FDM: send bits using EM waves

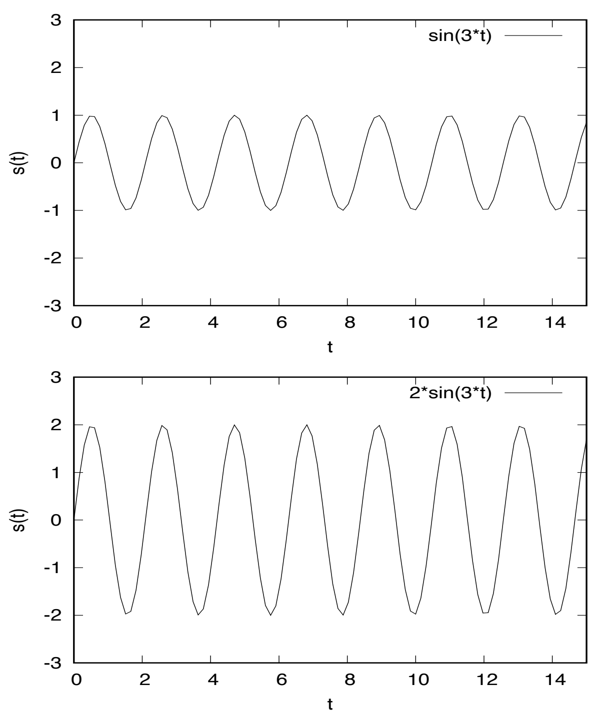
→ n users; one user sends one bit

Key variable: frequency f of sinusoid

→ $\sin ft$

→ f: carrier frequency

→ low amplitude: 0, high amplitude: 1



To transmit multiple bit streams concurrently:

→ use multiple carrier frequencies $f_1, f_2, \ldots, f_n$

→ dividing up frequency

→ FDM (frequency division multiplexing)

→ called WDM (wave division multiplexing) for optical fiber

Two primary application scenarios:

- multi-user: one user gets one frequency

  → FDMA (frequency division multiple access)

- single-user: one user gets all frequencies

  → ship bits in parallel

  → completion time to ship group of bits is reduced

Other applications: wireless

- confidentiality: protect against eavesdropping by randomly jumping around multiple frequencies

  → frequency hopping

  → transmission is sequential

- anti-jamming: spreading bits over multiple frequencies makes jamming harder
- frequency-selective fading

  → some frequencies suffer more distortion than others

  → use error correction

Referred to as spread spectrum

→ bits are spread over a wide range of frequencies

→ width from f_1 to f_n called bandwidth (Hz)

→ e.g., $n = 10$, $f_1 = 1$ GHz, $f_{10} = 1.9$ GHz, bandwidth 0.9 GHz

Fundamental engineering caveat: nature cannot be accurately captured by real numbers

$\rightarrow$ must use complex numbers

$\rightarrow$ not just because solving $x^2 = -1$ demands $i = \sqrt{-1}$

Use complex sinusoids

$\rightarrow \cos ft + i \sin ft$

$\rightarrow$ by Euler's formula

$$e^{ift} = \cos ft + i \sin ft$$

Correspondence to linear algebra used in CDMA . . .

Linear algebra used in CDMA:

- finite dimension n: number of users

  $\rightarrow$ each user sends a single bit

- fix basis vectors $\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^n$: code vectors

  $\rightarrow$ orthogonal

  $\rightarrow$ span n-dimensional vector space

- any vector $\mathbf{z}$ expressed as weighted sum of basis vectors

  $\rightarrow \mathbf{z} = \sum_{k=1}^{n} a_k \mathbf{x}^k$

- to send n bits construct $\mathbf{z}$ by hiding the n bits in the scalar weights a_k for $k = 1, \ldots, n$

  $\rightarrow$ called synthesis

  $\rightarrow$ $\mathbf{z}$ encodes n bits

  $\rightarrow$ $\mathbf{z}$: message

- user i decodes sent bit from message $\mathbf{z}$ by computing

  $\rightarrow \mathbf{z} \circ \mathbf{x}^i = a_i$

  $\rightarrow$ by orthogonality of basis vectors (i.e., codes)

  $\rightarrow$ called analysis

In FDMA with complex sinusoids:

- infinite dimensional space

  → time is unbounded and continuous

- basis elements:

  → complex sinusoids e^{ift} for different f

- signals $s(t)$ of interest are weighted sum of e^{ift} with weight a_f

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} a_f e^{ift} df$$

Note: analogous to $\mathbf{z} = \sum_{k=1}^{n} a_k \mathbf{x}^k$

→ $\sum$ becomes $\int$ since frequency f is continuous

→ called inverse Fourier transform

→ synthesis

→ as before: hide bits in the weights a_f

Given signal $s(t)$—our message (corresponds to $\mathbf{z}$)—in whose sinusoid coefficients we have hidden bits

