Fundamentals of information transmission and coding (a.k.a. communication theory)

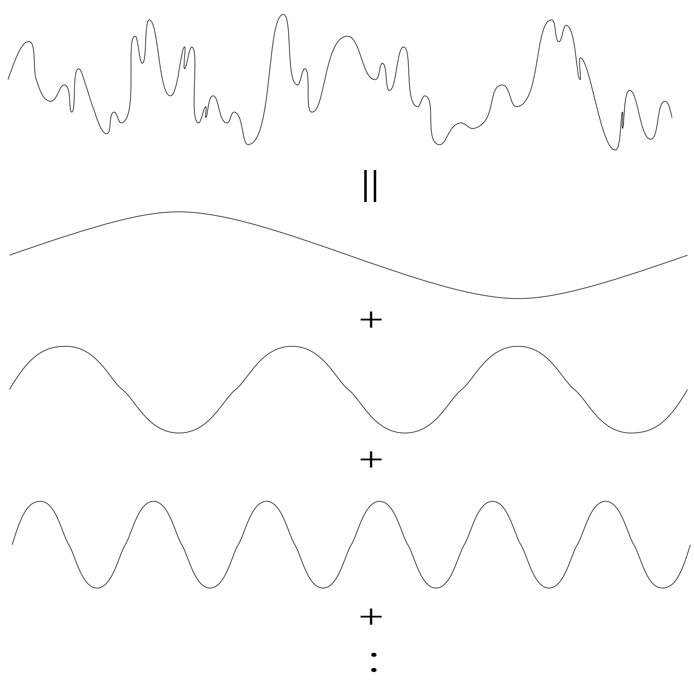
Signals and functions

Elementary operation of communication: send *signal* on medium from point A to point B.

- media—copper wire, optical fiber, air/space, etc.
- signals—voltage and currents, light pulses, radio waves, microwaves, etc.

Signal can be viewed as a time-varying function $s(t)$.

If $s(t)$ is "sufficiently nice" (Dirichlet conditions), then $s(t)$ can be represented as a linear combination of complex sinusoids.



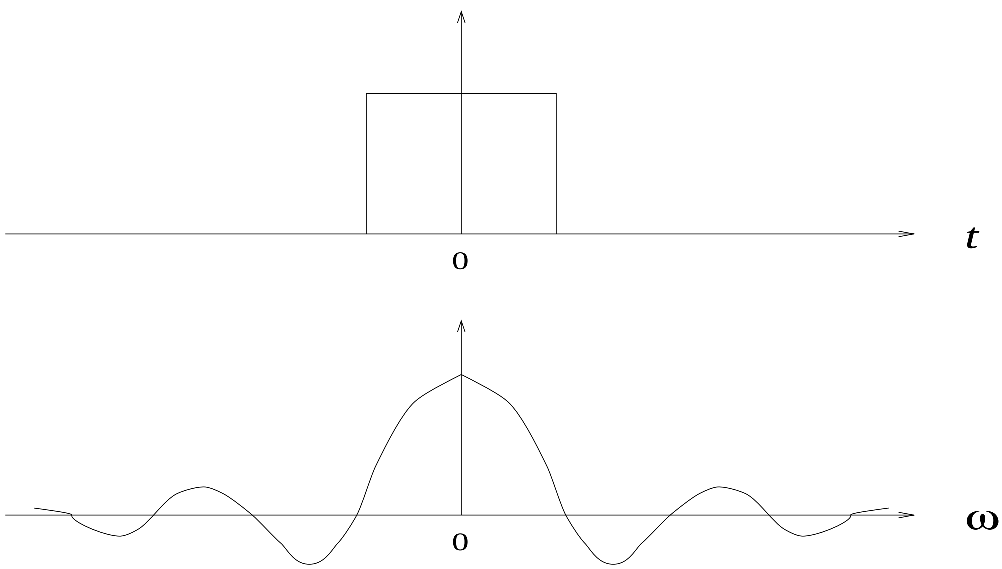
Fourier expansion and transform:

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega t} d\omega,$$

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt.$$

$\longrightarrow$ time domain vs. frequency domain

Example (square wave):



Random function (i.e., white noise) has "flat-looking" spectrum.

