Constraint-based Search I

CS560: Reasoning About Programs

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Partly based on slides by Armando Solar-Lezama

Roadmap

Previously

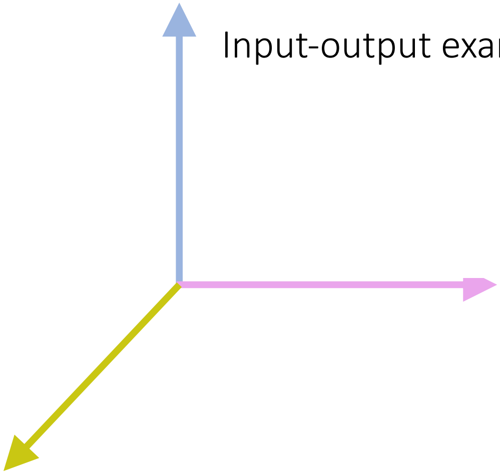
- Inductive synthesis
- CFGs (SyGuS) and DSLs (FlashFill)
- Enumerative search and Representation-based search

Today

- Inductive synthesis ➜ Functional synthesis
- Generators (Sketch)
- Constraint-based Search

User intent

Input-output examples



Search strategy

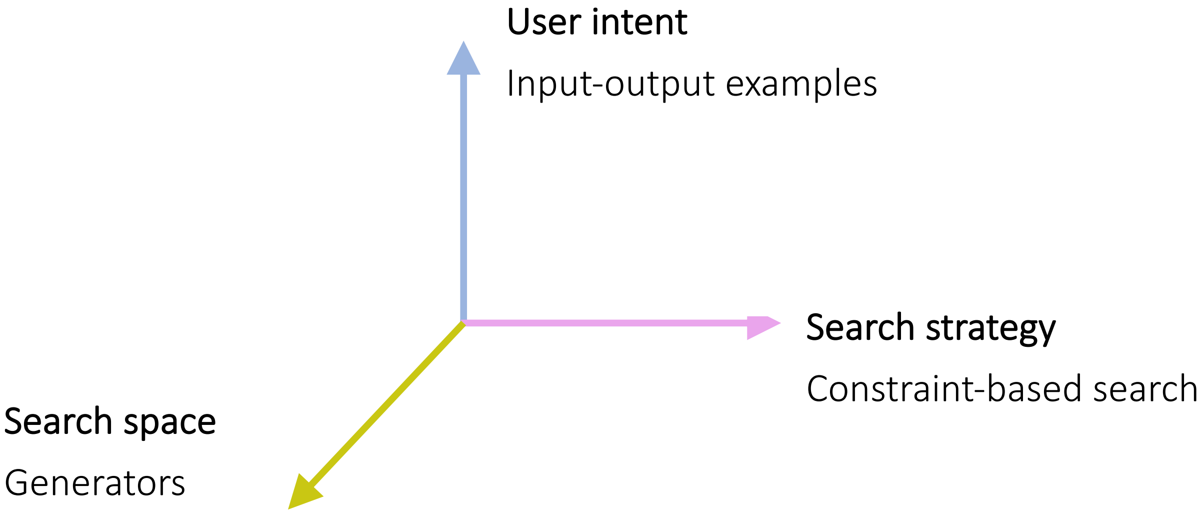
Enumerative search

Representation-based (version space algebras)

Search space

Grammars

DSLs


User intent
Input-output examples
Search strategy
Constraint-based search
Search space
Generators

Constraint-based search: Sketch

Key idea 1:

- Search as “curve fitting”
- “curve” is a parametric family of programs

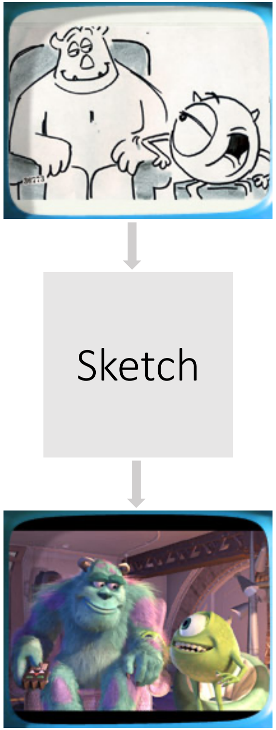
Key idea 2:

- Define a language to describe parametric programs

Key idea 3:

- “Solve” instead of search

sketch



implementation
(completed sketch)

- Sketch: a language for parametric programs
- Turning synthesis problems into constraints
- Efficient constraint solving

Inductive
↓
Functional

- Sketch: a language for parametric programs
- Turning synthesis problems into constraints
- Efficient constraint solving

Language design strategy

Extend base language with *one* construct
Unknown constant hole: ??

