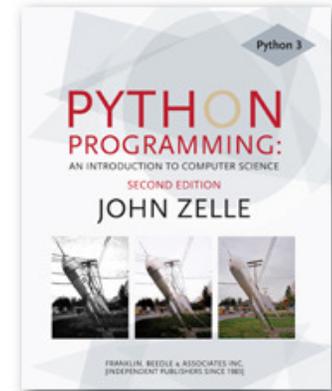
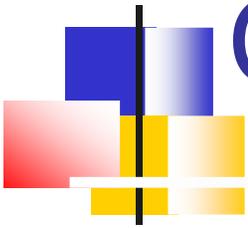
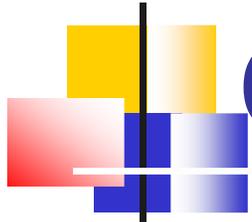


Python Programming: An Introduction to Computer Science

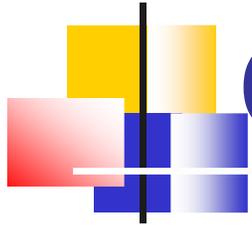


Chapter 3 Computing with Numbers



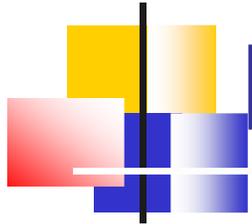
Objectives

- To understand the concept of data types.
- To be familiar with the basic numeric data types in Python.
- To understand the fundamental principles of how numbers are represented on a computer.



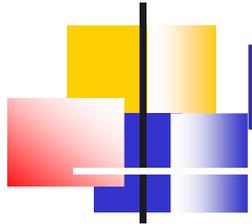
Objectives (cont.)

- To be able to use the Python math library.
- To understand the accumulator program pattern.
- To be able to read and write programs that process numerical data.



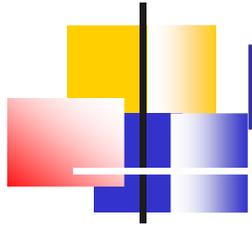
Numeric Data Types

- The information that is stored and manipulated by computers programs is referred to as data.
- There are two different kinds of numbers!
 - (5, 4, 3, 6) are whole numbers – they don't have a fractional part
 - (.25, .10, .05, .01) are decimal fractions



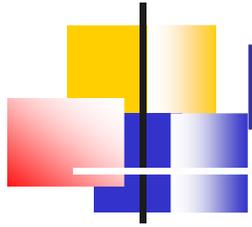
Numeric Data Types

- Inside the computer, whole numbers and decimal fractions are represented quite differently!
- We say that decimal fractions and whole numbers are two different data types.
- The data type of an object determines what values it can have and what operations can be performed on it.



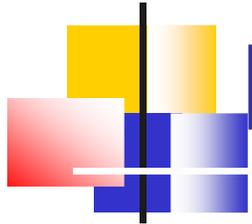
Numeric Data Types

- Whole numbers are represented using the integer (int for short) data type.
- These values can be positive or negative whole numbers.



Numeric Data Types

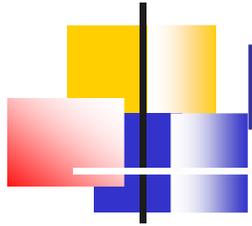
- Numbers that can have fractional parts are represented as floating point (or float) values.
- How can we tell which is which?
 - A numeric literal without a decimal point produces an int value
 - A literal that has a decimal point is represented by a float (even if the fractional part is 0)



Numeric Data Types

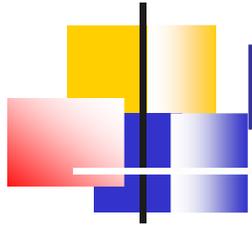
- Python has a special function to tell us the data type of any value.

```
>>> type(3)
<class 'int'>
>>> type(3.1)
<class 'float'>
>>> type(3.0)
<class 'float'>
>>> myInt = 32
>>> type(myInt)
<class 'int'>
>>>
```



Numeric Data Types

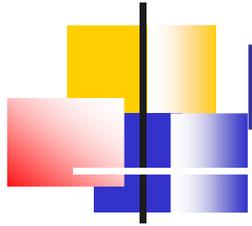
- Why do we need two number types?
 - Values that represent counts can't be fractional (you can't have 3 $\frac{1}{2}$ quarters)
 - Most mathematical algorithms are very efficient with integers
 - The float type stores only an approximation to the real number being represented!
 - Since floats aren't exact, use an int whenever possible!