$\longrightarrow$ unbounded bandwidth

Luckily, most "interesting" functions arising in practice are far from random; in fact, *bandwidth limited*.

E.g., speech: 20 Hz–20 kHz; telephone systems: 300 Hz–3300 Hz

$\longrightarrow$ bandwidth 3000 Hz

Digital data vs. analog data

Digital data: bits.

$\longrightarrow$ discrete signal (both in time and amplitude)

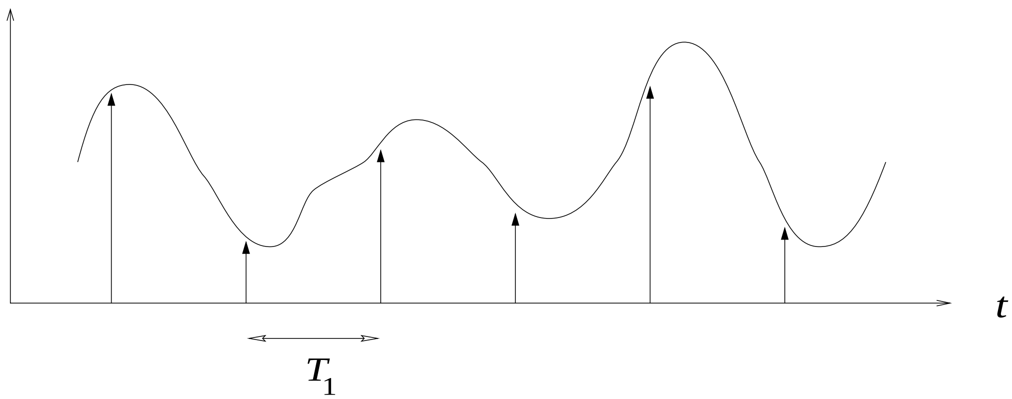
Analog data: audio/voice, video/image; analog data is oftentimes *digitized* so that bits form the starting point.

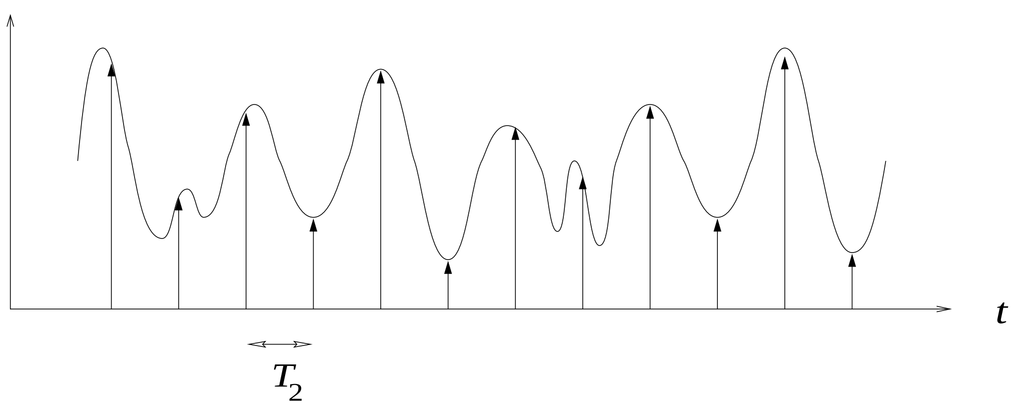
$\longrightarrow$ continuous signal

Sampling theorem (Nyquist): Given continuous bandlimited signal $s(t)$ with $S(\omega) = 0$ for $|\omega| > W$, $s(t)$ can be reconstructed from its samples if

$$\nu > 2W$$

where ν is the sampling rate.





$$\nu_1 = \frac{1}{T_1} \quad < \quad \nu_2 = \frac{1}{T_2}$$

Information transmission over noiseless channel

Set-up:

- source s emits symbols from finite alphabet set Σ
- symbol $a \in \Sigma$ is generated with probability $p_a > 0$
- a code book F of Σ assigns code word $w_a = F(a)$ for each $a \in \Sigma$
- F is invertible
- $|w_a|$ denotes length of w_a in bits
- average code length L associated with $\langle \Sigma, \mathbf{p}, F \rangle$

$$L = \sum_{a \in \Sigma} p_a |w_a|$$

Question: Given $\langle \Sigma, \mathbf{p} \rangle$, are there "good" code books F with small L?

