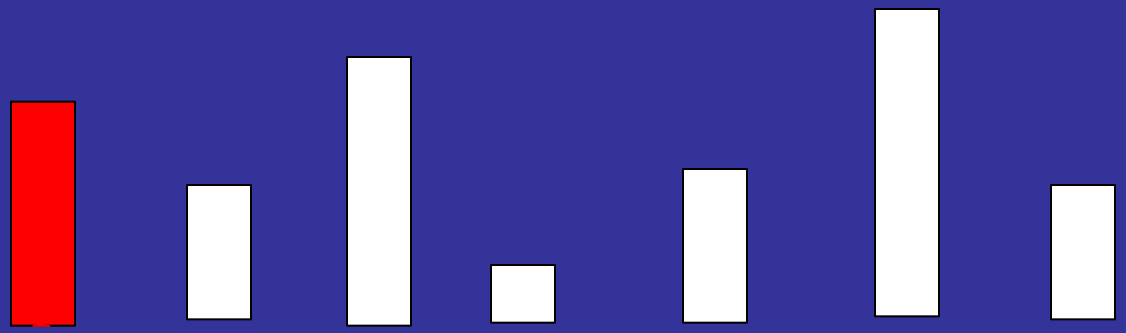
Quicksort and Function Pointers

CS 240

The Quicksort Algorithm

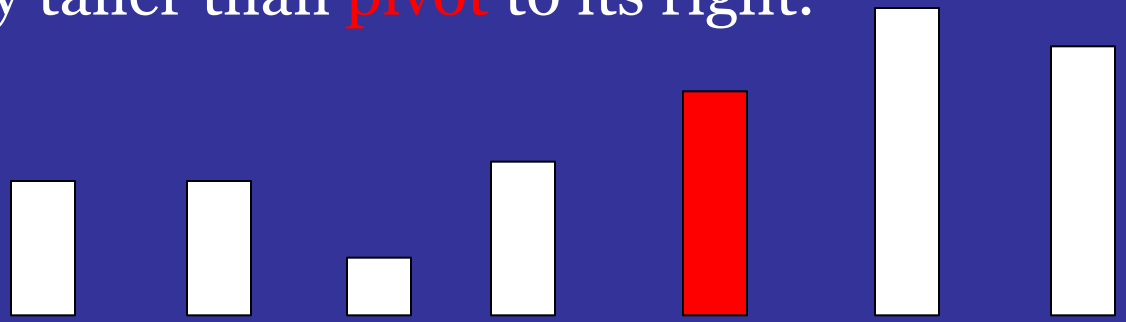
- Developed by **Tony Hoare**
- Sorts an array of items using a DIVIDE-AND-CONQUER approach.
  1. Divide the problem into two, smaller subproblems
  2. Solve each subproblem recursively
  3. Merge the partial solutions in a way that solves the original problem
- When merging takes less time than solving the two subproblems, the algorithm is efficient.

Quicksort from short to tall



Pick any one, call it the "pivot" (say the RED one).

Make one pass, putting any shorter than pivot to its left and any taller than pivot to its right.



Call Quicksort again,

first on the left part

then on the right part

RECURSION

Quicksort Pseudocode

```
Quicksort(L,  low,  high)
/* L = array, low = left-end, high = right-end */
{
    if ( low < high)
    {
        pivotindex = split(L, low, high)  // pivot in right place
        /* split is the workhorse;
        to its left is everything < pivot,
        to its right is everything >= pivot */

        Quicksort(L,  low,  pivotindex – 1);  // sort left sub-array
        Quicksort(L,  pivotindex + 1, high);  // sort right sub-array
    }
}
```

Want to sort: Q U I C K S O R T

The algorithm has two parts:

split: workhorse, splits the array into two sub-arrays

- one array element (pivot) is in its correct place
- the sub-arrays to the left and right are both unsorted

recursion: the easy / bossy part, gives split more work to do

```
quicksort(array A) {
    split A into two unsorted parts,
      leaving one element in its final, sorted place;
    quicksort(left part without the sorted element);
    quicksort(right part without the sorted element);
}
```

split's action, pivot = A[0] = Q

Q U I C K S O R T

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Q	U	I	C	K	S	O	R	T
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Q	I	C	K	O	S	U	R	T
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O	I	C	K	Q	S	U	R	T
O	I	C	K	Q	S	U	R	T

