formula ϕ_{spec}, if the algorithm generates P^s, then $P^s \models \phi_{spec}$.

[Completeness]:
Given unsynchronized processes P_1, P_2 and an **L**CTL formula ϕ_{spec}, the algorithm generates P^s such that $P^s \models \phi_{spec}$ if $\phi = \phi_{spec} \wedge \phi_P$ is satisfiable.

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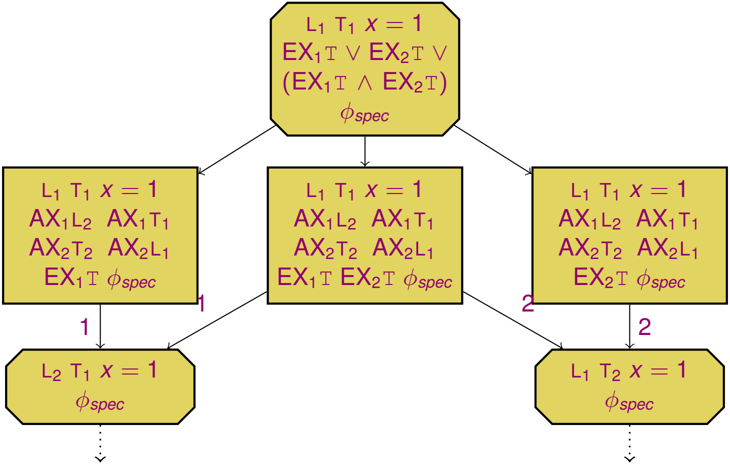

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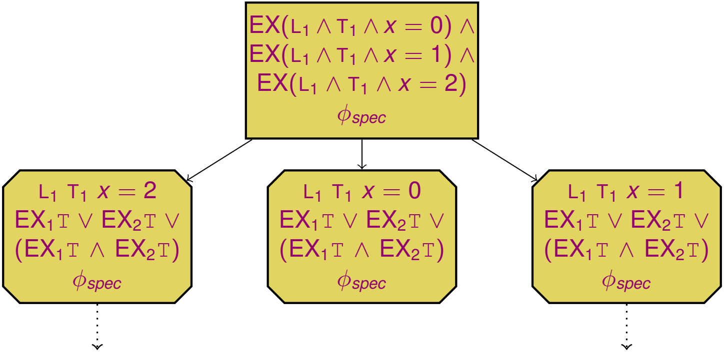

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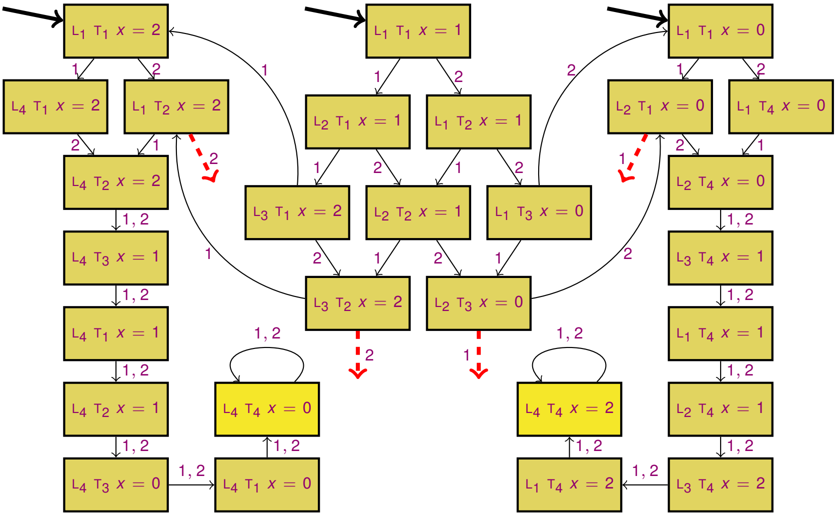

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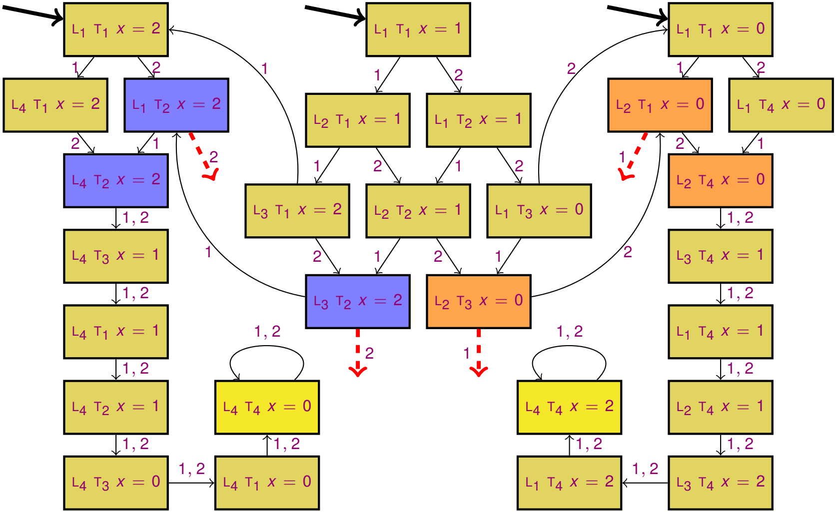

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l : bool with $l = 1$
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Related Work

- Inference of high-level synchronization, guarded commands: [EC82, VYY09]
- Synthesis of low-level synchronization for relaxed memory models: [KVV10]
- Sketching: [S-LRBE05]
- Open systems [PR89]

Ongoing Work

- Efficient tableau construction?
- Infinite-state programs?
- Pleasant guards?
- Other synchronization primitives?