To recover the bits: compute

$$a_f = \int_{-\infty}^{\infty} s(t)e^{-ift}dt$$

→ define as $a_f = s(t) \circ e^{ift}$

→ analogous to $\mathbf{z} \circ \mathbf{x}^i$

→ called Fourier transform

→ analysis: find coefficient a_f

→ possible because complex sinusoids are mutually orthogonal

Computing Fourier transform quickly is important

→ fast Fourier transform (FFT)

→ same goes for synthesis: IFFT

→ subject of CS 580

To send n bits in parallel:

→ use n frequencies $f_1, f_2, \ldots, f_n$

→ message: discrete and finite sum

$$s(t) = \sum_{k=1}^{n} a_k e^{i f_k t}$$

→ synthesis

How to choose n carrier frequencies $f_1, f_2, \ldots, f_n$?

- traditional method
  → insert sufficient gap between neighboring carrier frequencies

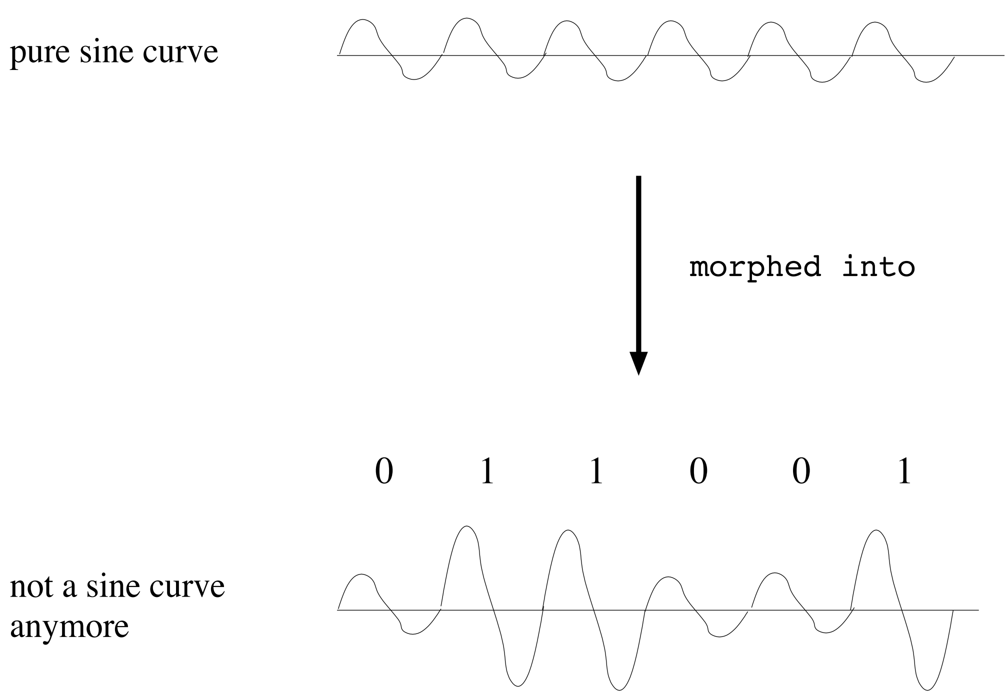
  → guard band

  → we will see why
- modern method: OFDM (orthogonal FDM)
  → gap can be significantly smaller

  → more carrier frequencies

Traditional method (FDM):

Consider AM modulation of single sinusoid of frequency f, say $f = 100$ MHz



$\rightarrow$ one user sends multiple bits: 011001

$\rightarrow$ previously: one bit per user

$\rightarrow$ new signal $s(t)$

Fact:

- resembles sinusoid but not the same
- what is it?

→ precise characterization possible

→ take a different viewpoint of $s(t)$

Switch from synthesis view to and analysis view

→ signal $s(t)$ is linear combination (i.e., weighted sum) of complex sinusoids

→ signature of $s(t)$

→ its "DNA"

Synthesis:

- complex sinosoids form a basis for a large family of functions $s(t)$
- includes signals encountered in engineered and natural systems
- hence, $s(t)$ can be expressed as a weighted sum

