Back to sending multiple bits using FDM

→ using AM to send bits causes frequency spreading

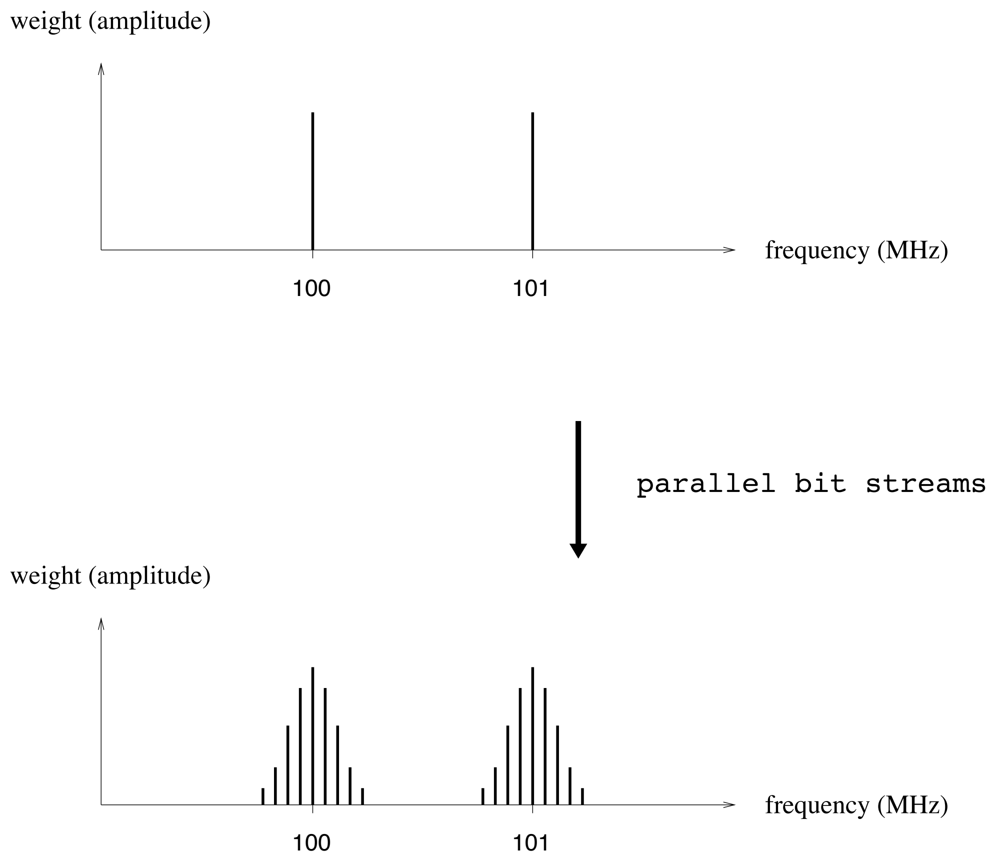
→ problematic because may cause interference

Inter-channel interference (ICI):