Answer: Yes.

The entropy H of $\langle \Sigma, \mathbf{p} \rangle$ is defined as

$$H = \sum_{a \in \Sigma} p_a \log \frac{1}{p_a}$$

Source-coding Theorem (Shannon): For all F,

$$H \leq L.$$

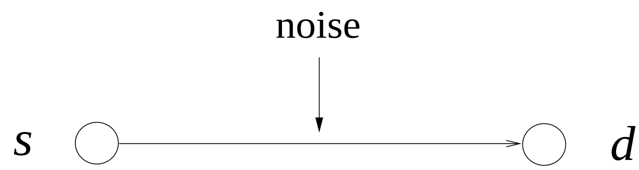
Moreover, there exist F such that

$$L \leq H + \epsilon$$

where $0 < \epsilon < 1$.

$\longrightarrow$ holds for extension codes (blocks of n symbols)

Information transmission over noisy channel



Channel capacity C: maximum achievable "reliable" data transmission rate (bps) over a noisy channel.

Channel Coding Theorem (Shannon): Given signal power P_S, noise power P_N, and channel subject to white Gaussian noise (detailed conditions omitted),

$$C = \frac{1}{2}\nu \log(1 + \frac{P_S}{P_N}) \text{ bps.}$$

Here P_S/P_N: *signal-to-noise ratio.*

By the Sampling Theorem,

$$C = W \log(1 + P_S/P_N) \text{ bps.}$$

Signal-to-noise ratio (SNR) is expressed as

$$\text{dB} = 10 \log_{10}(P_S/P_N).$$

Example: Assuming a decibel level of 30, what is the channel capacity of a telephone line?

Answer: First, $W = 3000$ Hz, $P_S/P_N = 1000$. Using Channel Coding Theorem,

$$C = 3000 \log 1001 \approx 30 \text{ kbps.}$$

Compare against 28.8 kbps modems.

Digital vs. analog signals

In essence, *square wave* versus everything else.

Two forms of *transmission*:

- digital transmission: data transmission using square waves
- analog transmission: data transmission using all other waves

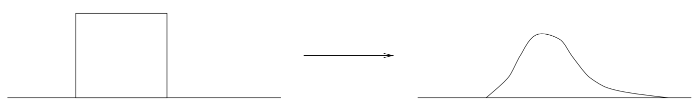
Analog data via analog transmission is straightforward; e.g., telephone at home.

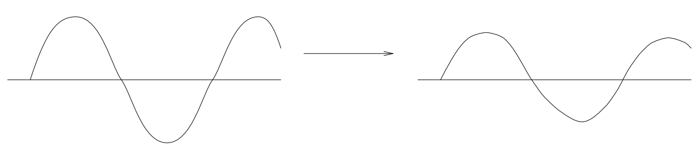
Need to consider:

- analog data via digital transmission
- digital data via analog transmission
- digital data via digital transmission

First, why is digital transmission "superior" to analog transmission?

Common to both: problem of *attenuation*.





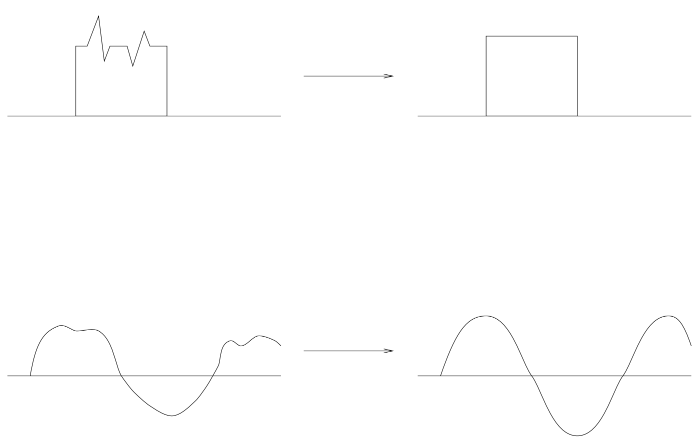
- decrease in signal strength as a function of distance
- increase in attenuation as a function of frequency

Rejuvenation of signal via amplifiers (analog) and repeaters (digital).