Numeric Data Types

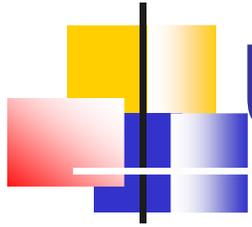
- Operations on ints produce ints, operations on floats produce floats (except for /).

```
>>> 3.0+4.0
7.0
>>> 3+4
7
>>> 3.0*4.0
12.0
>>> 3*4
12
>>> 10.0/3.0
3.3333333333333335
>>> 10/3
3.3333333333333335
>>> 10 // 3
3
>>> 10.0 // 3.0
3.0
```



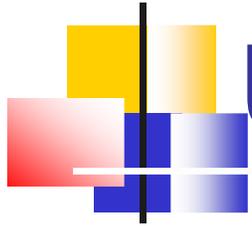
Numeric Data Types

- Integer division produces a whole number.
- That's why $10//3 = 3$!
- Think of it as 'gozinta', where $10//3 = 3$ since 3 gozinta (goes into) 10 3 times (with a remainder of 1)
- $10\%3 = 1$ is the remainder of the integer division of 10 by 3.
- $a = (a/b)*(b) + (a\%b)$



Using the Math Library

- Besides (+, -, *, /, //, **, %, abs), we have lots of other math functions available in a math library.
- A library is a module with some useful definitions/functions.

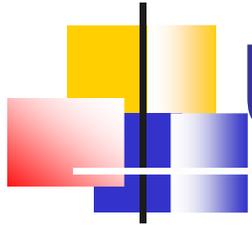


Using the Math Library

- Let's write a program to compute the roots of a quadratic equation!

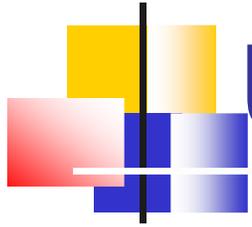
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The only part of this we don't know how to do is find a square root... but it's in the math library!



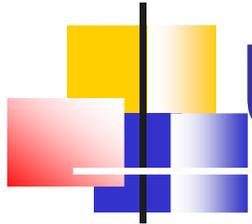
Using the Math Library

- To use a library, we need to make sure this line is in our program:
`import math`
- Importing a library makes whatever functions are defined within it available to the program.



Using the Math Library

- To access the sqrt library routine, we need to access it as `math.sqrt(x)`.
- Using this dot notation tells Python to use the sqrt function found in the math library module.
- To calculate the root, you can do
`discRoot = math.sqrt(b*b - 4*a*c)`



Using the Math Library

```
# quadratic.py
# A program that computes the real roots of a quadratic equation.
# Illustrates use of the math library.
# Note: This program crashes if the equation has no real roots.

import math # Makes the math library available.

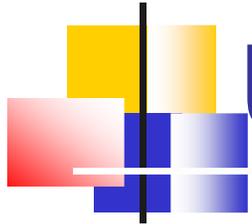
def main():
    print("This program finds the real solutions to a quadratic")
    print()

    a, b, c = eval(input("Please enter the coefficients (a, b, c): "))

    discRoot = math.sqrt(b * b - 4 * a * c)
    root1 = (-b + discRoot) / (2 * a)
    root2 = (-b - discRoot) / (2 * a)

    print()
    print("The solutions are:", root1, root2 )

main()
```



Using the Math Library

This program finds the real solutions to a quadratic

Please enter the coefficients (a, b, c): 3, 4, -1

The solutions are: 0.215250437022 -1.54858377035

■ What do you suppose this means?

