Hoare Logic, Part II

CS560: Reasoning About Programs

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Partly based on slides by Isil Dillig

Roadmap

Previously

- Hoare logic: Hoare triples, inference rules for partial correctness

Today

- (Semi-)automating Hoare logic
- Verification condition (VC) generation
- Predicate transformers

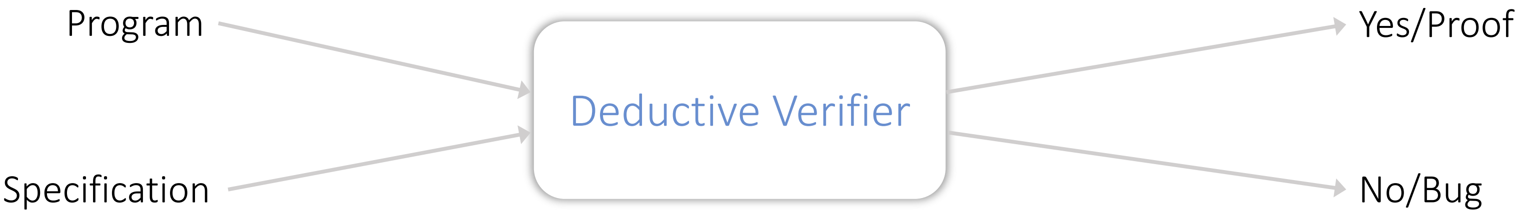
Hoare logic proofs can be tedious!

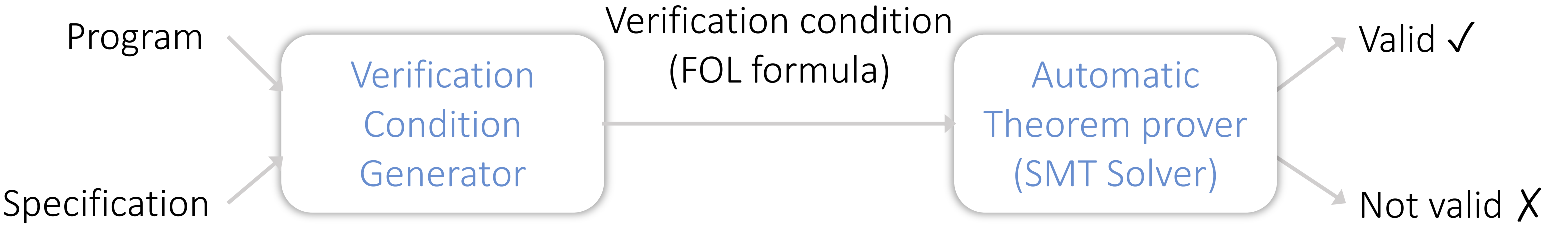
- What is a good loop invariant?

- What rule to apply when?

Let's assume an oracle provides loop invariants for now and automate the rest!