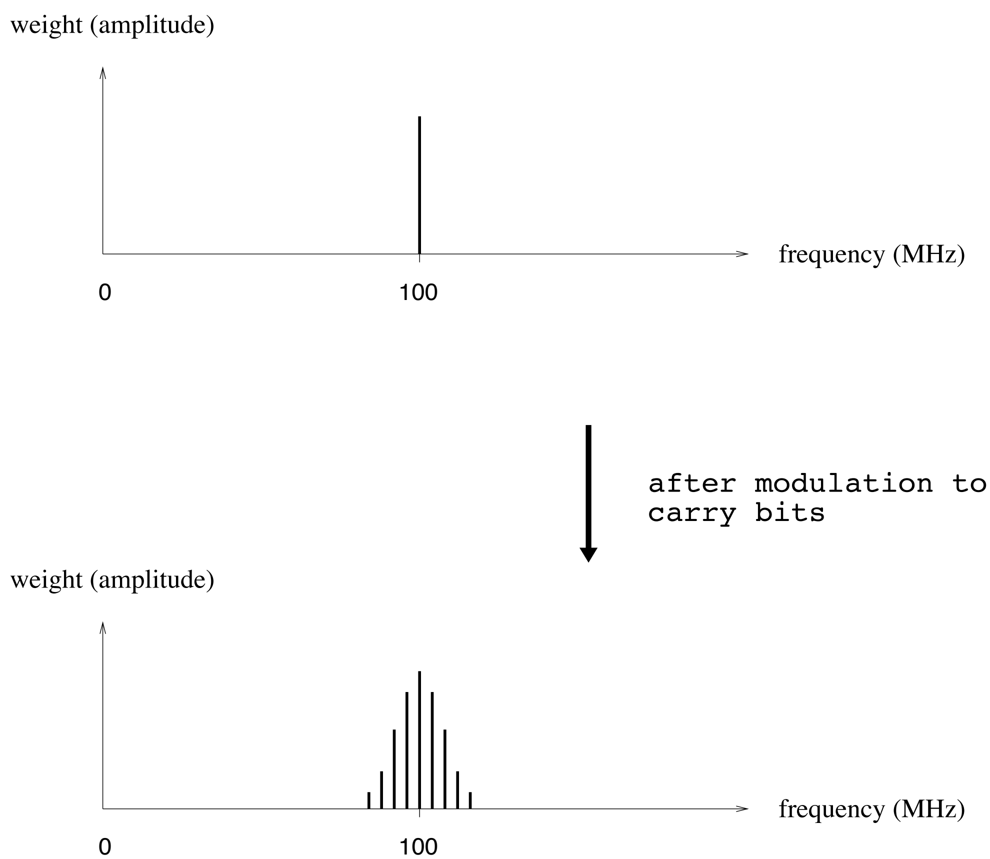
$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} a_f e^{ift} df$$

→ weight a_f signifies how important frequency f is for synthesis

→ for pure sinusoid of frequency f, $a_g = 0$ for all $g \neq f$

→ sending sequence of bits over carrier frequency f makes some $a_g \neq 0$

Sending single bit vs. stream of bits using carrier frequency $f = 100$ MHz



→ signature changes from single impulse to bump

→ a_g values near a_f become non-zero

→ new signature of modulated signal $s(t)$

What other sinusoids (besides 100 MHz) need to be added together to get signal $s(t)$?

If $s(t)$ only needs a bounded range of frequencies

→ bandlimited

→ practical real-world signals

→ bounded range is called $s(t)$'s bandwidth (Hz)

"Bandwidth" overloaded term: not to be confused with

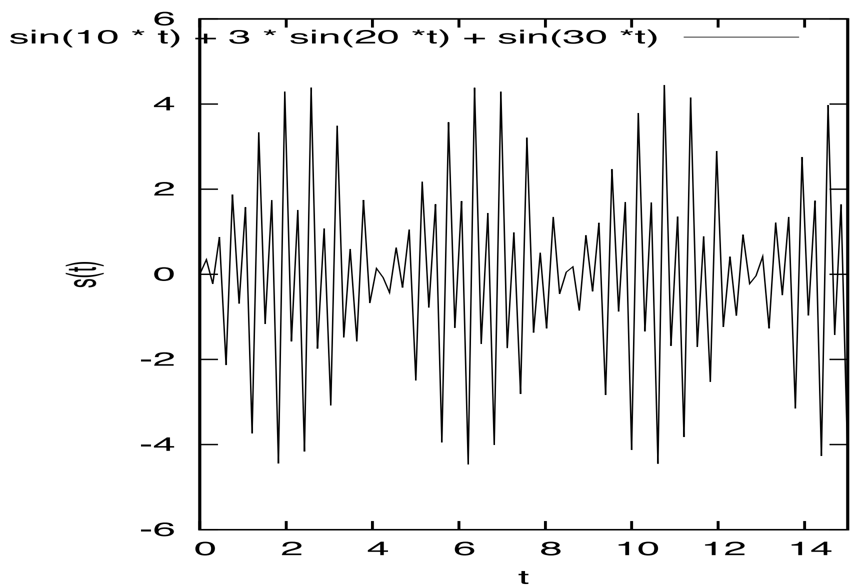
- bps bandwidth (e.g., 1 Gbps link)
- transmission medium (optical fiber, copper, wireless) characterized by bandwidth (Hz)

