Modern approach to FDM: orthogonal FDM
→ use sinusoids that are mutually orthogonal
→ over finite time window $\tau$

Complex sinusoids $e^{if_it}$ and $e^{if_jt}$ are orthogonal ($f_i \neq f_j$):

$$e^{if_it} \circ e^{if_jt} = \int_{-\infty}^{\infty} e^{if_it} e^{-if_jt} dt = 0$$

Want:

$$\int_{-\tau}^{\tau} e^{if_it} e^{-if_jt} dt = 0$$

→ orthogonal $f_1, f_2, \ldots, f_n$

→ $\tau$ is similar to baud, called symbol period
Transmit multiple bits on \( n \) orthogonal sinusoids with frequencies \( f_1, f_2, \ldots, f_n \):

\[
s(t) = \sum_{k=1}^{n} a_k e^{i f_k t}
\]

\( \rightarrow \) hide bits in \( a_k \)

\( \rightarrow \) AM, PM, combination

To decode bit on carrier \( f_k \):

\( \rightarrow \) \( a_k = s(t) \circ e^{i f_k t} \)

\( \rightarrow \) perform Fourier transform over time window \( \tau \)

\( \rightarrow \) orthogonality eliminates ICI issue
OFDM’s advantage over FDM:

FDM:

\[
\begin{array}{c}
0 \\
\hline
f_1 \\
\hline
f_2 \\
\hline
f_3
\end{array}
\]

\[
B \text{ overlap}
\]

OFDM:

\[
\begin{array}{c}
0 \\
\hline
f_1 \\
\hline
f_2 \\
\hline
f_3
\end{array}
\]

→ spectra allowed to overlap
Can pack more carrier frequencies within given a frequency band

→ enhanced spectral efficiency

Technique discussed since the mid-1960s

→ only recently practically feasible: DSP advances
→ FFT, inverse fast Fourier transform (IFFT)
→ demand for ever increasing bandwidth
→ additional benefits
→ popular in both wireless and wired networks
How to get \( n \) mutually orthogonal sinusoids over finite time interval:

→ choose harmonics of base frequency

Given available frequency between \( f_a \) and \( f_b \)

→ bandwidth: \( W = f_b - f_a \)

- Choose \( n \) carrier frequencies:
  → carrier frequency spacing \( W/n \)
  → \( f_a, f_a + (W/n), f_a + 2(W/n), \ldots, f_a + (n - 1)(W/n) \)

- Symbol period \( \tau = n/W \)
  → increasing \( n \) results in increased symbol period and decrease in bps
  → increasing \( W \) results in decreased symbol period and increase in bps
  → no free lunch
Example:

- Available bandwidth: \( f_a = 2.4 \) GHz, \( f_b = 2.5 \) GHz, \( W = 100 \) MHz

- Number of carrier frequencies: \( n = 100 \)
  \[ \text{carrier spacing } 100 \text{ MHz} / 100 = 1 \text{ MHz} \]
  \[ \rightarrow 2.4 \text{ GHz, 2.401 GHz, 2.402 GHz, \ldots, 2.499 GHz} \]

- Symbol period: \( \tau = 100/100 \text{ MHz} = 1 \mu\text{sec} \)
  \[ \rightarrow \text{AM with 2 levels: 1 Mbps per user, 100 Mbps total} \]

In the above example, there are 100 orthogonal carrier frequencies but each carries one bit during time interval of length 1 \( \mu\text{sec} \)

\[ \rightarrow \text{not at clock rate 2.4 GHz} \]

\[ \rightarrow \text{why not transmit bits using TDMA at 2.4 GHz?} \]
Wireless network example: IEEE 802.11g WLANs
→ 2.4 GHz band
→ uses OFDM
→ $W = 20$ MHz, $n = 64$
→ carrier spacing $20$ MHz / $64 = 312.5$ kHz
→ symbol time $\tau = 3.2 \ \mu s$

But: OFDM not used to support 64 users
→ i.e., not OFDMA
→ one user uses all 64 frequencies
→ multiple access (MA), i.e., sharing of bandwidth among users solved using higher layer protocol
→ CSMA/CA

Similar approach for 802.11a/n in 5 GHz band.
Wired network example: ADSL

→ part of ITU G.992.1 standard

→ UTP (unshielded twisted pair) copper wire

→ $W = 1.104 \text{ MHz}, n = 256$

→ carrier spacing $4.3125 \text{ kHz}$

→ again OFDM, not OFDMA
How much throughput can we squeeze out from a network link

→ upper bound on capacity: reliable throughput

→ information transmission under noise

Impact of noise:

→ encoding/decoding: \( a \mapsto w_a \mapsto w \mapsto ? \)

→ \( w_a \) gets corrupted, i.e., becomes \( w \)

→ if \( w = w_b \), incorrectly conclude \( b \) as symbol
• Detect $w$ is corrupted
  $\rightarrow$ error detection
• Correct $w$ back to $w_a$
  $\rightarrow$ error correction

Shannon showed that there is a fundamental limit to achieving reliable data transmission.

• the wider the bandwidth (Hz) the higher the reliable throughput
  $\rightarrow$ bandwidth of physical medium (i.e., channel)
• the noisier the channel, the smaller the reliable throughput
  $\rightarrow$ overhead incurred dealing with corrupted bits

Quantitative captured in a formula.
Channel Coding Theorem (Shannon): Given bandwidth $W$, signal power $P_S$, noise power $P_N$, channel subject to white noise,

$$C = W \log \left( 1 + \frac{P_S}{P_N} \right) \text{ bps}$$

→ $P_S/P_N$: signal-to-noise ratio (SNR)

→ increasing power yields logarithmic gain
Implications for networking:

- Increase bandwidth $W$ (Hz) to proportionally increase reliable throughput
  $\rightarrow$ e.g., FDM, OFDM, TDM

- Power control (e.g., handheld wireless devices)
  $\rightarrow$ trade-off w.r.t. battery power
  $\rightarrow$ trade-off w.r.t. multi-user interference: doesn’t work if everyone increases power
  $\rightarrow$ signal-to-interference ratio (SIR)
Signal-to-noise ratio (SNR) expressed as

$$\text{dB} = 10 \log_{10}(P_S/P_N)$$

Example: assuming a decibel level of 30, what is the channel capacity of a telephone line?

First, $W = 3000$ Hz, $P_S/P_N = 1000$. Using Channel Coding Theorem,

$$C = 3000 \log 1001 \approx 30 \text{ Kbps}.$$  

→ compare against 28.8 Kbps modems

→ what about 56 Kbps modems?

→ inaccurate assumptions
Nyquist’s sampling criterion:

→ digitize analog signal: time and amplitude

→ key issue: digitizing time

→ continuous time signal to discrete time samples

Sampling Theorem (Nyquist): Given continuous bandlimited signal $s(t)$ with bandwidth $W$ (Hz), $s(t)$ can be reconstructed from its samples if

$$\nu > 2W$$

where $\nu$ is the sampling rate.

→ $\nu$: samples per second
Human auditory system:

→ sensitivity: 20 Hz–20 KHz range (roughly 20 KHz)
→ voice: 300 Hz–3.3 KHz (roughly 4 KHz)
→ 8000 samples per second

T1 TDMA line: 1.544 Mbps

→ frame size 193 (24 users, 8 bits-per-user, 1 preamble bit)
→ 8000 samples per second
→ $193 \times 8000 = 1.544$ Mbps

CD quality audio: 44100 samples per second

→ also denoted Hz (44.1 KHz)