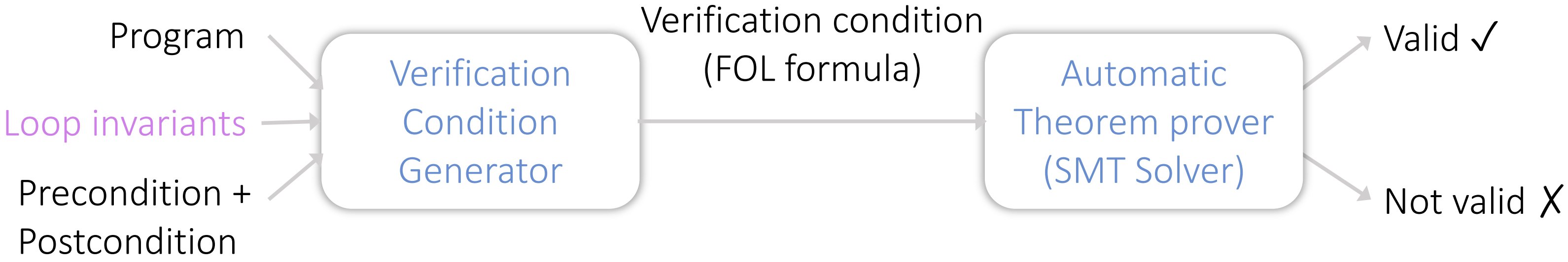
semi-automated


Program
Specification
Deductive Verifier
Yes/Proof
No/Bug



Verification condition is a formula that is valid iff program is correct

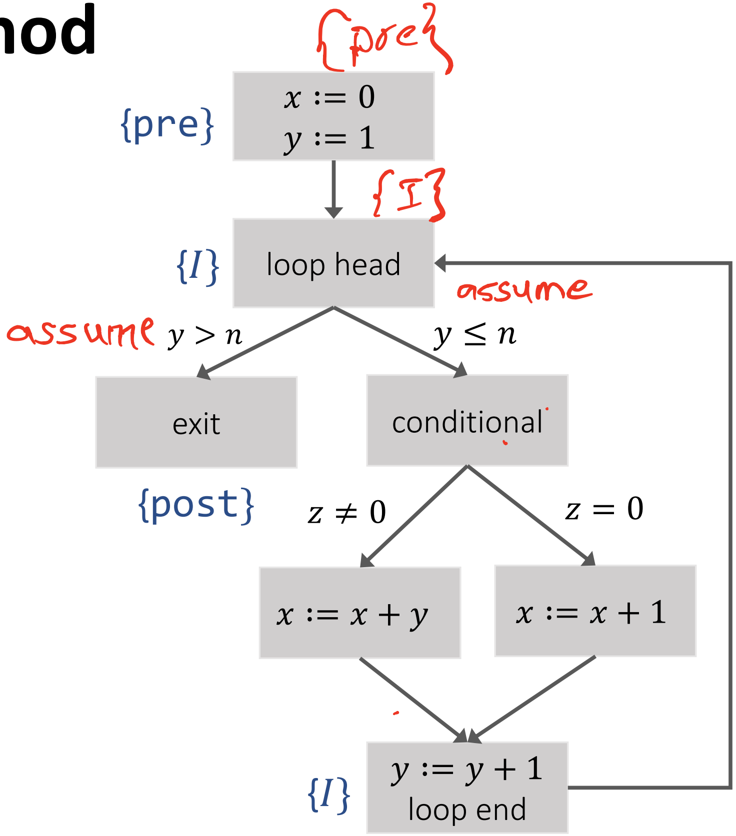
Inductive assertion method



Verification condition is a formula that is valid iff program is partially correct

Inductive assertion method

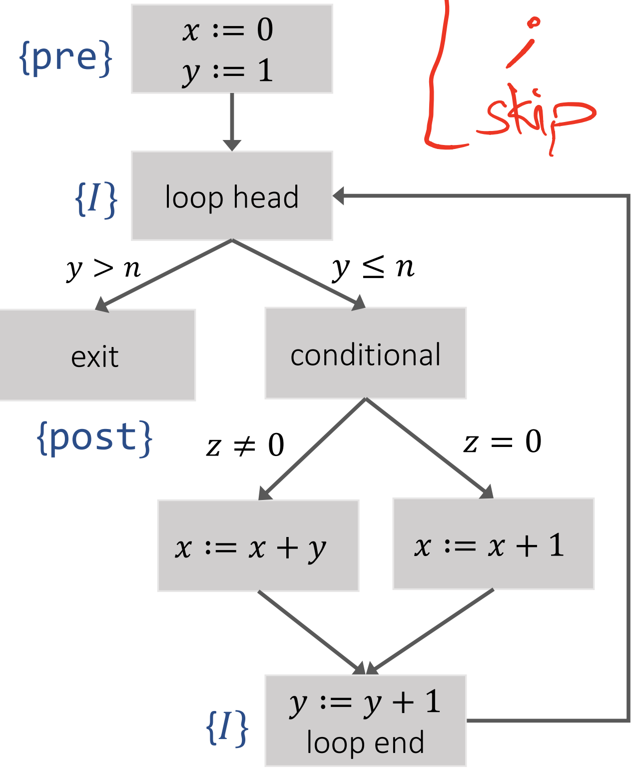
```
{pre}
x := 0
y := 1
while {I} (y <= n) do
    if (z == 0) then
        x := x + 1
    else
        x := x + y
    y := y + 1
{post}
```



