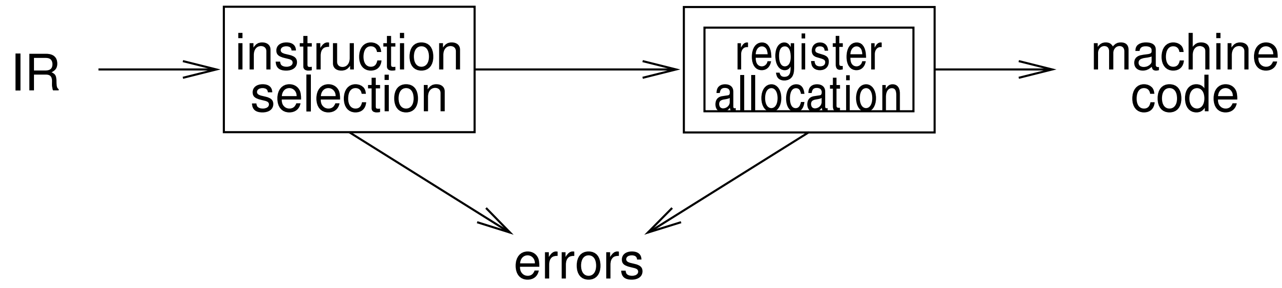
Register allocation



Register allocation:

- have value in a register when used
- limited resources
- changes instruction choices
- can move loads and stores
- optimal allocation is difficult
  $\Rightarrow$ NP-complete for $k \geq 1$ registers

Liveness analysis

Problem:

- IR contains an unbounded number of temporaries
- machine has bounded number of registers

Approach:

- temporaries with disjoint *live* ranges can map to same register
- if not enough registers then *spill* some temporaries
  (i.e., keep them in memory)

The compiler must perform *liveness analysis* for each temporary:

It is *live* if it holds a value that may be needed in future

Control flow analysis

Before performing liveness analysis, need to understand the control flow by building a *control flow graph* (CFG):

- nodes may be individual program statements or basic blocks
- edges represent potential flow of control

Out-edges from node n lead to *successor* nodes, *succ*$[n]$
In-edges to node n come from *predecessor* nodes, *pred*$[n]$
Example:

$$
\begin{array}{rl}
 & a \leftarrow 0 \\
L_1: & b \leftarrow a + 1 \\
 & c \leftarrow c + b \\
 & a \leftarrow b \times 2 \\
 & \text{if } a < N \text{ goto } L_1 \\
 & \text{return } c
\end{array}
$$

Liveness analysis

Gathering liveness information is a form of *data flow analysis* operating over the CFG:

- liveness of variables "flows" around the edges of the graph
- assignments *define* a variable, v:
  - *def*$(v) = \text{set of graph nodes that define } v$
  - *def*$[n] = \text{set of variables defined by } n$
- occurrences of v in expressions *use* it:
  - *use*$(v) = \text{set of nodes that use } v$
  - *use*$[n] = \text{set of variables used in } n$

Liveness: v is *live* on edge e if there is a directed path from e to a *use* of v that does not pass through any *def*(v)

v is *live-in* at node n if live on any of n's in-edges

v is *live-out* at n if live on any of n's out-edges

$v \in$ *use*$[n] \Rightarrow v \text{ live-in at } n$

$v \text{ live-in at } n \Rightarrow v \text{ live-out at all } m \in$ *pred*$[n]$

$v \text{ live-out at } n, v \notin$ *def*$[n] \Rightarrow v \text{ live-in at } n$

Liveness analysis

Define:

in$[n]$: variables live-in at n

out$[n]$: variables live-out at n

Then:

$$\mathit{out}[n] = \bigcup_{s \in \mathit{succ}(n)} \mathit{in}[s]$$

$$\mathit{succ}[n] = \phi \Rightarrow \mathit{out}[n] = \phi$$

Note:

$$\mathit{in}[n] \supseteq \mathit{use}[n]$$

$$\mathit{in}[n] \supseteq \mathit{out}[n] - \mathit{def}[n]$$

use$[n]$ and *def*$[n]$ are constant (independent of control flow)

Now, $v \in \mathit{in}[n]$ iff. $v \in \mathit{use}[n]$ or $v \in \mathit{out}[n] - \mathit{def}[n]$

Thus, $\mathit{in}[n] = \mathit{use}[n] \cup (\mathit{out}[n] - \mathit{def}[n])$

Iterative solution for liveness

foreach n

  in$[n] \leftarrow \phi$

  out$[n] \leftarrow \phi$

repeat

    foreach n

      in$'[n] \leftarrow$ in$[n]$;

      out$'[n] \leftarrow$ out$[n]$;

      in$[n] \leftarrow$ use$[n] \cup ($out$[n] -$ def$[n])$

      out$[n] \leftarrow \bigcup_{s \in \text{succ}[n]}$ in$[s]$

until in$'[n] =$ in$[n] \wedge$ out$'[n] =$ out$[n], \forall n$

Notes:

- should order computation of inner loop to follow the "flow"
- liveness flows *backward* along control-flow arcs, from *out* to *in*
- nodes can just as easily be basic blocks to reduce CFG size
- could do one variable at a time, from *uses* back to *defs*, noting liveness along the way

Iterative solution for liveness

Complexity: for input program of size N

- $\leq N$ nodes in CFG
  $\Rightarrow \leq N$ variables
  $\Rightarrow N$ elements per *in/out*
  $\Rightarrow O(N)$ time per set-union
- **for** loop performs constant number of set operations per node
  $\Rightarrow O(N^2)$ time for **for** loop
- each iteration of **repeat** loop can only add to each set
  sets can contain at most every variable
  $\Rightarrow$ sizes of all in and out sets sum to $2N^2$,
  bounding the number of iterations of the **repeat** loop

$\Rightarrow$ worst-case complexity of $O(N^4)$

- ordering can cut **repeat** loop down to 2-3 iterations
  $\Rightarrow O(N)$ or $O(N^2)$ in practice

Least fixed points

There is often more than one solution for a given dataflow problem (see example).

Any solution to dataflow equations is a *conservative approximation*:

- v has some later use downstream from n
  $\Rightarrow v \in \mathit{out}(n)$
- but not the converse

Conservatively assuming a variable is live does not break the program; just means more registers may be needed.

Assuming a variable is dead when it is really live *will* break things.

May be many possible solutions but want the “smallest”: the least fixpoint.

The iterative liveness computation computes this least fixpoint.