→ distortion introduced by spreading can make decoding bits difficult

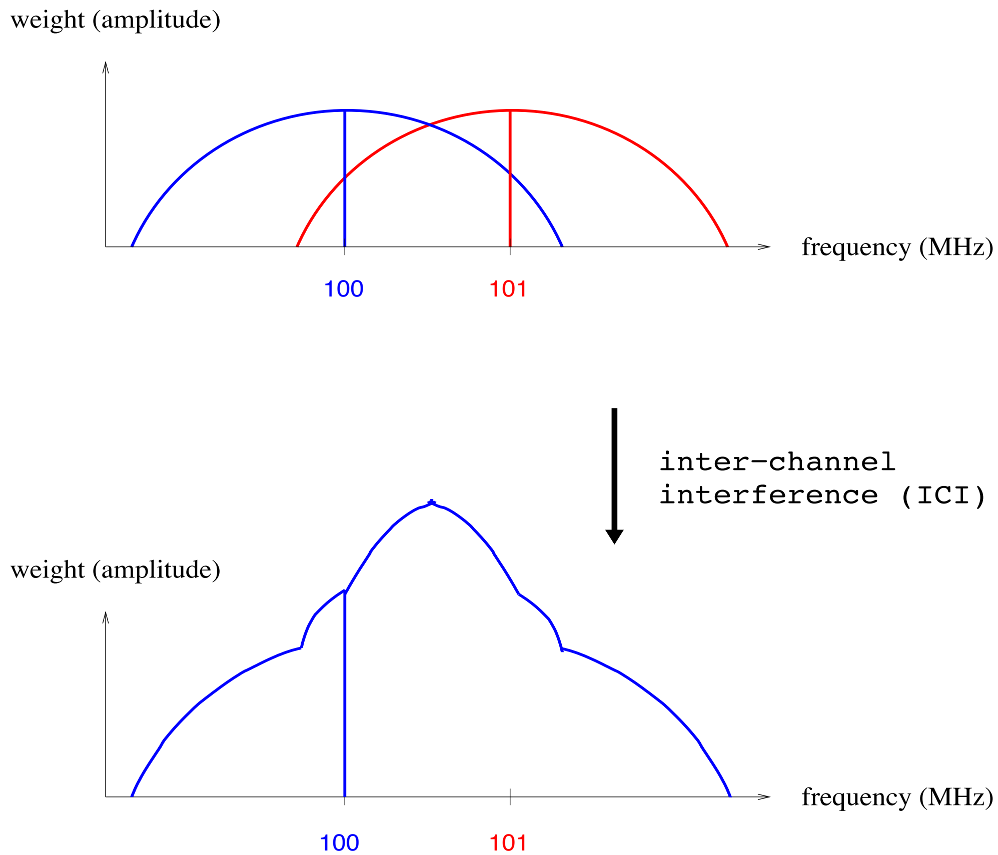
Example: two parallel bit streams carried on carrier frequencies 100 MHz and 101 MHz

Good case: ICI does not impact decoding bits



→ signal spreading around 100 MHz and 101 MHz does not overlap

Bad case: ICI impacts decoding bits



$\rightarrow$ superposition of overlapping spectra

$\rightarrow$ Alice and Bob will have difficulty decoding their respective bit streams

To prevent ICI: sufficiently separate neighboring carrier frequencies

→ guardband

→ traditional FDM approach

Drawback: limits how many carrier frequencies can be squeezed in a given frequency range

