Information Transmission under Noise

Uncertainty introduced by noise:

\[ S \xrightarrow{\text{F}} F \xrightarrow{\text{noise}} F^{-1} \xrightarrow{\text{d}} \]

\[ \text{encoding/decoding: } a \xleftrightarrow{\text{w}_a} w \xleftrightarrow{[?]} \]

\[ \text{w}_a \text{ gets corrupted, i.e., becomes } w \]

\[ \text{if } w = w_b, \text{ incorrectly conclude } b \text{ as symbol} \]

\[ \text{detect } w \text{ is corrupted: error detection} \]

\[ \text{correct } w \text{ to } w_a: \text{ error correction} \]
Would like: If received code word $w = w_c$ for some symbol $c \in \Sigma$, then probability that actual symbol sent is indeed $c$ is high.

\[ \rightarrow \ Pr\{\text{symbol sent} = c \mid w = w_c\} \approx 1 \]

\[ \rightarrow \text{ noiseless channel: special case (prob = 1)} \]

In practice, $w$ may not match any legal code word:

\[ \rightarrow \text{ for all } c \in \Sigma, \ w \neq w_c \]

\[ \rightarrow \text{ then what?} \]
Fundamental limitation to reliable data transmission:

Channel capacity $C$: maximum achievable reliable data transmission rate (bps) over a noisy channel (dB) with bandwidth $W$ (Hz).

**Channel Coding Theorem (Shannon):** Given bandwidth $W$, signal power $P_S$, noise power $P_N$, channel subject to white noise,

$$C = W \log \left(1 + \frac{P_S}{P_N}\right) \text{ bps.}$$

$P_S/P_N$: signal-to-noise ratio (SNR)

$\rightarrow$ upper bound achieved by using longer codes

$\rightarrow$ detailed set-up/conditions omitted
Increasingly important for modern day networking:

- Power control (e.g., pocket PCs)
  → trade-off w.r.t. battery power
  → trade-off w.r.t. multi-user interference
  → signal-to-interference ratio (SIR)

- Recent trend: software radio
  → hardware-to-software migration
  → configurable
Signal-to-noise ratio (SNR) is expressed as
\[ dB = 10 \log_{10}(P_S/P_N). \]

**Example:** Assuming a decibel level of 30, what is the channel capacity of a telephone line?

*Answer:* First, \( W = 3000 \text{ Hz}, \ P_S/P_N = 1000. \) Using Channel Coding Theorem,
\[ C = 3000 \log 1001 \approx 30 \text{ kbps}. \]

\[ \rightarrow \text{ compare against 28.8 kbps modems} \]
\[ \rightarrow \text{ what about 56 kbps modems?} \]
\[ \rightarrow \text{ DSL lines?} \]
Digital vs. Analog Transmission

Two forms of transmission:

- digital transmission: data transmission using square waves
- analog transmission: data transmission using all other waves

Four possibilities to consider:

- analog data via analog transmission
  → “as is” (e.g., radio)
- analog data via digital transmission
  → sampling (e.g., voice, audio, video)
- digital data via analog transmission
  → broadband & wireless (“high-speed networks”)
- digital data via digital transmission
  → baseband (e.g., Ethernet)
Why consider digital transmission?

Common to both: problem of *attenuation*.

- decrease in signal strength as a function of distance
- increase in attenuation as a function of frequency

Rejuvenation of signal via amplifiers (analog) and repeaters (digital).
Delay distortion: different frequency components travel at different speeds.

Most problematic: effect of noise

\[\rightarrow \text{thermal, interference, ...}\]

- Analog: Amplification also amplifies noise—filtering out just noise, in general, is a complex problem.
- Digital: Repeater just generates a new square wave; more resilient against ambiguity.
Analog Transmission of Digital Data

Three pieces of information to manipulate: amplitude, frequency, phase.

- Amplitude modulation (AM): encode bits using amplitude levels.
- Frequency modulation (FM): encode bits using frequency differences.
- Phase modulation (PM): encode bits using phase shifts.
FM radio uses . . . FM!

AM radio uses . . . AM!

iPod & radio experiment uses . . . ?

Why is FM radio clearer ("high fidelity") than AM radio?

Broadband uses . . . ?
Baud Rate vs. Bit Rate

*Baud rate*: Unit of time within which carrier wave can be altered for AM, FM, or PM.

→ signalling rate

→ e.g., clock

Not synonymous with bit rate: e.g., AM with 8 levels, PM with 8 phases

→ bit rate (bps) = 3 × baud rate

... less than one bit per baud?
Broadband vs. Baseband

Presence or absence of carrier wave: allows many channels to co-exist at the same time

→ frequency division multiplexing (FDM)

Ex.: AM radio (535 kHz–1705 kHz)

→ tuning to specific frequency: Fourier transform
→ coefficient (magnitude) carries bit information
Ex.: FM radio

→ 88 MHz–108 MHz
→ 200 kHz slices
→ how does it work?
→ better or worse than AM?

Ex.: Digital radio

→ digital audio radio service
→ GEO satellites (a.k.a. satellite radio)
→ uses 2.3 GHz spectrum (a.k.a. S-band)
→ e.g., XM, Sirius
In the absence of carrier wave, can still use multiplexing:

\[ \rightarrow \text{ time-division multiplexing (TDM)} \]

- digital transmission of analog data
  \[ \rightarrow \text{ first digitize} \]
  \[ \rightarrow \text{ PCM (e.g., PC sound cards), modem} \]
- digital transmission of digital data
  \[ \rightarrow \text{ e.g., telephony backbone network} \]
**Example:** T1 carrier (1.544 Mbps)

One Frame (193 bits)

<table>
<thead>
<tr>
<th>Channel 1</th>
<th>Channel 2</th>
<th>Channel 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8</td>
<td>1 2 3 4 5 6 7 8</td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
</tbody>
</table>

- 24 simultaneous users
- 7 bit quantization

Assuming 4 kHz telephone channel bandwidth, Sampling Theorem dictates 8000 samples per second.

\[ \rightarrow 125 \mu \text{sec inter-sample interval} \]

Bandwidth = 8000 \times 193 = 1.544 \text{ Mbps}
Digital transmission of digital data

Direct encoding of square waves using voltage differentials; e.g., -15V–+15V for RS-232-C.

- NRZ-L (non-return to zero, level)
- NRZI (NRZ invert on ones)
- Manchester (biphase or self-clocking codes)

→ baud rate vs. bit rate of Manchester?
Trade-offs:

- NRZ codes—long sequences of 0’s (or 1’s) causes synchronization problem; need extra control line (clock) or sensitive signalling equipment.

- Manchester codes—synchronization achieved through self-clocking; however, achieves only 50% efficiency vis-à-vis NRZ codes.

4B/5B code

Encode 4 bits of data using 5 bit code where the code word has at most one leading 0 and two trailing 0’s.

0000 ↔ 11110, 0001 ↔ 01001, etc.

→ at most three consecutive 0’s

→ efficiency: 80%
Multiplexing techniques:

- TDM
- FDM
- mixture (FDM + TDM); e.g., TDMA
- CDMA (code division multiple access) or spread spectrum
  → wireless communication
  → competing scheme with TDMA