Type inferred from context

```
int bar (int x)
{
    int t = x * ??;
    assert t == x + x;
    return t;
}
```

Synthesizer replaces ?? with an integer constant

```
int bar (int x)
{
    int t = x * 2;
    assert t == x + x;
    return t;
}
```

High-level constructs defined in terms of ??

Sketch is bounded!

```
harness void test()
{
    bar(2);
    bar(10);
    bar(1);
}
```

+

```
int bar (int x)
{
    int t = x * ??;
    assert t == x + x;
    return t;
}
```

↓

```
int bar (int x)
{
    int t = x * 2;
    assert t == x + x;
    return t;
}
```

Assertion validity is modulo test harness

Inductive synthesis: no inputs to harness

Integer holes → sets of expressions

Expressions with `??` = sets of expressions

- linear expressions `x*?? + y*??`
- polynomials `x*x*?? + x*?? + ??`
- sets of variables `?? ? x : y`

Example: Least significant zero bit

0010 0101 → 0000 0010

0010 0000 → 0000 0001

```
harness void test()
{
    assert isolateSk(00100101) = 00000010;
    assert isolateSk(00100000) = 00000000;
}
```

Example: Least significant zero bit

0010 0101 → 0000 0010

0010 0000 → 0000 0001

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harness void test()
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    assert isolateSk(00100101) = 00000010;
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}
```

Trick for an optimized program:

Adding 1 to a string of ones turns the next zero to a one:

000111 + 1 = 001000

```
bit[W] isolateSk (bit[W] x) {

    int t = !(x + ??) & (x + ??) ;
    return t;
}
```

Example: Least significant zero bit

0010 0101 → 0000 0010

0010 0000 → 0000 0001

```
harness void test()
{
    assert isolateSk(00100101) = 00000010;
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Trick for an optimized program:

Adding 1 to a string of ones turns the next zero to a one:

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```
bit[W] isolateSk (bit[W] x) {

    int t = !(x + ??) & (x + ??) ;
    return t;
}
```

`!(x + ??) & (x + ??)` →

`!(x + 1) & (x + 0)`	`!(x + 0) & (x + 0)`
`!(x + 0) & (x + 1)`	`!(x + 1) & (x + 1)`

Integer holes → sets of expressions

Expressions with `??` = sets of expressions

- linear expressions `x*?? + y*??`
- polynomials `x*x*?? + x*?? + ??`
- sets of variables `?? ? x : y`

Semantically powerful but syntactically clunky

Regular Expressions are a more convenient way of defining sets

Regular expression generators (syntactic sugar)

- `{| RegExp |}`
- RegExp supports choice ‘|’ and optional ‘?’
  - can be used arbitrarily within an expression
    - to select operands `{| (x | y | z) + 1 |}`
    - to select operators `{| x (+ | -) y |}`
    - to select fields `{| n(.prev | .next)? |}`
    - to select arguments `{| foo( x | y, z) |}`
- Set must respect the type system
  - all expressions in the set must type-check
  - all must be of the same type

Least significant zero bit revisited

How did I know the solution would take the form `!(x + ??) & (x + ??)`

What if all you know is that the solution involves `x, +, & and !`

```
bit[W] tmp=0;
{| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
{| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
return tmp;
```

tmp = !x
x = tmp + x

This is now a set of statements
(and a really big one too)

Sets of statements

Statements with holes = sets of statements

Higher level constructs for statements too: repeat

```
bit[W] tmp=0;
repeat(3){
 {| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
}
return tmp;
```

repeat

Avoid copying and pasting

- `repeat(n){ s}` ➔ `s;s;…s;` (n copies)
- each of the n copies may resolve to a distinct stmt
- n can be a hole too.