Now, call Quicksort recursively on

O I C K and S U R T

because Q is already in its sorted position

O I C K
O I C K
O I C K
K I C O Now O is in its final place

S U R T
S U R T
S R U T
S R U T
R S U T Now S is in its final place

Quicksort, pivot = A[0]

Q U I C K S O R T	
O I C K Q S U R T	pi = 4 (pivot index)
K I C O Q S U R T	pi = 3
C I K O Q S U R T	pi = 2
C I K O Q S U R T	pi = 0
C I K O Q R S U T	pi = 6
C I K O Q R S T U	pi = 8

Strangely, when you sort ‘QUICKSORT’ you end up with Prof. W. C. S’s middle name. 

Quicksort, pivot = A[0]

8 1 3 7 2 5 9 4	
4 1 3 7 2 5 8 9	pi = 6
2 1 3 4 7 5 8 9	pi = 3
1 2 3 4 7 5 8 9	pi = 1
1 2 3 4 5 7 8 9	pi = 5

Quicksort behaves poorly when sub-arrays are not “balanced” in size

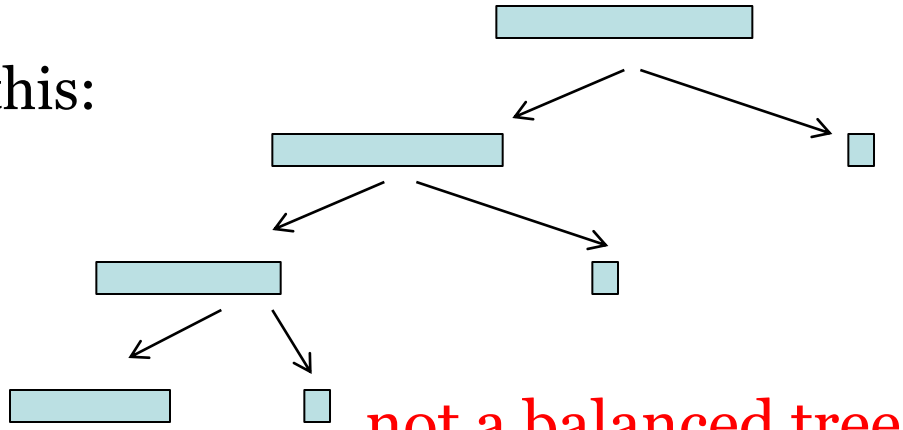
9 8 7 6 5 4 3 2 1	
1 8 7 6 5 4 3 2 9	pi = 8
1 8 7 6 5 4 3 2 9	pi = 0
1 2 7 6 5 4 3 8 9	pi = 7
1 2 7 6 5 4 3 8 9	pi = 1
1 2 3 6 5 4 7 8 9	pi = 6
1 2 3 6 5 4 7 8 9	pi = 2
1 2 3 4 5 6 7 8 9	pi = 5
1 2 3 4 5 6 7 8 9	pi = 3

When Quicksort runs, each sub-array becomes a node in a binary tree. So, what are helpful trees? What are unhelpful trees?

A = {9, 8, 7, 6, 5, 4, 3, 2, 1} does this:

tall, unbalanced tree

height is n or close to n

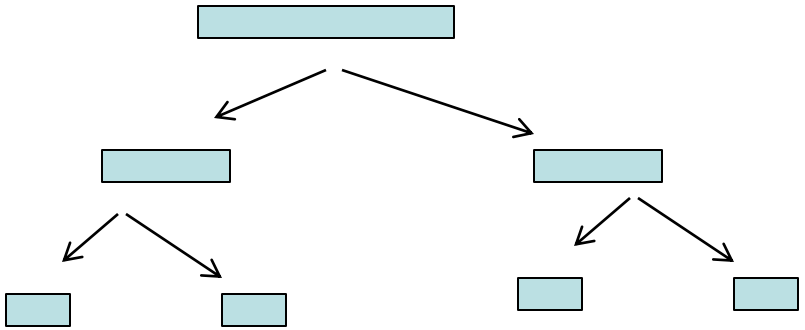


not a balanced tree

But what if Split gives you balanced sub-arrays?

balanced tree

height is lg(n)



a balanced tree