Information Transmission under Noise



Uncertainty introduced by noise:

- \longrightarrow encoding/decoding: $a \mapsto w_a \mapsto w \mapsto [?]$
- $\longrightarrow w_a$ gets corrupted, i.e., becomes w
- \longrightarrow if $w = w_b$, incorrectly conclude b as symbol
- \longrightarrow detect w is corrupted: error detection
- \longrightarrow correct w to w_a : error correction

Would like: If received code word $w = w_c$ for some symbol $c \in \Sigma$, then probability that actual symbol sent is indeed c is high.

$$\longrightarrow$$
 Pr{symbol sent = $c \mid w = w_c$ } ≈ 1

 \longrightarrow noiseless channel: special case (prob = 1)

In practice, w may not match any legal code word:

$$\longrightarrow$$
 for all $c \in \Sigma, w \neq w_c$

$$\longrightarrow$$
 then what?

Fundamental limitation to reliable data transmission:

Channel capacity C: maximum achievable reliable data transmission rate (bps) over a noisy channel (dB) with bandwidth W (Hz).

Channel Coding Theorem (Shannon): Given bandwidth W, signal power P_S , noise power P_N , channel subject to white noise,

$$C = W \log \left(1 + \frac{P_S}{P_N}\right)$$
 bps.

 P_S/P_N : signal-to-noise ratio (SNR)

 \longrightarrow upper bound achieved by using longer codes

 \longrightarrow detailed set-up/conditions omitted

Increasingly important for modern day networking:

- Power control (e.g., pocket PCs)
 - \rightarrow trade-off w.r.t. battery power
 - \rightarrow trade-off w.r.t. multi-user interference
 - \rightarrow signal-to-interference ratio (SIR)
- Recent trend: software radio
 - \rightarrow hardware-to-software migration
 - \rightarrow configurable

Signal-to-noise ratio (SNR) is expressed as

$$\mathrm{dB} = 10 \log_{10}(P_S/P_N).$$

Example: Assuming a decibel level of 30, what is the channel capacity of a telephone line?

Answer: First, W = 3000 Hz, $P_S/P_N = 1000$. Using Channel Coding Theorem,

 $C = 3000 \log 1001 \approx 30$ kbps.

- \longrightarrow compare against 28.8 kbps modems
- \longrightarrow what about 56 kbps modems?
- \longrightarrow DSL lines?

Digital vs. Analog Transmission

Two forms of *transmission*:

- digital transmission: data transmission using square waves
- analog transmission: data transmission using all other waves

Four possibilities to consider:

• analog data via analog transmission

 \rightarrow "as is" (e.g., radio)

• analog data via digital transmission

 \rightarrow sampling (e.g., voice, audio, video)

- digital data via analog transmission
 - \rightarrow broadband & wireless ("high-speed networks")
- digital data via digital transmission

 \rightarrow baseband (e.g., Ethernet)

Why consider digital transmission?

Common to both: problem of attenuation.





- decrease in signal strength as a function of distance
- increase in attenuation as a function of frequency

Rejuvenation of signal via amplifiers (analog) and repeaters (digital). Delay distortion: different frequency components travel at different speeds.

Most problematic: effect of noise

 \longrightarrow thermal, interference, . . .

- Analog: Amplification also amplifies noise—filtering out just noise, in general, is a complex problem.
- Digital: Repeater just generates a new square wave; more resilient against ambiguitity.



Three pieces of information to manipulate: amplitude, frequency, phase.

- Amplitude modulation (AM): encode bits using amplitude levels.
- Frequency modulation (FM): encode bits using frequency differences.
- Phase modulation (PM): encode bits using phase shifts.



FM radio uses ... FM!

AM radio uses ... AM!

iPod & radio experiment uses ...?

Why is FM radio clearer ("high fidelity") than AM radio?

Broadband uses ... ?

Baud Rate vs. Bit Rate

Baud rate: Unit of time within which carrier wave can be altered for AM, FM, or PM.

- \longrightarrow signalling rate
- \longrightarrow e.g., clock

Not synonymous with bit rate: e.g., AM with 8 levels, PM with 8 phases

 \longrightarrow bit rate (bps) = 3 × baud rate

... less than one bit per baud?

Presence or absence of carrier wave: allows many channels to co-exist at the same time

 \longrightarrow frequency division multiplexing (FDM)



Ex.: AM radio (535 kHz–1705 kHz)

- \longrightarrow tuning to specific frequency: Fourier transform
- \longrightarrow coefficient (magnitude) carries bit information

Ex.: FM radio

- \longrightarrow 88 MHz–108 MHz
- \longrightarrow 200 kHz slices
- \longrightarrow how does it work?
- \longrightarrow better or worse than AM?

- Ex.: Digital radio
 - \longrightarrow digital audio radio service
 - \longrightarrow GEO satellites (a.k.a. satellite radio)
 - \longrightarrow uses 2.3 GHz spectrum (a.k.a. S-band)
 - \longrightarrow e.g., XM, Sirius

In the absence of carrier wave, can still use multiplexing:

 \longrightarrow time-division multiplexing (TDM)



- digital transmission of analog data
 - \rightarrow first digitize
 - \rightarrow PCM (e.g., PC sound cards), modem
- digital transmission of digital data
 - \rightarrow e.g., telephony backbone network



- 24 simultaneous users
- 7 bit quantization

Assuming 4 kHz telephone channel bandwidth, Sampling Theorem dictates 8000 samples per second.

 $\rightarrow 125 \ \mu \text{sec}$ inter-sample interval

Bandwidth = $8000 \times 193 = 1.544$ Mbps

Digital transmission of digital data

Direct encoding of square waves using voltage differentials; e.g., -15V-+15V for RS-232-C.

- NRZ-L (non-return to zero, level)
- NRZI (NRZ invert on ones)
- Manchester (biphase or self-clocking codes)



 \rightarrow baud rate vs. bit rate of Manchester?

Trade-offs:

- NRZ codes—long sequences of 0's (or 1's) causes synchronization problem; need extra control line (clock) or sensitive signalling equipment.
- Manchester codes—synchronization achieved through self-clocking; however, achieves only 50% efficiency vis-à-vis NRZ codes.

4B/5B code

Encode 4 bits of data using 5 bit code where the code word has at most one leading 0 and two trailing 0's.

 $0000 \leftrightarrow 11110, 0001 \leftrightarrow 01001,$ etc.

- \longrightarrow at most three consecutive 0's
- \longrightarrow efficiency: 80%

Multiplexing techniques:

- TDM
- FDM
- mixture (FDM + TDM); e.g., TDMA
- CDMA (code division multiple access) or spread spectrum
 - \rightarrow wireless communication
 - \rightarrow competing scheme with TDMA