This program finds the real solutions to a quadratic

Please enter the coefficients (a, b, c): 1, 2, 3

Traceback (most recent call last):

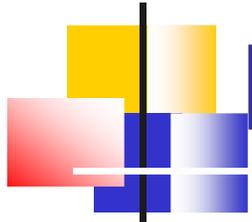
```
File "<pyshell#26>", line 1, in -toplevel-  
    main()
```

```
File "C:\Documents and Settings\Terry\My Documents\Teaching\W04\CS  
120\Textbook\code\chapter3\quadratic.py", line 14, in main
```

```
    discRoot = math.sqrt(b * b - 4 * a * c)
```

```
ValueError: math domain error
```

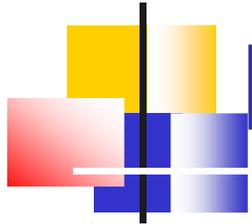
```
>>>
```



Math Library

- If $a = 1$, $b = 2$, $c = 3$, then we are trying to take the square root of a negative number!
- Using the `sqrt` function is more efficient than using `**`. How could you use `**` to calculate a square root?

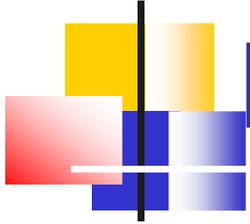
Accumulating Results:



Factorial

- Say you are waiting in a line with five other people. How many ways are there to arrange the six people?
- 720 -- 720 is the factorial of 6 (abbreviated 6!)
- Factorial is defined as:
$$n! = n(n-1)(n-2)\dots(1)$$
- So, $6! = 6*5*4*3*2*1 = 720$

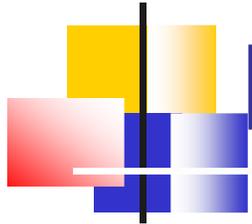
Accumulating Results:



Factorial

- How we could we write a program to do this?
- Input number to take factorial of, n
Compute factorial of n , $fact$
Output $fact$

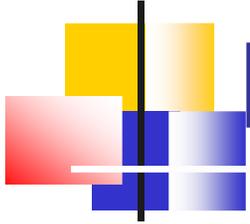
Accumulating Results:



Factorial

- How did we calculate 6!?
- $6 * 5 = 30$
- Take that 30, and $30 * 4 = 120$
- Take that 120, and $120 * 3 = 360$
- Take that 360, and $360 * 2 = 720$
- Take that 720, and $720 * 1 = 720$

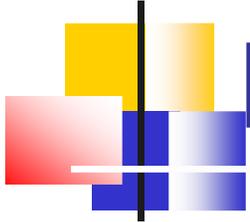
Accumulating Results:



Factorial

- What's really going on?
- We're doing repeated multiplications, and we're keeping track of the running product.
- This algorithm is known as an accumulator, because we're building up or accumulating the answer in a variable, known as the accumulator variable.

Accumulating Results:



Factorial

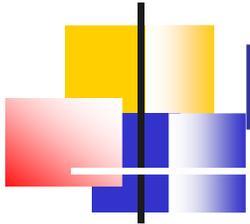
- The general form of an accumulator algorithm looks like this:

Initialize the accumulator variable

Loop until final result is reached

update the value of accumulator variable

Accumulating Results:



Factorial

- It looks like we'll need a loop!

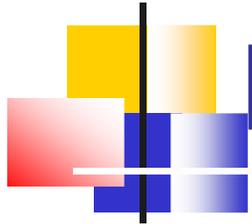
```
fact = 1
```

```
for factor in [6, 5, 4, 3, 2, 1]:
```

```
    fact = fact * factor
```

- Let's trace through it to verify that this works!

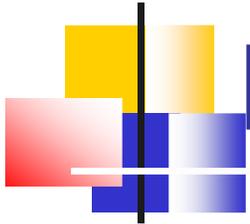
Accumulating Results:



Factorial

- Why did we need to initialize fact to 1? There are a couple reasons...
 - Each time through the loop, the previous value of fact is used to calculate the next value of fact. By doing the initialization, you know fact will have a value the first time through.
 - If you use fact without assigning it a value, what does Python do?