```
bit[W] tmp=0;
repeat(??){
    {| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
}
return tmp;
```

The synthesizer won't try to minimize n
Use --unrollamnt to set the maximum value of n

Procedures and Sets of Procedures

Two types of procedures

- standard procedures
  - represents a single procedure
  - all call sites resolve to the same procedure
  - identified by the keyword **static**
- generators
  - represents a set of procedures
  - each call site resolves to a different procedure in the set
  - **can recursively define arbitrary families of programs**
  - default in the Sketch implementation

Generators are very expressive!

Example: Least significant zero bit

```
generator bit[W] gen(bit[W] x, int bnd){
    assert bnd > 0;
    if(??) return x;
    if(??) return ??;
    if(??) return !gen(x, bnd-1);
    if(??){
      return {| gen(x, bnd-1) (& | +) gen(x, bnd-1) |}; }
}
```

```
bit[W] tmp=0;
repeat(??){
    {| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
}
return tmp;
```

High order generators

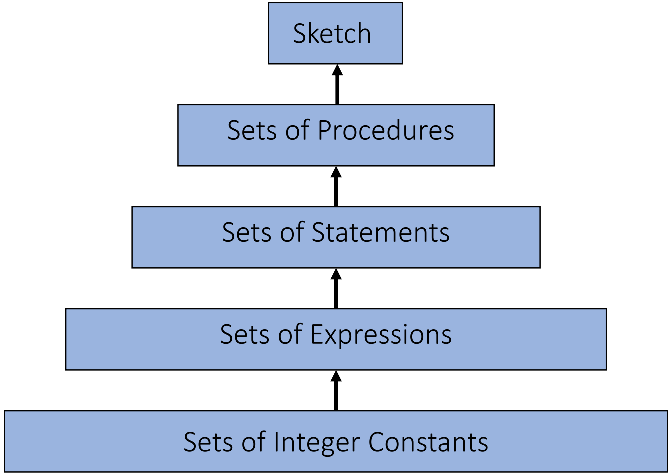
```
/*
 * Generate code from f n times
 */
generator void rep(int n, fun f){
    if(n>0){
        f();
        repeat(n-1, f);
    }
}
```

Program space – Contrast with SyGuS?

special notation

compiled

parametric family



Defining sets of code fragments is the key to Sketching effectively

- Overview of the Sketch language
- Turning synthesis problems into constraints
- Efficient constraint solving

Step 1: Turn holes into special inputs

The ?? operator is modeled as a special control input