Networking deals with bandlimited signals

→ when not bandlimited approximate as bandlimited

Example: signal created by

$\rightarrow s(t) = \sin 10t + 3\, \sin 20t + \sin 30t$



Spectrum of signal

$\rightarrow$ 10 Hz: 1

$\rightarrow$ 20 Hz: 3 (contributes most)

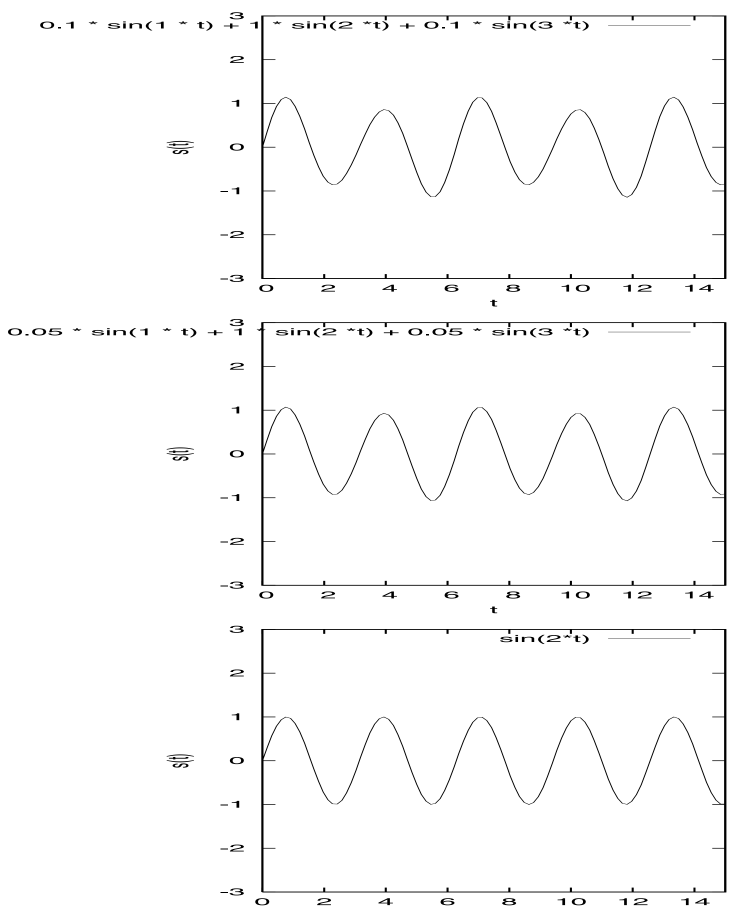
$\rightarrow$ 30 Hz: 1

Example: three signals created by

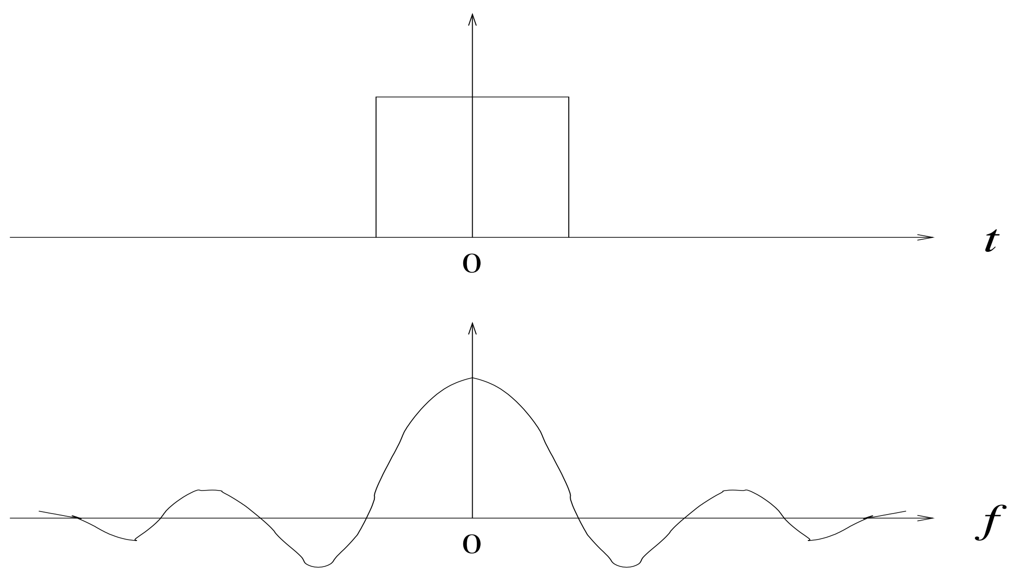
$\rightarrow s(t) = 0.1 \sin 1t + \sin 2t + 0.1 \sin 3t$