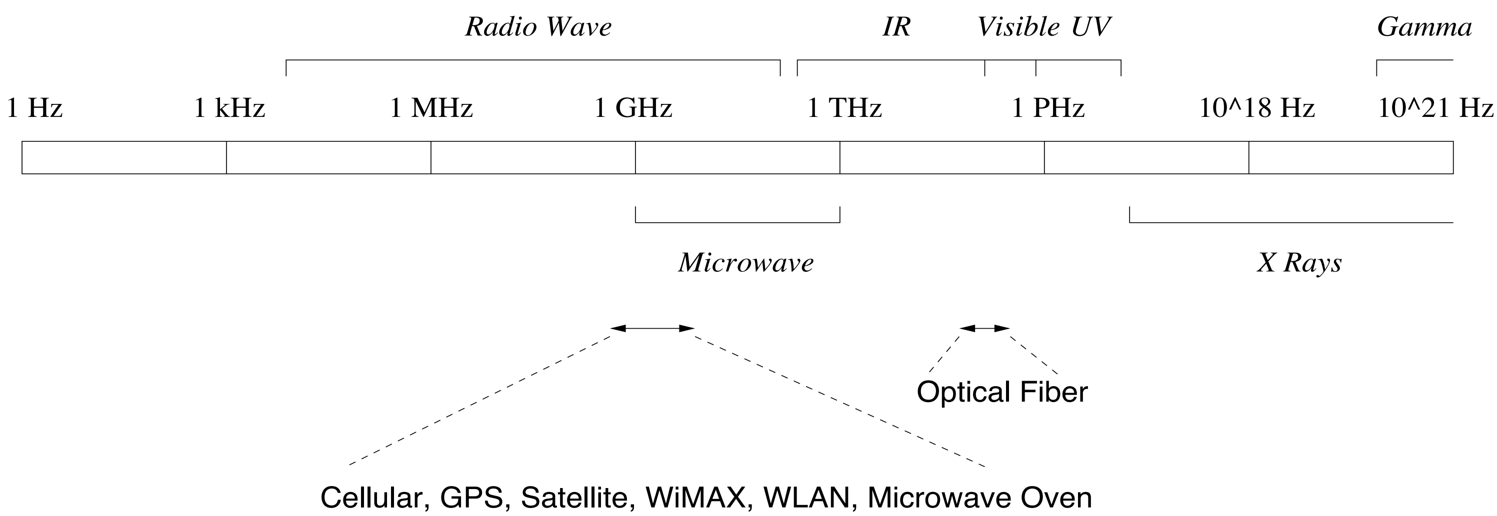
→ i.e., spectral efficiency is low

→ network bandwidth is a scarce resource

Electromagnetic spectrum (wireless):

→ some of its data communication use today

→ logarithmic scale



→ crowded

→ spectral efficiency a premium

Modern approach to FDM: orthogonal FDM

→ use sinusoids that are mutually orthogonal

→ over finite time window τ

Complex sinusoids e^{if_it} and e^{if_jt} are orthogonal ($f_i \neq f_j$):

$$e^{if_it} \circ e^{if_jt} = \int_{-\infty}^{\infty} e^{if_it} e^{-if_jt} dt = 0$$

Want:

$$\int_{-\tau/2}^{\tau/2} e^{if_it} e^{-if_jt} dt = 0$$

→ orthogonal $f_1, f_2, \ldots, f_n$

→ τ is similar to baud, called symbol period

→ duration of single bit

Transmit multiple bits using n orthogonal sinusoids with frequencies $f_1, f_2, \ldots, f_n$:

$$s(t) = \sum_{k=1}^{n} a_k e^{if_k t}$$

$\rightarrow$ hide bits in a_k

$\rightarrow$ note: a_k is complex number

$\rightarrow$ AM, PM, combination

To decode bit on carrier f_k:

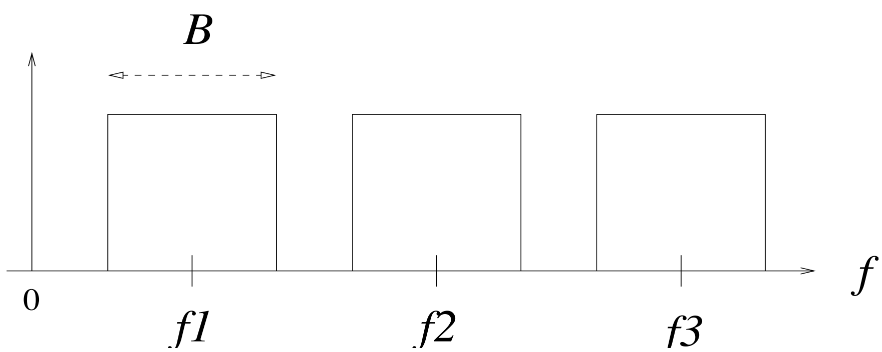
$\rightarrow$ $a_k = s(t) \circ e^{if_k t}$

$\rightarrow$ perform Fourier transform over time window τ

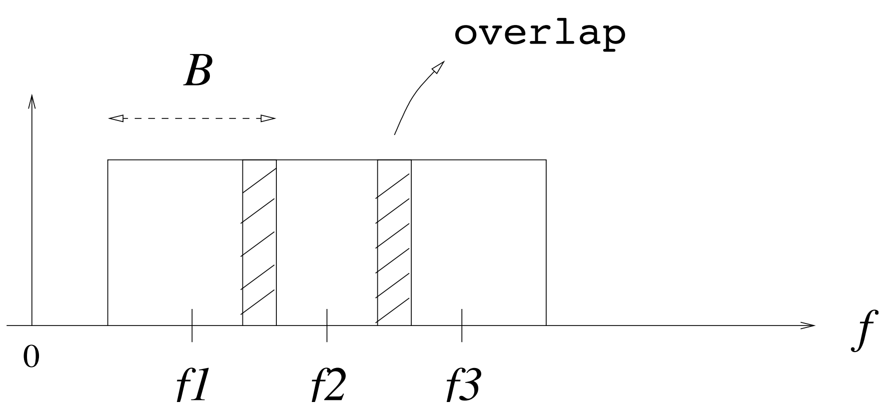
$\rightarrow$ orthogonality eliminates ICI issue