Delay distortion: different frequency components (in guided media) travel at different speeds.

Most problematic: effect of noise (thermal, cross talk, etc.)

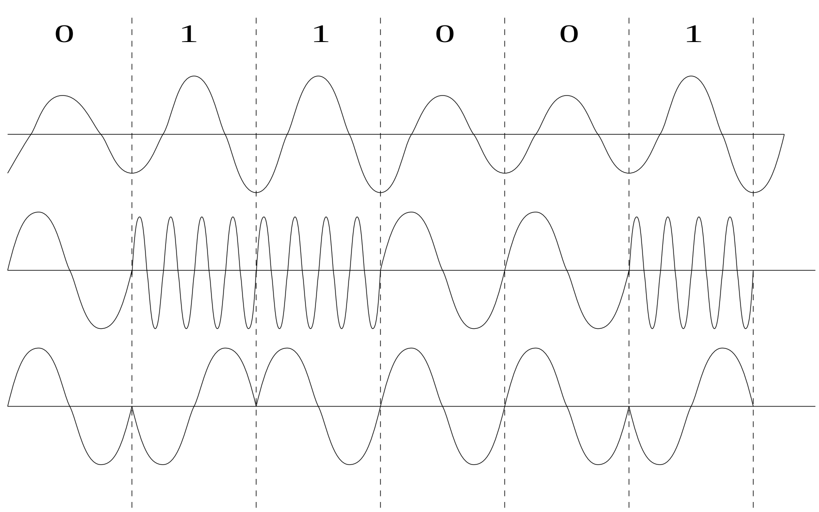
- Analog: Amplification also amplifies noise—filtering out just noise, in general, is a hard problem; e.g., voice.
- Digital: Repeater just generates a new square wave. Noise cannot be confused with data (unless too high).



Analog transmission of digital data

Three pieces of information to manipulate: amplitude, frequency, phase.

- Amplitude modulation (AM)—sensitive to power fluctuations.
- Frequency modulation (FM)—allow full duplex communication; need four frequencies.
- Phase modulation (PM)—more sophisticated.



Baud rate vs. bit rate

Baud rate: Unit of time within which carrier wave can be altered for AM, FM, or PM.

At every signalling event, potentially more than 1 bit of information can be encoded; e.g., PM with multiple phases, say, four.

Thus 2 bits of information per signalling period.

$\longrightarrow$ bit rate (bps) $= 2 \times$ baud rate

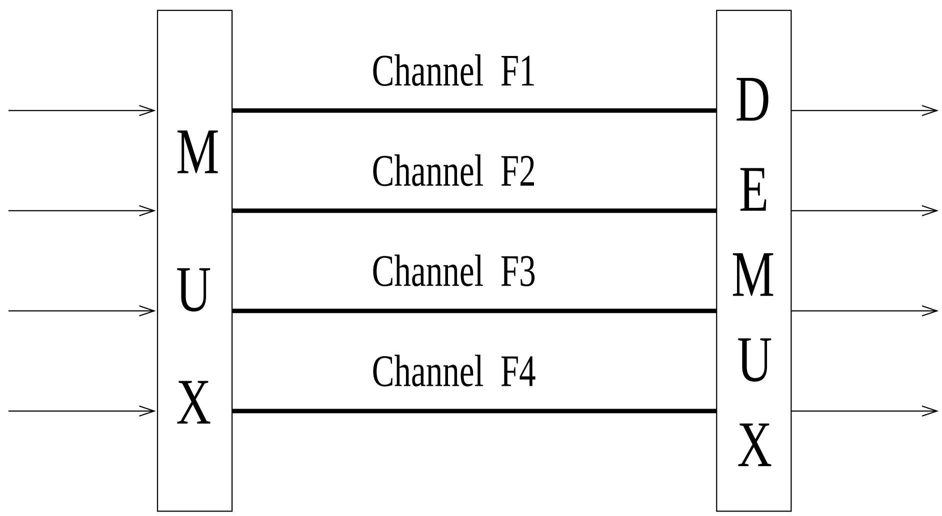
Combine the three; e.g., QAM—8 phase angles and 2 amplitudes for a total of 16 detectable events (CCITT v.29 standard, 9600 bps, 2400 baud).