Accumulating Results:



Factorial

- Since multiplication is associative and commutative, we can rewrite our program as:

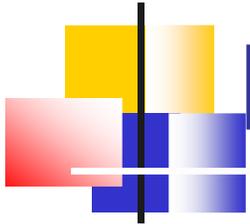
```
fact = 1
```

```
for factor in [2, 3, 4, 5, 6]:
```

```
    fact = fact * factor
```

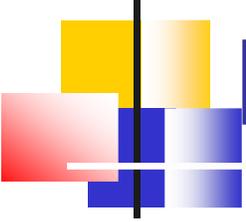
- Great! But what if we want to find the factorial of some other number??

Accumulating Results:



Factorial

- What does `range(n)` return?
`0, 1, 2, 3, ..., n-1`
- `range` has another optional parameter!
`range(start, n)` returns
`start, start + 1, ..., n-1`
- But wait! There's more!
`range(start, n, step)`
`start, start+step, ..., n-1`
- `list(<sequence>)` to make a list



Accumulating Results: Factorial

- Let's try some examples!

```
>>> list(range(10))
```

```
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

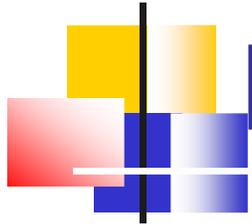
```
>>> list(range(5,10))
```

```
[5, 6, 7, 8, 9]
```

```
>>> list(range(5,10,2))
```

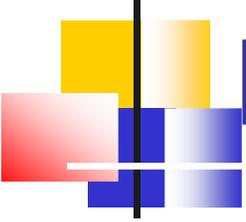
```
[5, 7, 9]
```

Accumulating Results:



Factorial

- Using this souped-up range statement, we can do the range for our loop a couple different ways.
 - We can count up from 2 to n:
`range(2, n+1)`
(Why did we have to use `n+1`?)
 - We can count down from n to 2:
`range(n, 1, -1)`



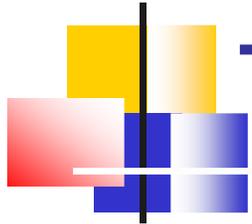
Accumulating Results: Factorial

- Our completed factorial program:

```
# factorial.py
# Program to compute the factorial of a number
# Illustrates for loop with an accumulator

def main():
    n = eval(input("Please enter a whole number: "))
    fact = 1
    for factor in range(n,1,-1):
        fact = fact * factor
    print("The factorial of", n, "is", fact)

main()
```



The Limits of Int

- What is 100!?

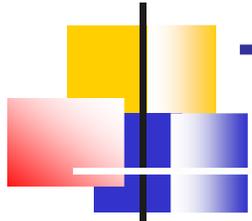
```
>>> main()
```

```
Please enter a whole number: 100
```

```
The factorial of 100 is
```

```
9332621544394415268169923885626670049071596826438162  
1468592963895217599993229915608941463976156518286253  
6979208272237582511852109168640000000000000000000000  
00
```

- Wow! That's a pretty big number!



The Limits of Int

- Newer versions of Python can handle it, but...

Python 1.5.2 (#0, Apr 13 1999, 10:51:12) [MSC 32 bit (Intel)] on win32

Copyright 1991-1995 Stichting Mathematisch Centrum, Amsterdam

```
>>> import fact
```

```
>>> fact.main()
```

```
Please enter a whole number: 13
```

```
13
```

```
12
```

```
11
```

```
10
```

```
9
```

```
8
```

```
7
```

```
6
```

```
5
```

```
4
```

```
Traceback (innermost last):
```

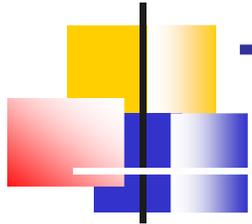
```
File "<pyshell#1>", line 1, in ?
```

```
fact.main()
```

```
File "C:\PROGRA~1\PYTHON~1.2\fact.py", line 5, in main
```

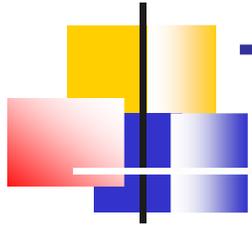
```
fact=fact*factor
```