Inductive assertion method

IMP* [assign
assume
;
skip

- Represent program as control-flow graph with annotations/inductive assertions

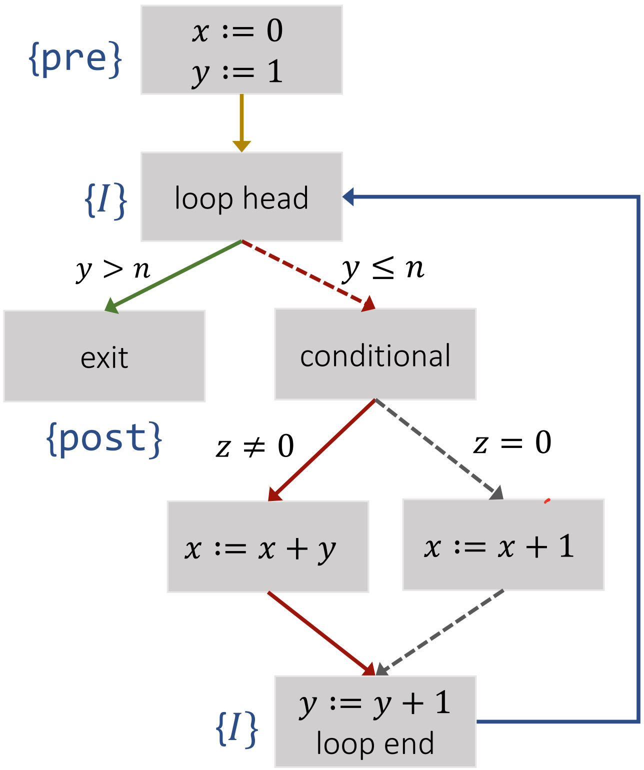


An annotation can be a precondition, postcondition, loop invariant or *assertion*.

Inductive assertion method

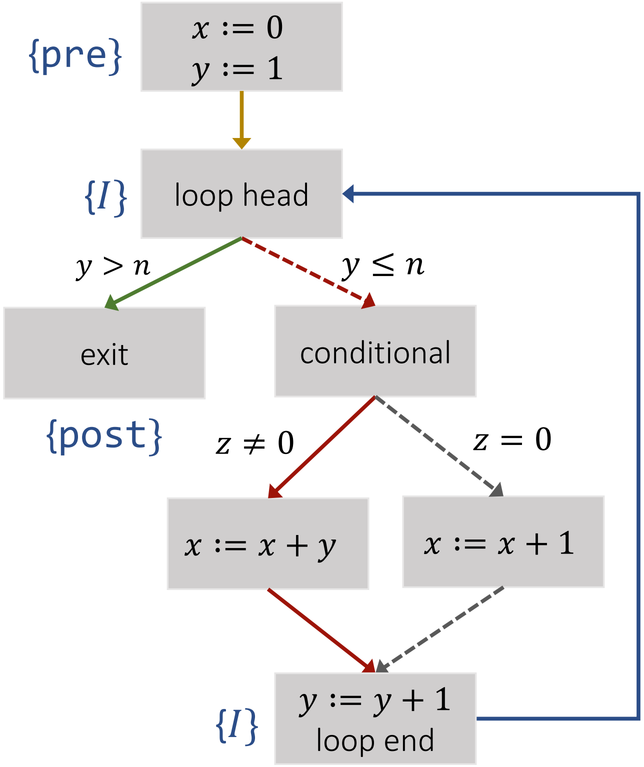
- Represent program as control-flow graph with annotations/inductive assertions
- Identify basic paths

- Basic path starts at a precondition/loop invariant, and ends at a postcondition/loop invariant/assertion.
- Loop invariants only at the start/end of basic paths



Inductive assertion method

- Represent program as control-flow graph with annotations/inductive assertions
- Identify basic paths
- For each basic path:
  check if corresponding Hoare triple is valid



Inductive assertion method

- Represent program as control-flow graph with annotations/inductive assertions
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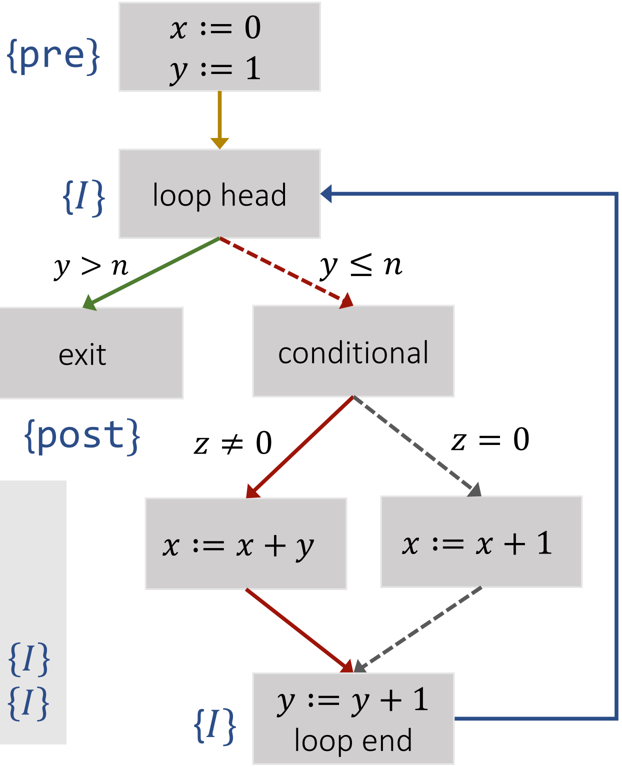