```
bit[W] isolSk(bit[W] x)
{
    return ~(x + ??) & (x + ??);
}
```

→

```
bit[W] isolSk(bit[W] x, bit[W] c1, c2)
{
    return ~(x + c1) & (x + c2);
}
```

Bounded candidate spaces are important

- bounded unrolling of `repeat` is important
- bounded inlining of generators is important

Step 2: Constraining the set of controls

- Correct control
  - causes the spec & sketch to match for all inputs
  - causes all assertions to be satisfied for all inputs
- Constraints are collected into a predicate

$$Q(x, c)$$

- -showDAG will show you the constraints!

A Sketch as a constraint system

Synthesis reduces to constraint satisfaction

$$\exists c. \forall x \text{ in test harness. } Q(x, c)$$

Constraints are too hard for standard techniques

- Universal quantification over inputs
- Too many inputs
- Too many constraints
- Too many holes

$\forall x.\ Q(x)$

Verification.

$\exists f\ \forall x.\ f(x) \neq 0$

Synthesis.

Quantifier alternation

$\exists c.\ Q(x_1, c) \wedge Q(x_2, c) \wedge \ldots Q(x_k, c)$

- Overview of the Sketch language
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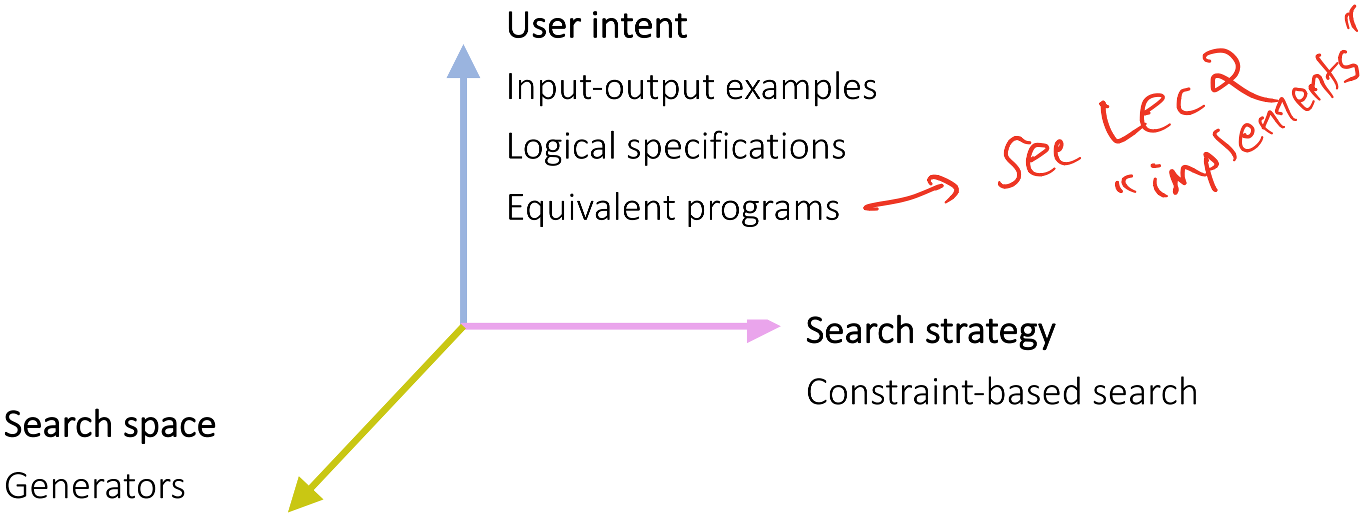

User intent
Input-output examples
Logical specifications
Equivalent programs
See Lec2
"implements"
Search strategy
Constraint-based search
Search space
Generators

A Sketch as a constraint system

Technique 1:

Synthesis reduces to constraint satisfaction

$$\exists c.\forall x.\quad Q(x,c)$$

Quantifier Elimination

– Can blow up

Constraints are too hard for standard techniques

- Universal quantification over inputs
- Too many inputs
- Too many constraints
- Too many holes

Technique 2:

CEGIS

Insight

Sketches are not arbitrary constraint systems

- They express the high level structure of a program

A small set of inputs can fully constrain the solution

- focus on corner cases

$$\exists c. \forall x \text{ in } E. \quad Q(x, c)$$

$$\text{where } E = \{x_1, x_2, \ldots, x_k\}$$

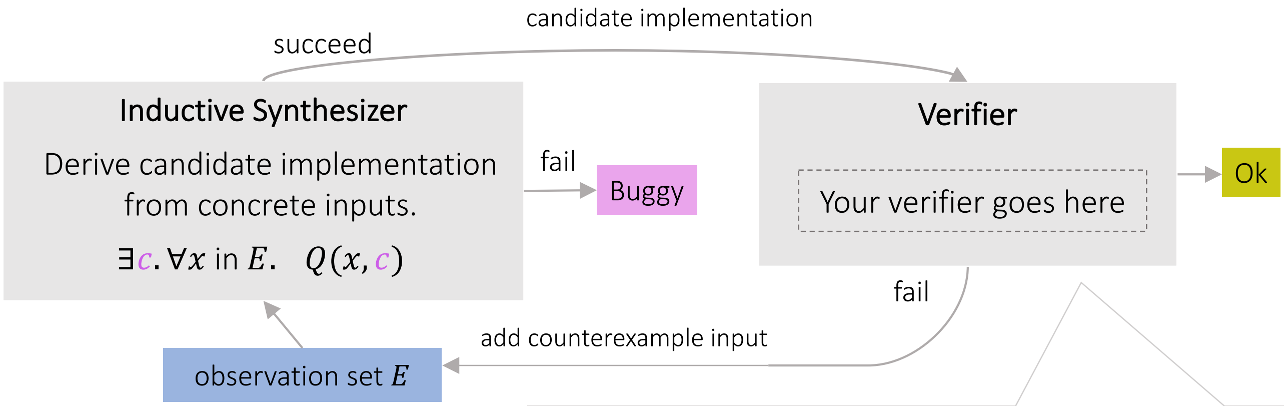
This is an inductive synthesis problem!

- how do we find the set E?
- how do we solve the inductive synthesis problem?

Step 3: Counterexample Guided Inductive Synthesis

Idea: Couple inductive synthesizer with a verifier

- Verifier is charged with detecting convergence



- Standard implementation uses Sat based bounded verifier
- Any verifier/checker that produces counterexamples works

Summary

Today

- Inductive synthesis → Functional synthesis
- Generators (Sketch)
- Constraint-based Search

Next

- Another constraint-based approach for functional synthesis