$\rightarrow s(t) = 0.05 \sin 1t + \sin 2t + 0.05 \sin 3t$

$\rightarrow s(t) = \sin 2t$



Example: spectrum of square wave



→ signal considered difficult to synthesize using sinusoids

→ infinite spectrum: sinc function

Practical approach: since sinusoids with small weights don't contribute much

→ ignore: approximation

→ i.e., treat as if weights are zero

→ cut-off and approximate

Back to one sender transmitting multiple bits:

→ using AM to send bits causes frequency spreading

→ from single impulse to bump in frequency domain

Causes problem for multiple senders if their bumps overlap

→ interference

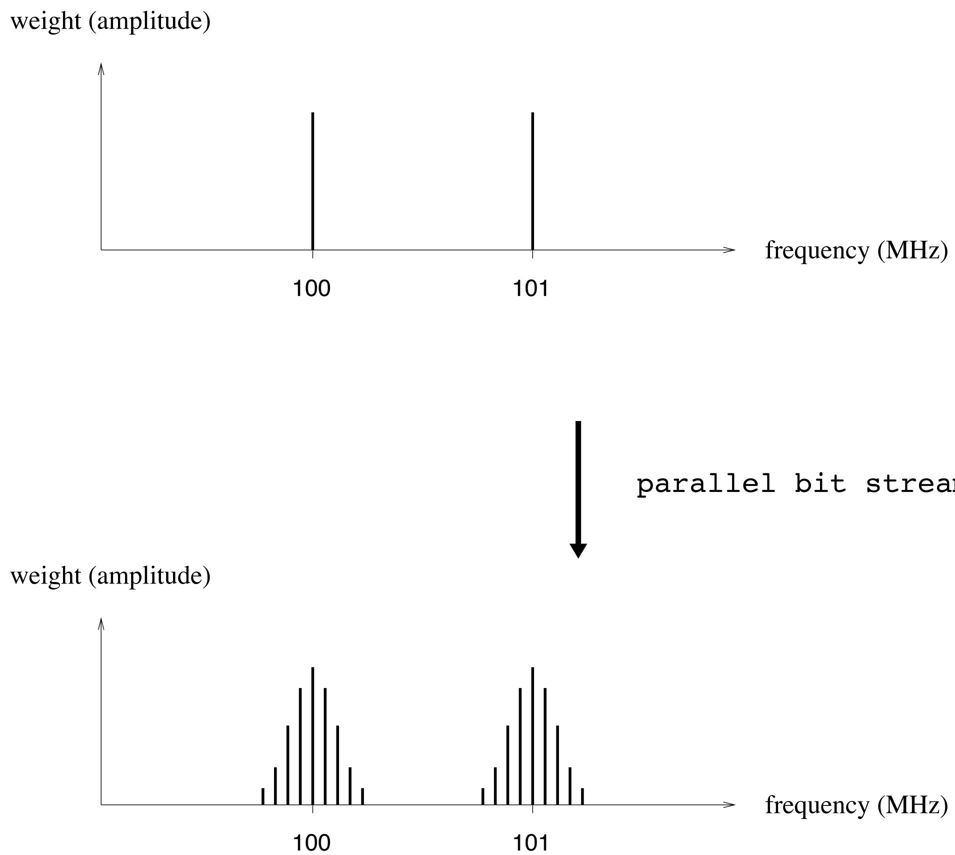
Inter-channel interference (ICI):