VCs

$\{\text{pre}\}\ x := 0; y := 1\ \{I\}$

$\{I\}$ assume $y > n$ $\{\text{post}\}$

$\{I\}$ skip $\{I\}$

$\{I\}$ assume $y \leq n$; assume $z \neq 0; x := x + y; y := y + 1\ \{I\}$

$\{I\}$ assume $y \leq n$; assume $z = 0; x := x + 1; y := y + 1\ \{I\}$

Generating VCs: Forwards vs. Backwards

Forwards Analysis

- Starts from precondition and tries to prove postcondition
- Computes strongest postconditions (sp)

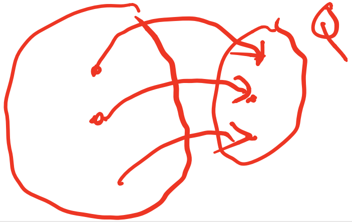
Backwards Analysis

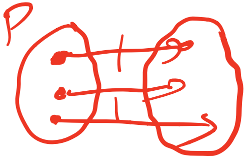
- Starts from postcondition and tries to prove precondition
- Computes weakest liberal preconditions (wp)

Predicate transformers: FOL x stmts → FOL

Incorporate effects of program statements into FOL formulas

WP and SP





$wp(S, Q)$: the weakest predicate that guarantees Q will hold after executing S from any state satisfying the predicate

$sp(S, P)$: the strongest predicate that holds after executing S from any state satisfying P

"largest" set of states

"smallest" set of states

$\{P\}\ S\ \{Q\}$ is valid iff:

$$P \Rightarrow wp(S, Q) \text{ , or, } sp(S, P) \Rightarrow Q$$

Computing $sp(S, P)$

$sp(\text{skip}, P) = P$

- $sp(\texttt{assume}\ C, P) = C \land P$
- $sp(S_1; S_2, P) = sp(S_2, sp(S_1, P))$
- $sp(x := E, P) = \exists x^0. x = E[x^0/x] \land P[x^0/x]$

"Previous value"

$x := x+1$

$x := x+y$

SSA

$x_1 := x_0 + 1$

$x_2 := x_1 + y_0$

$sp(x := x+1, x > 0) =$

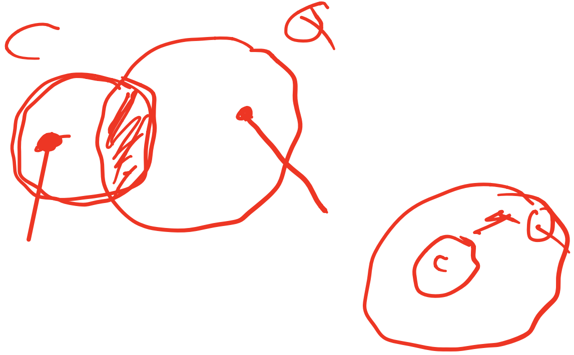
$\exists x^0.\ x = x^0 + 1 \land x^0 > 0$

$= x > 1$ (Quantifier Elimination)

Computing $wp(S, Q)$

- $wp(\texttt{assume}\ C, Q) = C \rightarrow Q$
- $wp(S_1; S_2, Q) = wp(S_1, wp(S_2, Q))$
- $wp(x := E, Q) = Q[E/x]$

$\vdash \{Q[E/x]\}\ x := E\ \{Q\}$



assume C;

assert C;

Roadmap

Today

- (Semi-)automating Hoare logic
- Verification condition (VC) generation
- Predicate transformers

Next

- How to fully automate Hoare logic: automatic invariant generation