OFDM's advantage over FDM:

FDM:



OFDM:



$\rightarrow$ spectra allowed to overlap

Can pack more carrier frequencies within given a frequency band

→ enhanced spectral efficiency

Technique considered since the mid-1960s

→ only recently practically feasible: DSP advances

→ FFT, inverse fast Fourier transform (IFFT)

→ demand for ever increasing bandwidth

→ popular in both wireless and wired networks

How to find n mutually orthogonal sinusoids over finite time interval:

$\rightarrow$ choose harmonics of base frequency

Given available frequency band between f_a and f_b

$\rightarrow$ bandwidth: $W = f_b - f_a$

- Choose n carrier frequencies:

  $\rightarrow$ carrier frequency spacing W/n

  $\rightarrow$ $f_a + (W/n)$, $f_a + 2(W/n)$, $\ldots$, $f_a + n(W/n)$

- Symbol period $\tau = n/W$

  $\rightarrow$ increasing n results in increased symbol period and decrease in bps

  $\rightarrow$ increasing W results in decreased symbol period and increase in bps

  $\rightarrow$ no free lunch

Example:

- Available bandwidth range: $f_a = 2.4$ GHz, $f_b = 2.5$ GHz, $W = 100$ MHz
- Number of carrier frequencies: $n = 100$

  $\rightarrow$ carrier spacing 100 MHz / 100 = 1 MHz

  $\rightarrow$ 2.401 GHz, 2.402 GHz, $\ldots$, 2.5 GHz
- Symbol period: $\tau = 100/100$ MHz = 1 μsec

  $\rightarrow$ AM with 2 levels: 1 Mbps per user, 100 Mbps total

In the above example, there are 100 orthogonal carrier frequencies. Each carries one bit during time interval of length 1 μsec.

$\rightarrow$ not at clock rate 2.4 GHz

$\rightarrow$ 1 μsec symbol period implies 1 MHz baud rate

Note: primary is bandwidth

→ 0.1 MHz = 2.5 GHz - 2.4 GHz

→ not absolute frequency 2.4 GHz

OFDM information processing takes place at 1 μsec time granularity

→ 1 MHz

→ much slower than 2.4 GHz

Since only allowed to use 2.4-2.5 GHz band, final step involves multiplying 1 MHz bandwidth signal by 2.4 GHz sinusoid.

→ basic property: time domain multiplication becomes shift operation in frequency domain

→ 1 MHz data carrying signal gets shifted to 2.4-2.5 GHz band

Wireless network example: IEEE 802.11g WLANs

→ 2.4 GHz band

→ uses OFDM

→ $W = 20$ MHz, $n = 64$

→ carrier spacing 20 MHz / 64 = 312.5 kHz

→ symbol time $\tau = 3.2$ μs

But: OFDM not used to support 64 users

→ i.e., not OFDMA

→ one user uses all 64 frequencies

→ multiple access (MA), i.e., sharing of bandwidth among users solved using higher layer protocol

→ CSMA/CA

Similar approach for 802.11a/n in 5 GHz band.