```
OverflowError: integer multiplication
```



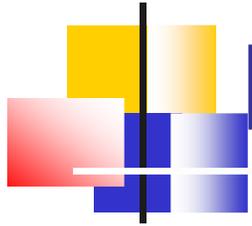
The Limits of Int

- What's going on?
 - While there are an infinite number of integers, there is a finite range of ints that can be represented.
 - This range depends on the number of bits a particular CPU uses to represent an integer value. Typical PCs use 32 bits.



The Limits of Int

- Typical PCs use 32 bits
- That means there are 2^{32} possible values, centered at 0.
- This range then is -2^{31} to $2^{31}-1$. We need to subtract one from the top end to account for 0.
- But our $100!$ is much larger than this. How does it work?



Handling Large Numbers

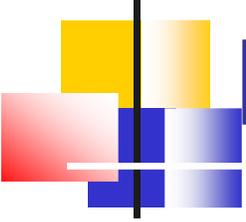
- Does switching to float data types get us around the limitations of ints?
- If we initialize the accumulator to 1.0, we get

```
>>> main()
```

```
Please enter a whole number: 15
```

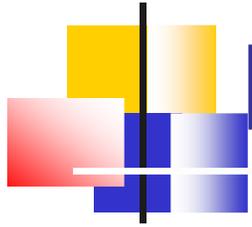
```
The factorial of 15 is 1.307674368e+012
```

- We no longer get an exact answer!



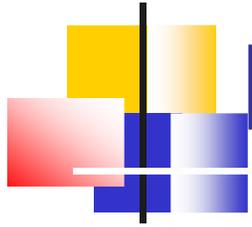
Handling Large Numbers: Long Int

- Very large and very small numbers are expressed in scientific or exponential notation.
- $1.307674368e+012$ means $1.307674368 * 10^{12}$
- Here the decimal needs to be moved right 12 decimal places to get the original number, but there are only 9 digits, so 3 digits of precision have been lost.



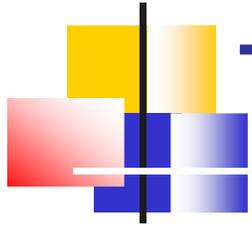
Handling Large Numbers

- Floats are approximations
- Floats allow us to represent a larger range of values, but with lower precision.
- Python has a solution, expanding ints!
- Python Ints are not a fixed size and expand to handle whatever value it holds.



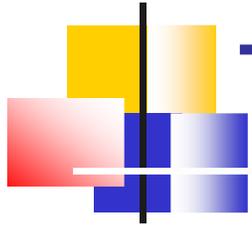
Handling Large Numbers

- Newer versions of Python automatically convert your ints to expanded form when they grow so large as to overflow.
- We get indefinitely large values (e.g. 100!) at the cost of speed and memory



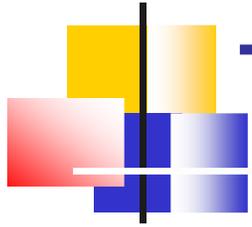
Type Conversions

- We know that combining an int with an int produces an int, and combining a float with a float produces a float.
- What happens when you mix an int and float in an expression?
 $x = 5.0 + 2$
- What do you think should happen?



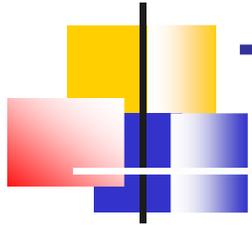
Type Conversions

- For Python to evaluate this expression, it must either convert 5.0 to 5 and do an integer addition, or convert 2 to 2.0 and do a floating point addition.
- Converting a float to an int will lose information
- Ints can be converted to floats by adding “.0”



Type Conversion

- In mixed-typed expressions Python will convert ints to floats.
- Sometimes we want to control the type conversion. This is called explicit typing.



Type Conversions

```
>>> float(22//5)
```

```
4.0
```

```
>>> int(4.5)
```

```
4
```

```
>>> int(3.9)
```

```
3
```

```
>>> round(3.9)
```

```
4
```

```
>>> round(3)
```

```
3
```