→ distortion introduced by spreading can make decoding bits difficult

→ simple filtering does not work

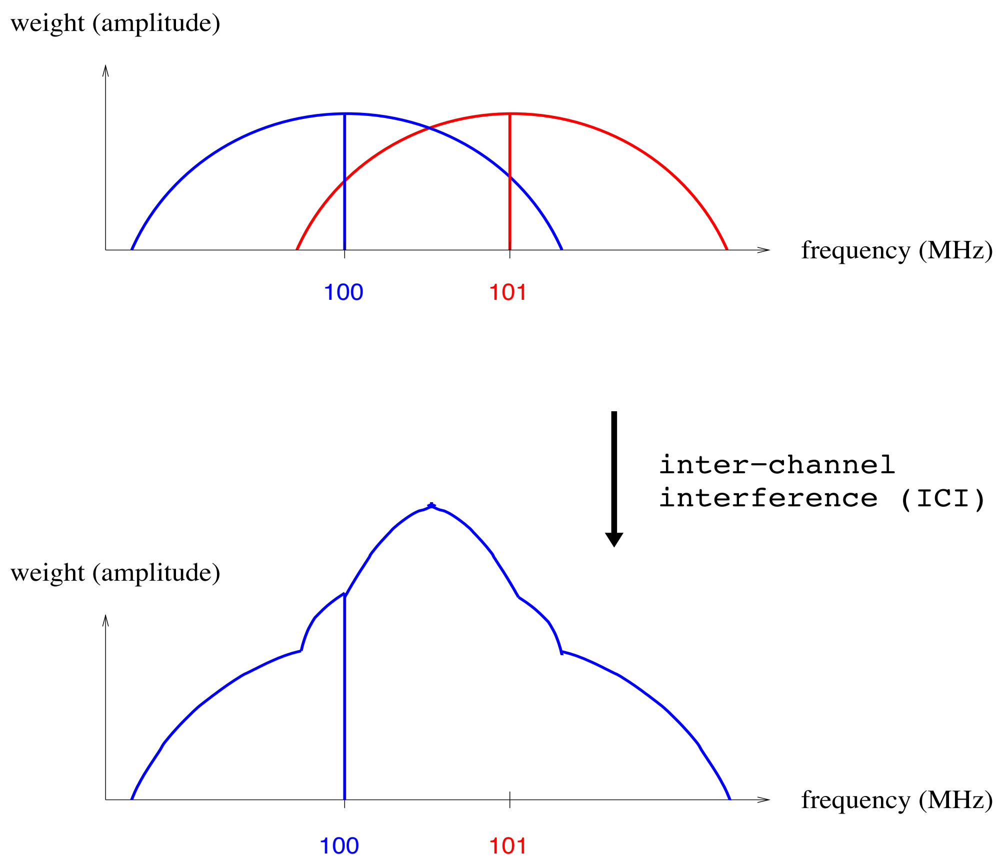
Example: two parallel bit streams carried on carrier frequencies 100 MHz and 101 MHz

Good case: ICI does not impact decoding bits



→ signal spreading around 100 MHz and 101 MHz does not overlap

Bad case: ICI impacts decoding bits



→ superposition of overlapping spectra

→ Alice and Bob will have difficulty decoding their respective bit streams

→ inverse problem

Thus: to prevent ICI sufficiently separate neighboring carrier frequencies

$\rightarrow$ guardband

$\rightarrow$ traditional FDM approach: still used today

Drawback: limits how many carrier frequencies can be squeezed in a given frequency range

$\rightarrow$ i.e., spectral efficiency is low

$\rightarrow$ network bandwidth is a scarce resource

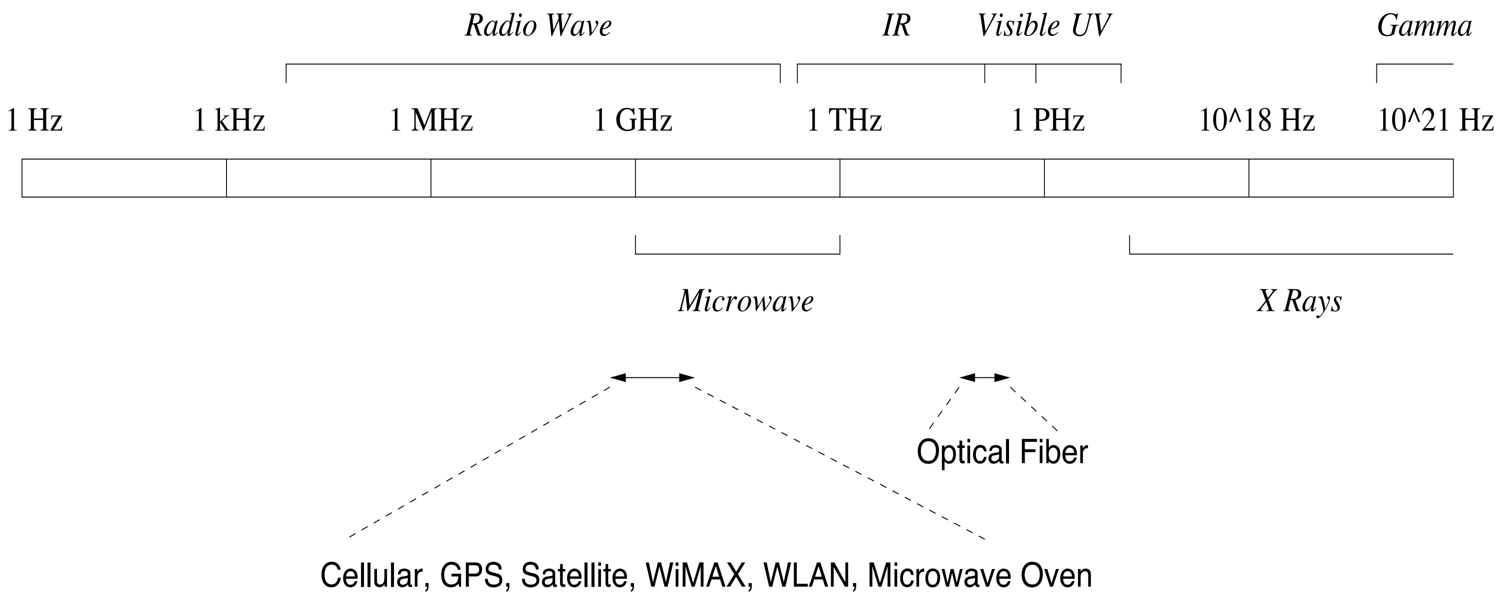
First: all physical media possess bandwidth range (Hz) within which signals can be feasibly communicated

