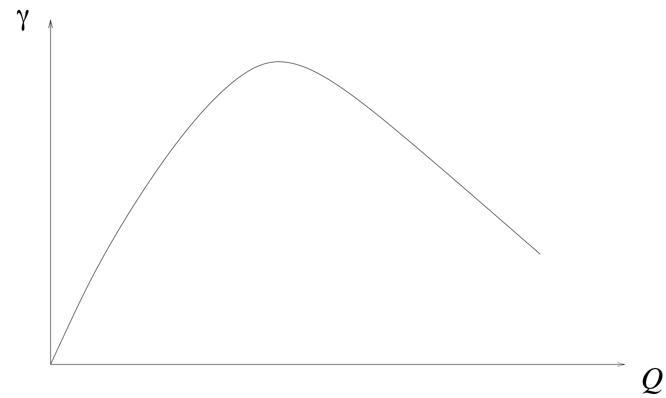
Congestion control for unimodal curve:

$\longrightarrow$ throughput decreases with excessive load



Consider performance of Method D:

$\longrightarrow$ $Q^* = 100$, $\varepsilon = 0.1$, $\beta = 0.5$

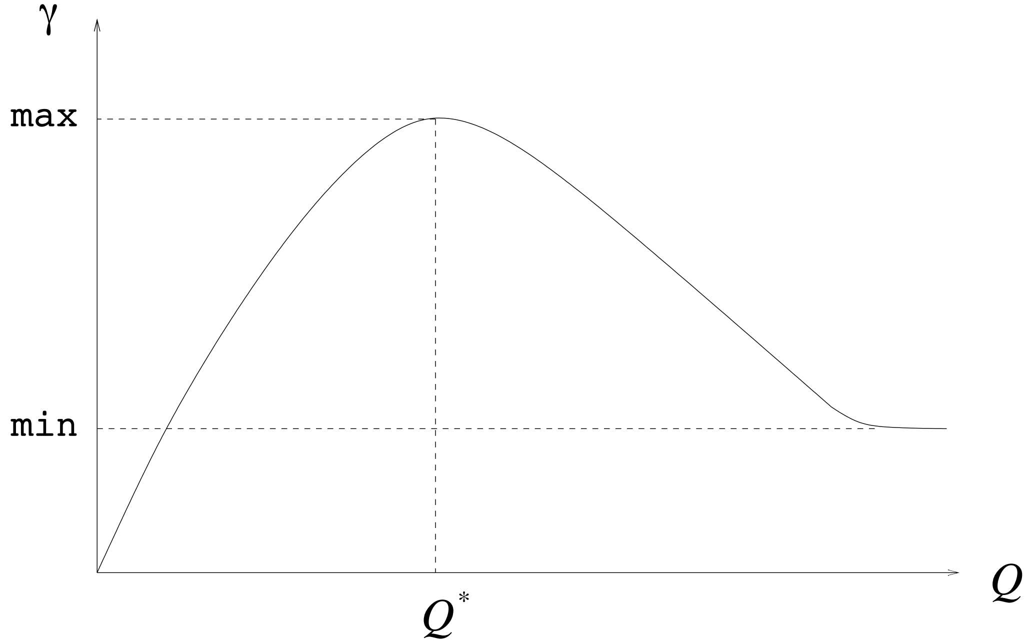
- Case 1: monotone

  $\rightarrow \gamma = 10$ for all Q

- Case 2: unimodal

  $\rightarrow \gamma = 10$ for $Q \leq Q^*$, but $\gamma = 10 - 2(Q^* - Q)$ for $Q > Q^*$ with minimum throughput $\gamma \geq 1$

Load-throughput curve:

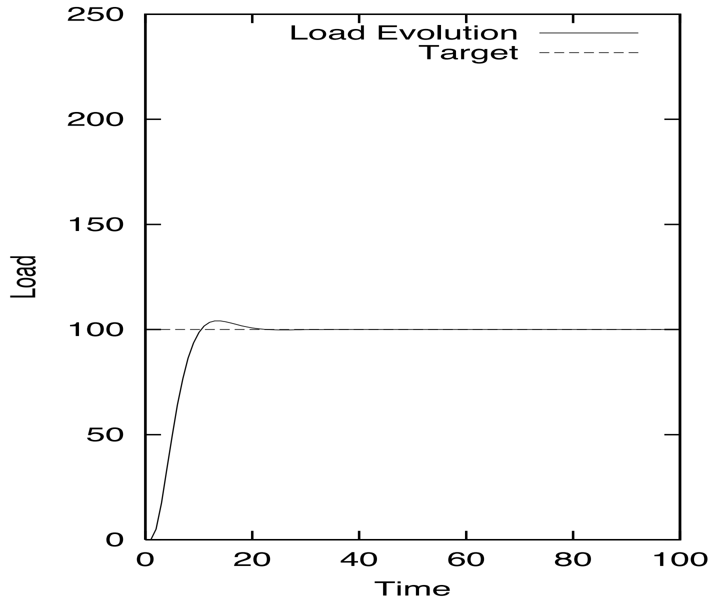


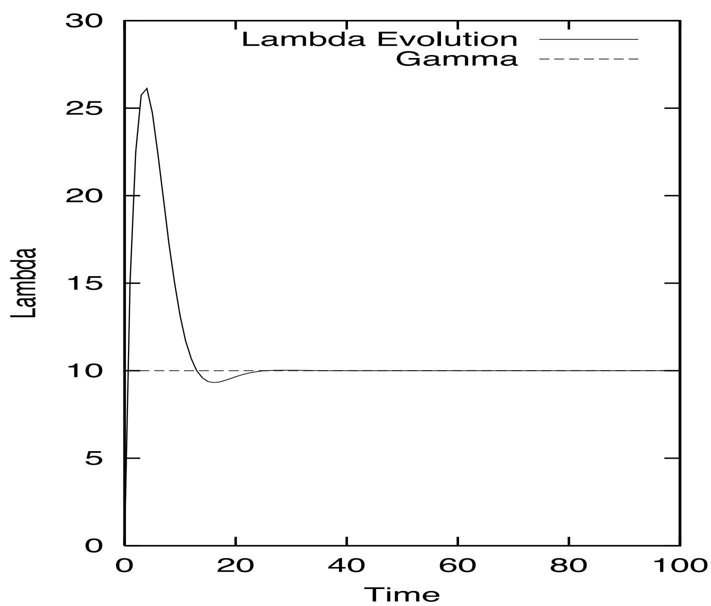
Compare time evolution of Method for Case 1 and 2

$\longrightarrow$ from (asymptotically) stable to unstable

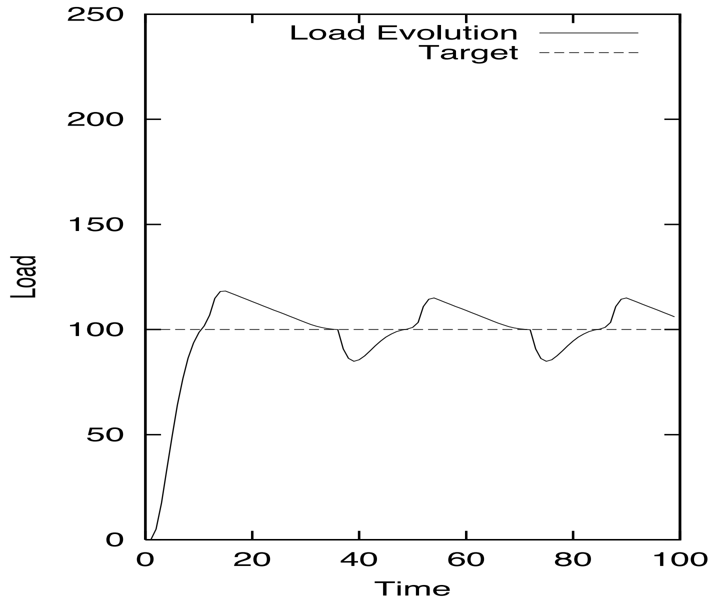
$\longrightarrow$ worst case: congestion collapse