→ Wi-Fi 5

IEEE 802.11ax supports OFDMA

→ Wi-Fi 6

→ fundamentally different from CSMA/CA which is collision based

→ same for Wi-Fi 7 (802.11be)

→ close to ratification in 2024

Wired network example: ADSL

→ part of ITU G.992.1 standard

→ UTP (unshielded twisted pair) copper wire

→ $W = 1.104$ MHz, $n = 256$

→ carrier spacing 4.3125 kHz

→ OFDM, not OFDMA

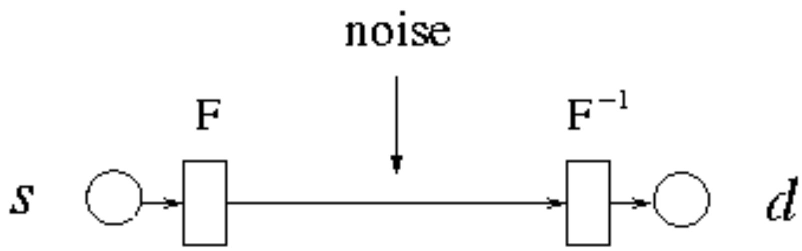
OFDMA under consideration for optical fiber communication

→ extends WDM

Fundamental question: How much throughput can we squeeze out from a network link

→ upper bound on capacity: reliable throughput

→ information transmission under noise



Impact of noise:

$\longrightarrow$ encoding/decoding: $a \mapsto w_a \mapsto w \mapsto ?$

→ w_a gets corrupted, i.e., becomes w

→ if $w = w_b$, incorrectly conclude b as symbol

- Detect w is corrupted

  $\rightarrow$ error detection

- Correct w back to w_a

  $\rightarrow$ error correction

Shannon showed that there is a fundamental limit to achieving reliable data transmission.

- the wider the bandwidth (Hz) the higher the reliable throughput

  $\rightarrow$ bandwidth of physical medium (i.e., channel)

- the noisier the channel, the smaller the reliable throughput

  $\rightarrow$ overhead incurred dealing with corrupted bits

Quantitative captured in a formula.

Channel Coding Theorem (Shannon): Given bandwidth W, signal power P_S, noise power P_N, channel subject to white noise,

$$C \;=\; W \log\left(1 + \frac{P_S}{P_N}\right) \text{ bps}$$

$\rightarrow$ P_S/P_N: signal-to-noise ratio (SNR)

$\rightarrow$ increasing power yields logarithmic gain

Implications for networking:

- Increase bandwidth W (Hz) to proportionally increase reliable throughput

  → e.g., FDM, OFDM, TDM

- Power control (e.g., handheld wireless devices)

  → trade-off w.r.t. battery power

  → trade-off w.r.t. multi-user interference: doesn't work if everyone increases power

  → signal-to-interference ratio (SIR)

Signal-to-noise ratio (SNR) expressed as

$$\text{dB} = 10 \log_{10}(P_S/P_N)$$

Example: assuming a decibel level of 10, what is the channel capacity of a telephone line?

First, $W = 3000$ Hz, $P_S/P_N = 1000$. Using Channel Coding Theorem,

$$C = 3000 \log 1001 \approx 30 \text{ Kbps.}$$

$\rightarrow$ compare against 28.8 Kbps modems

$\rightarrow$ what about 56 Kbps modems?

$\rightarrow$ inaccurate assumptions

Nyquist's sampling criterion:

$\rightarrow$ digitize analog signal: time and amplitude

$\rightarrow$ key issue: digitizing time

$\rightarrow$ continuous time signal to discrete time samples

Sampling Theorem (Nyquist): Given continuous bandlimited signal $s(t)$ with bandwidth W (Hz), $s(t)$ can be reconstructed from its samples if

$$\nu > 2W$$

where ν is the sampling rate.

$\rightarrow$ ν: samples per second

Human auditory system:

$\rightarrow$ sensitivity: 20 Hz–20 KHz range (roughly 20 KHz)

$\rightarrow$ voice: 300 Hz–3.3 KHz (roughly 4 KHz)

$\rightarrow$ 8000 samples per second

T1 TDMA line: 1.544 Mbps

$\rightarrow$ frame size 193 (24 users, 8 bits-per-user, 1 preamble bit)

$\rightarrow$ 8000 samples per second

$\rightarrow$ $193 \times 8000 = 1.544$ Mbps

CD quality audio: 44100 samples per second

$\rightarrow$ also denoted Hz (44.1 KHz)