$\longrightarrow$ 4 bits per baud

Broadband vs. baseband

Presence/absence of *carrier wave*; allows many channels to co-exist at the same time.

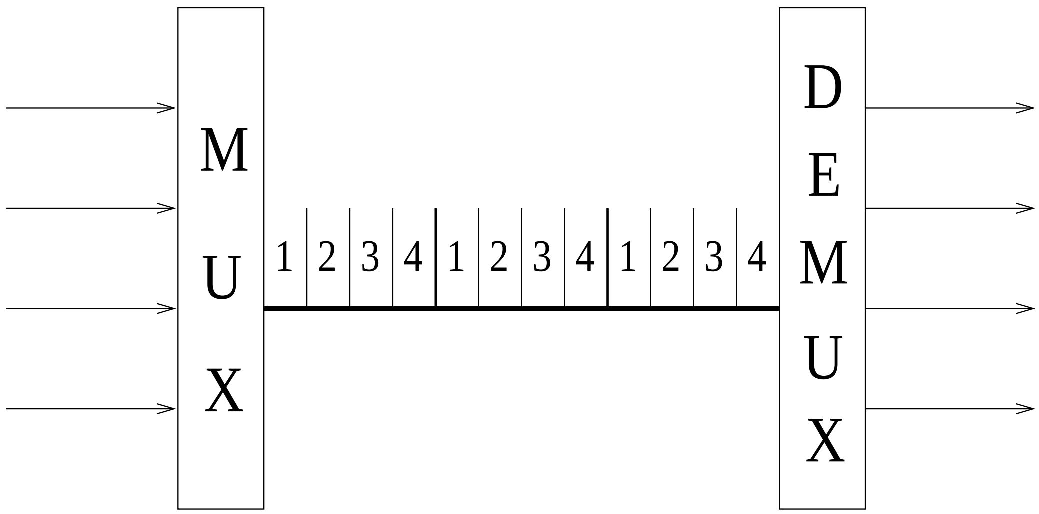
$\longrightarrow$ frequency division multiplexing (FDM)



Clearly, BW of medium $>4\times$ BW of signal.

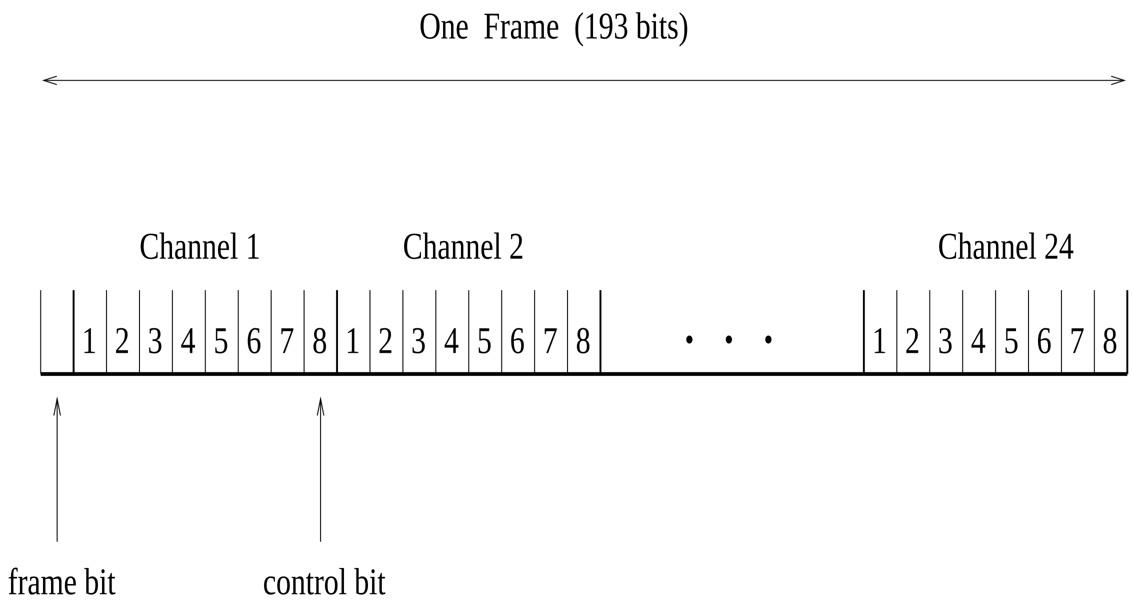
In the absence of carrier wave, can still use multiplexing:

$\longrightarrow$ time-division multiplexing (TDM)



Clearly, bit rate of medium > data rate of signal; however, mostly used for digital transmission of digital or analog data (PCM, codec).