$\rightarrow$ outside the physical medium's bandwidth: distortion too high

$\rightarrow$ technology limitation

Second: bandwidth constraint: regulation

→ wireless electromagnetic spectrum



→ logarithmic scale

→ wireless spectrum: crowded

→ spectral efficiency is premium

Modern approach to FDM: orthogonal FDM

$\rightarrow$ use sinusoids that are mutually orthogonal

$\rightarrow$ over finite time window τ

Complex sinusoids e^{if_it} and e^{if_jt} are orthogonal ($f_i \neq f_j$):

$$e^{if_it} \circ e^{if_jt} = \int_{-\infty}^{\infty} e^{if_it} e^{-if_jt} dt = 0$$

Want:

$$\int_{-\tau/2}^{\tau/2} e^{if_it} e^{-if_jt} dt = 0$$

$\rightarrow$ orthogonal $f_1, f_2, \ldots, f_n$ over $[-\tau/2, \tau/2]$

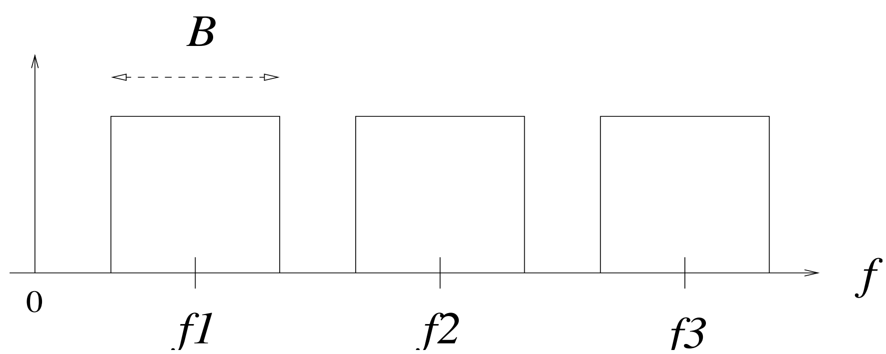
$\rightarrow$ finite support

$\rightarrow$ τ is similar to baud: called symbol period

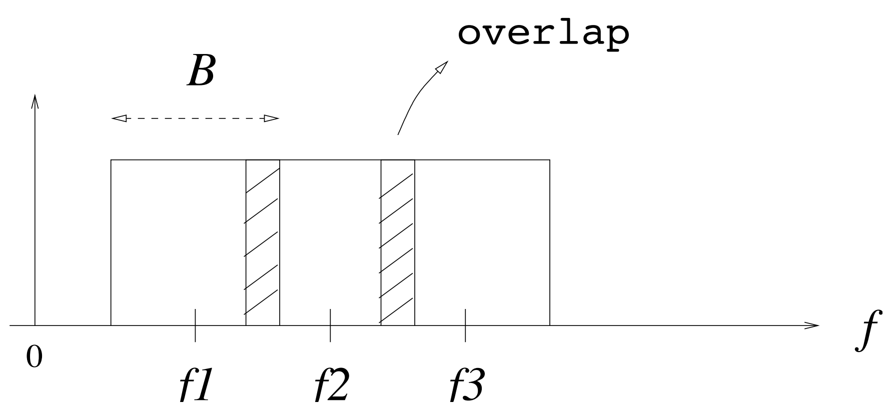
$\rightarrow$ duration of single bit

OFDM's advantage over FDM:

FDM:



OFDM:



→ spectra can overlap

Pack more carrier frequencies within given a frequency band

$\rightarrow$ enhanced spectral efficiency

Technique considered since the mid-1960s

$\rightarrow$ practically feasible: DSP advances

$\rightarrow$ FFT, inverse fast Fourier transform (IFFT)

$\rightarrow$ popular in both wireless and wired networks

How to find n mutually orthogonal sinusoids over finite time interval:

$\rightarrow$ choose harmonics of base frequency

Procedure: given target frequency band between f_a and f_b

$\rightarrow$ bandwidth: $W = f_b - f_a$ (Hz)

- choose n carrier frequencies:

  $\rightarrow f_a + (W/n),\ f_a + 2(W/n),\ \ldots,\ f_a + n(W/n)$

  $\rightarrow$ carrier frequency spacing W/n

- set symbol period as $\tau = n/W$

  $\rightarrow$ increasing n results in increased symbol period: decrease in bps

  $\rightarrow$ increasing W results in decreased symbol period: increase in bps

Note:

- harmonic carrier frequencies $f_1, f_2, \ldots, f_n$ are mutually orthogonal over finite time interval $[-\tau/2, \tau/2]$
- every carrier frequency transmits one bit over symbol period τ
- f_n (higher frequency carrier) transmits at same speed (bps) as f_1 (lower frequency carrier)

  $\rightarrow$ same symbol period τ

Trade-off:

- the more carrier frequencies n allocated within available bandwidth W, the longer the bit duration

  $\rightarrow$ time dilation
- the wider the bandwidth W, the shorter the bit duration

  $\rightarrow$ faster transmission

Conservation law: total bps over n parallel bit streams is conserved (i.e., remains invariant)

$\longrightarrow$ e.g., AM with 2 levels

- one carrier f_i: $1/\tau$ bps
- all carriers (i.e., total capacity bps): $n \times 1/\tau$ bps

$\rightarrow$ equals W since $\tau = n/W$

$\rightarrow$ independent of n

$\rightarrow$ no free lunch

Reflects Shannon's second theorem

$\rightarrow$ total capacity $\propto W$

$\rightarrow$ extra component: impact of noise

Example:

- Available bandwidth: $f_a = 2.4$ GHz, $f_b = 2.5$ GHz

  $\rightarrow W = 100$ MHz

- Number of carrier frequencies: $n = 100$

  $\rightarrow$ carrier spacing 100 MHz / 100 = 1 MHz

  $\rightarrow$ 2.401 GHz, 2.402 GHz, $\ldots$, 2.5 GHz

- Symbol period: $\tau = 100/100$ MHz $= 1$ μsec

  $\rightarrow$ AM with 2 levels: 1 Mbps per user

  $\rightarrow$ total: 100 Mbps

Note:

- speed (bps) determined by bandwidth W, not base frequency f

  → W = 100 MHz (2.5 GHz - 2.4 GHz)

  → not absolute frequency 2.4 GHz

  → same bps for W = 100 MHz (5.5 GHz - 5.4 GHz)

- OFDM information processing takes place at 1 μsec time granularity

  → computational load affected by W and n

  → much slower than 2.4 GHz

- actual carrier frequency and physical transmission is separate translation step

  → multiply 1 MHz bandwidth signal by 2.4 GHz sinusoid

  → 1 MHz data carrying signals get shifted to 2.4-2.5 GHz band

Wireless network example: IEEE 802.11g WLANs

→ 2.4 GHz band

→ uses OFDM

→ $W = 20$ MHz, $n = 64$

→ carrier spacing 20 MHz / 64 = 312.5 kHz

→ symbol time $\tau = 3.2\ \mu$s

But: OFDM not used to support 64 users

→ i.e., not OFDMA

→ one user uses all 64 frequencies

→ multiple access (MA), i.e., sharing of bandwidth among users solved using higher layer protocol

→ CSMA/CA

Similar approach for 802.11a/n in 5 GHz band.

→ Wi-Fi 5

IEEE 802.11ax supports OFDMA

→ Wi-Fi 6

→ fundamentally different from CSMA/CA which is collision based

→ same for Wi-Fi 7 (802.11be)

→ ratified in 2024

Wired network example: ADSL

$\rightarrow$ part of ITU G.992.1 standard

$\rightarrow$ UTP (unshielded twisted pair) copper wire

$\rightarrow$ $W = 1.104$ MHz, $n = 256$

$\rightarrow$ carrier spacing 4.3125 kHz

$\rightarrow$ OFDM, not OFDMA

OFDMA under consideration for optical fiber communication

$\rightarrow$ extends WDM