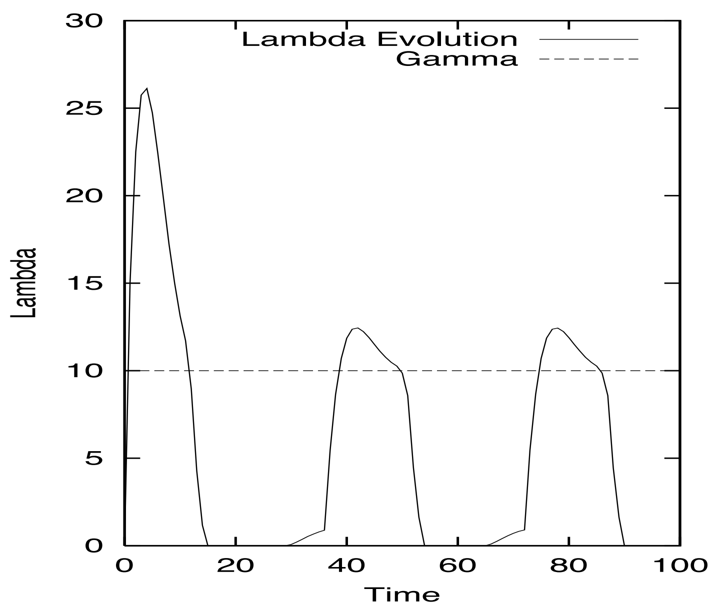
Monotone load-throughput curve:





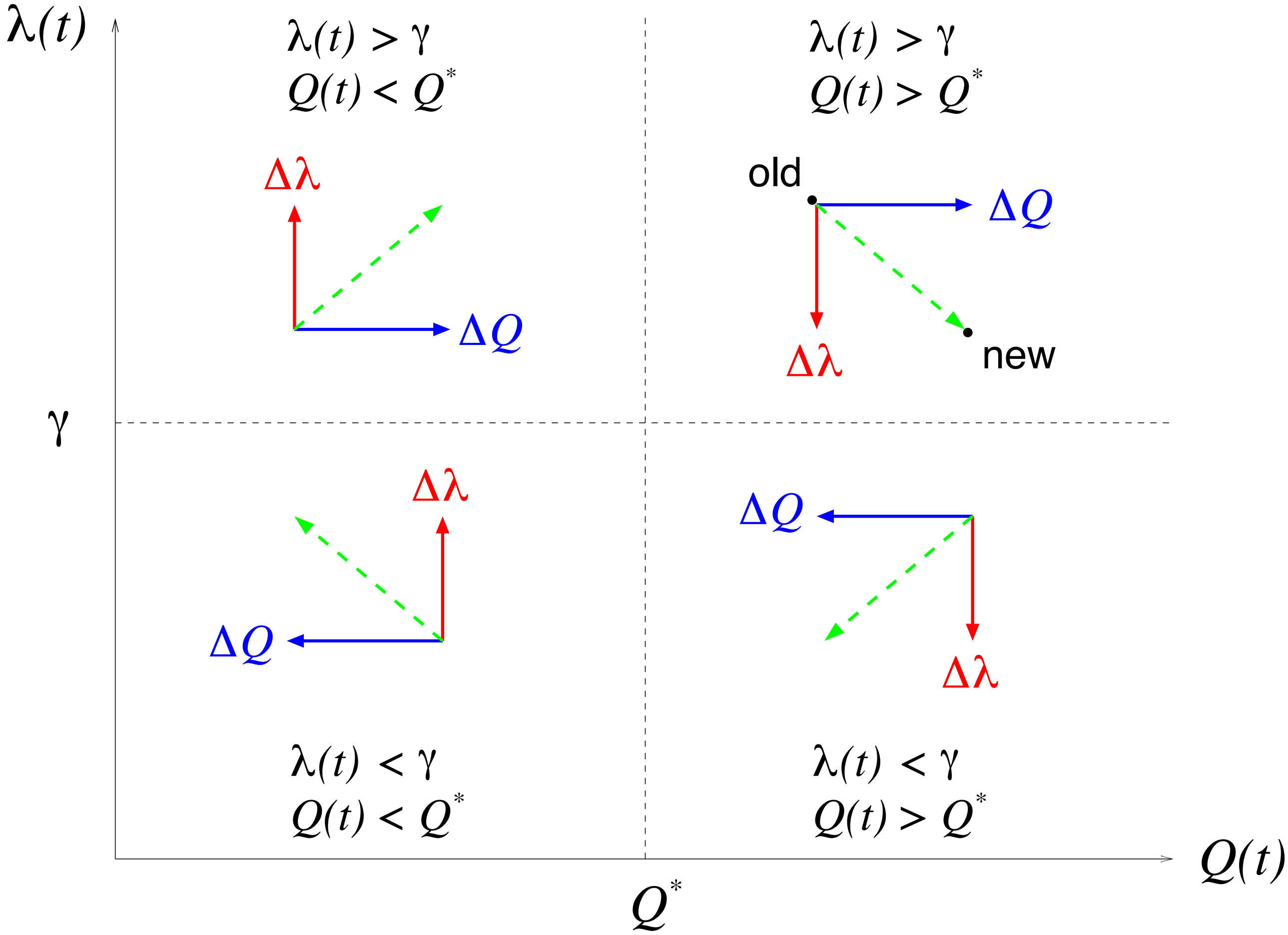
Unimodal load-throughput curve:





Working around in $(Q(t), \lambda(t))$-space:

$\longrightarrow$ phase space



Thus, in danger zone $Q(t) > Q^*$:

- Both $\lambda(t) \downarrow$ and $\gamma(Q) \downarrow$
- If $\lambda(t)$ doesn't decrease fast enough (relative to $\gamma(Q)$)

  $\rightarrow \lambda(t) - \gamma(Q) > 0$

  $\rightarrow Q(t)$ keeps on increasing

  $\rightarrow$ further aggrevates situation

  $\rightarrow$ one-way street to congestion collapse

Role of exponential decrease:

$\longrightarrow$ exponential is superfast

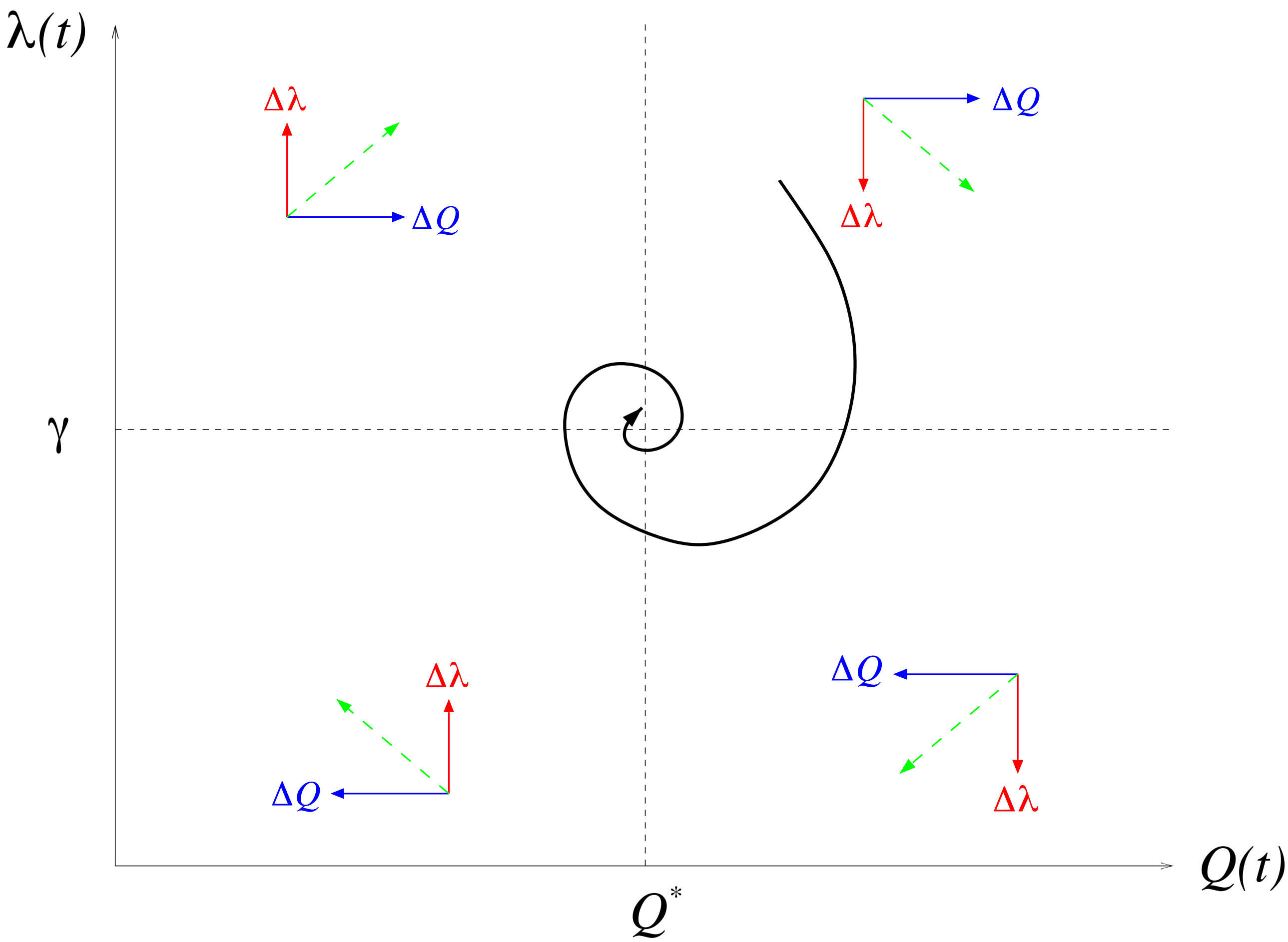
In general, need not be exponential

$\longrightarrow$ depends on shape of $\gamma(Q)$

$\longrightarrow$ e.g., Method D with different parameters

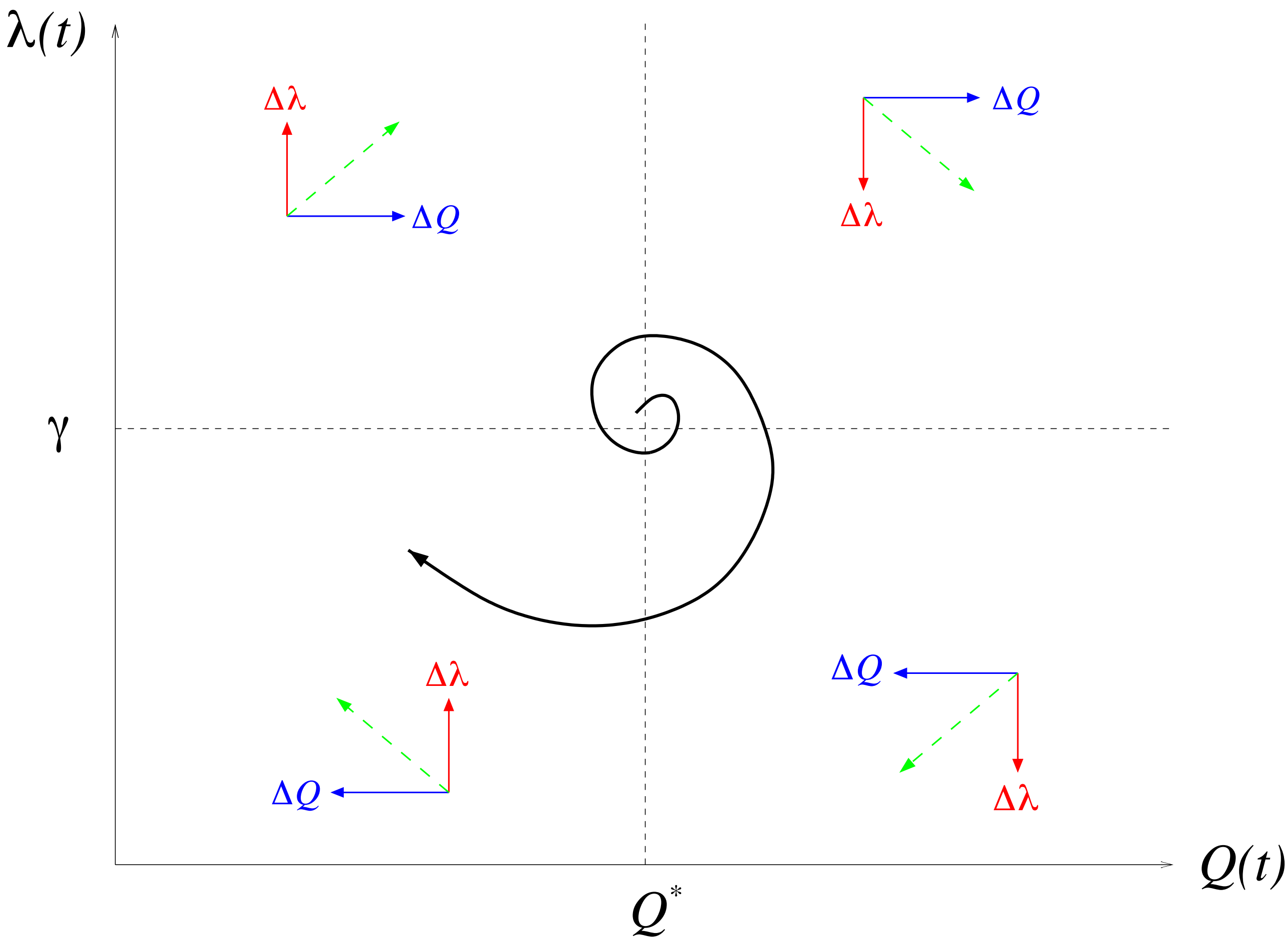
Convergent trajectory:

$\longrightarrow$ asymptotically stable & optimal



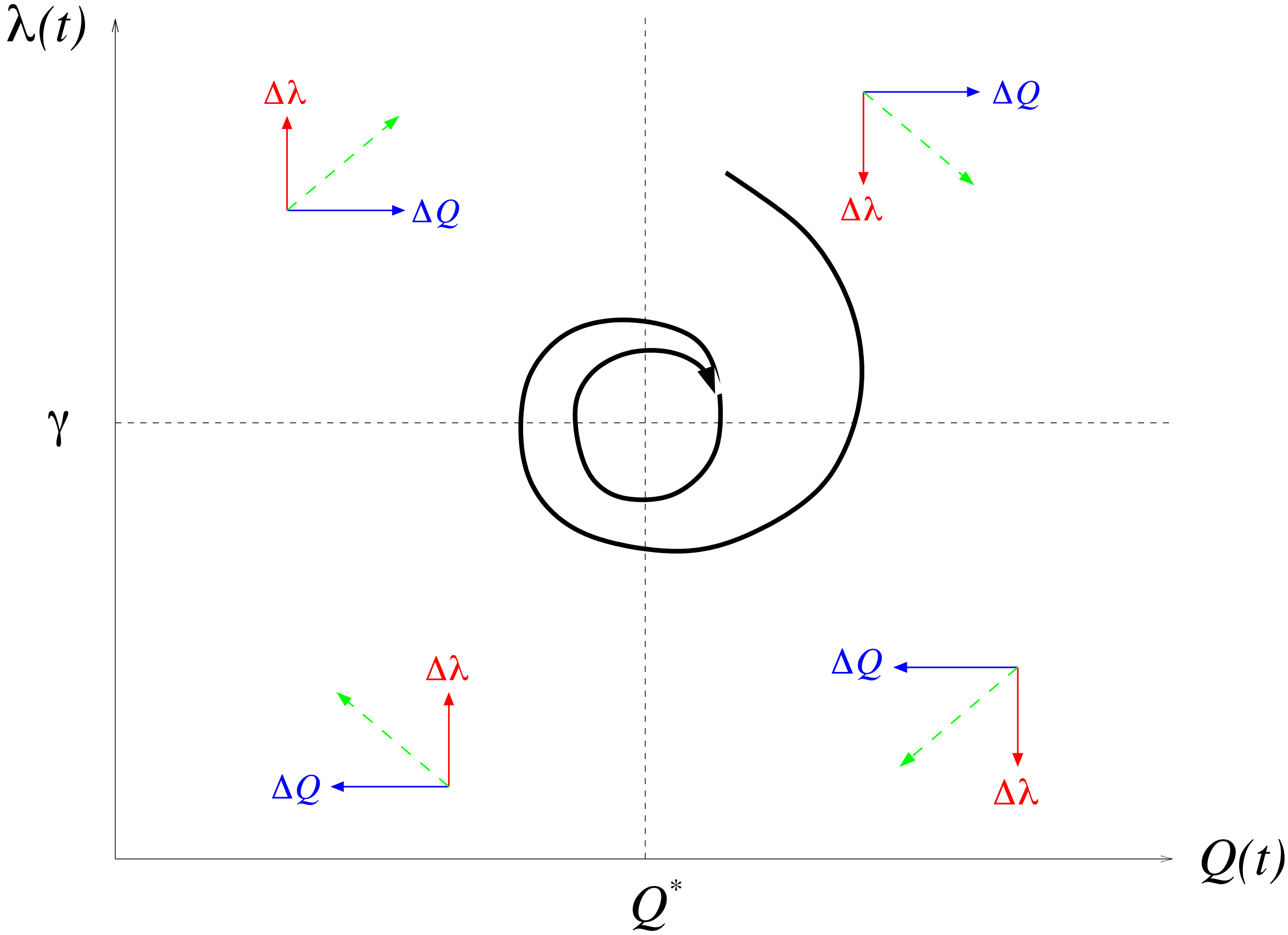
Divergent trajectory:

$\longrightarrow$ unstable



Stable (but not asymptotically so) trajectory:

$\longrightarrow$ limit cycle



System behavior for unimodal load-throughput curve:

$$\begin{aligned}\frac{dQ(t)}{dt} &= \lambda(t) - \gamma(Q) \\ \frac{d\lambda(t)}{dt} &= \varepsilon(Q^* - Q(t)) - \beta(\lambda(t) - \gamma(Q))\end{aligned}$$

$\longrightarrow$ nonlinear differential equation due to $\gamma(Q)$

Stability analysis using local method:

$\longrightarrow$ stable manifold theorem

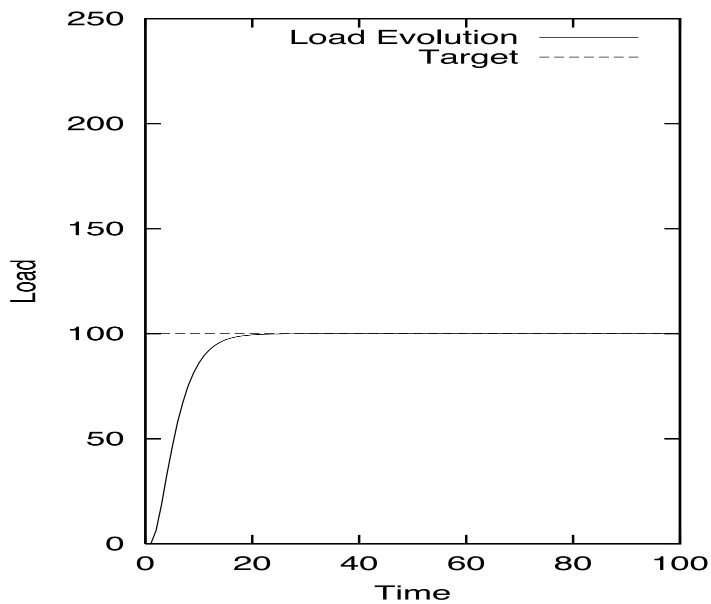
$\longrightarrow$ relates stability of nonlinear system to linearized one

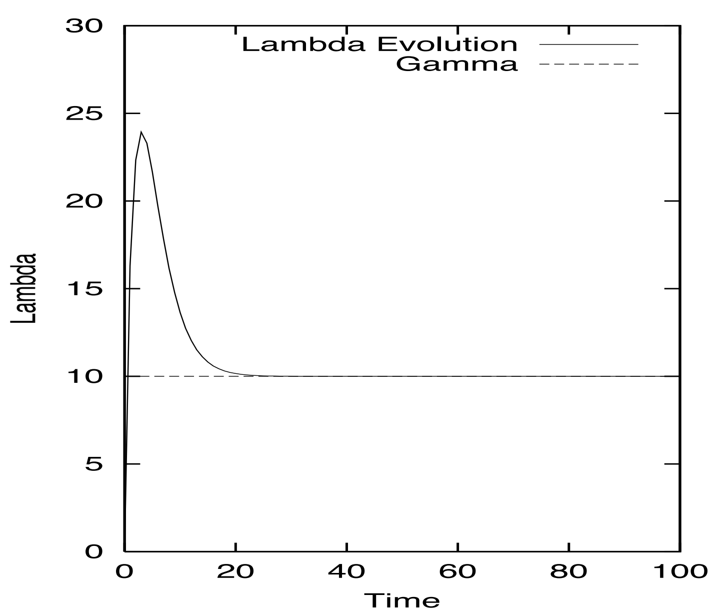
$\longrightarrow$ to do or not to do?

Result: Method D is stable if

$$-\beta \frac{d\gamma(Q)}{dQ} > 1$$

β is increased from 0.5 to 0.63:





In general: shape of $d\gamma(Q)/dQ$ unknown

→ difficult to estimate

→ also impact of feedback delay: delay × gain cannot be too large

→ large gain increases outdatedness of feedback information

Thus: exponential decrease acts as a sufficient condition for preventing instability and congestion collapse.

→ may be overkill

→ but widely employed control law to prevent collapse

→ Ethernet, WLAN, TCP