Example: T1 carrier.



Assuming 4 kHz telephone channel bandwidth, Sampling Theorem dictates 8000 samples per second (125 μsec/sample).

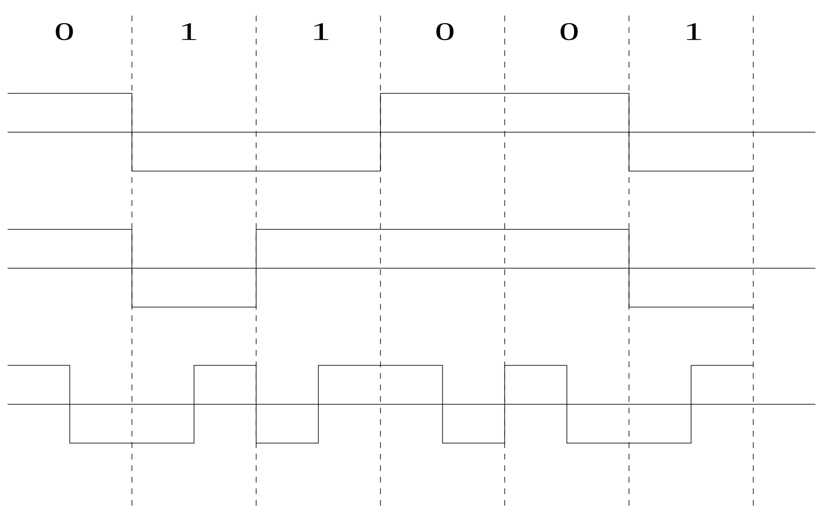
Bandwidth $= 8000 \times 193 = 1.544$ Mbps

Digital transmission of digital data

Direct encoding of square waves using voltage differentials; e.g., -15V–+15V for RS-232-C.

NRZ-L (non-return to zero, level), NRZI (NRZ invert on ones);

Manchester (biphase or self-clocking codes).



Trade-offs:

- NRZ codes—long sequences of 0's (or 1's) causes synchronization problem; need extra control line (clock) or sensitive signalling equipment.
- biphase codes—synchronization easily achieved through self-clocking; however, other things being equal, achieves only 50% efficiency vis-a-vis NRZ codes.

4B/5B code

Encode 4 bits of data using 5 bit code where the code word has at most one leading 0 and two trailing 0's.

0000 $\leftrightarrow$ 11110, 0001 $\leftrightarrow$ 01001, etc.

$\longrightarrow$ using 4B/5B, at most three consecutive 0's

$\longrightarrow$ efficiency: 80%

Multiplexing techniques:

- TDM
- FDM
- mixture (FDM + TDM); e.g., TDMA (time division multiple access) scheme in wireless media
- spread spectrum or CDMA (code division multiple access); competing scheme with TDMA for wireless media

Code division multiplexing

Direct sequence:

To send (i.e., encode) bit sequence $x = x_1, x_2, \ldots, x_n$, use pseudorandom bit sequence $y = y_1, y_2, \ldots, y_n$ to compute

$$\begin{aligned} z &= z_1, z_2, \ldots, z_n \\ &= x_1 \oplus y_1, x_2 \oplus y_2, \ldots, x_n \oplus y_n. \end{aligned}$$

To decode bit sequence $z = z_1, z_2, \ldots, z_n$, compute

$$x = z \oplus y.$$

- data rate usually slower than code rate (spreading)
- multiplexing N sources achieved via a set of chipping codes

$$\{y^1, y^2, \ldots, y^N\}$$

Frequency hopping:

Use pseudorandom number sequence as key to index a set of carrier frequencies $f_1, f_2, \ldots, f_m$ (spreading).

Receiver with access to pseudorandom sequence can decode transmitted signal.

$\longrightarrow$ code narrowband input as broadband output

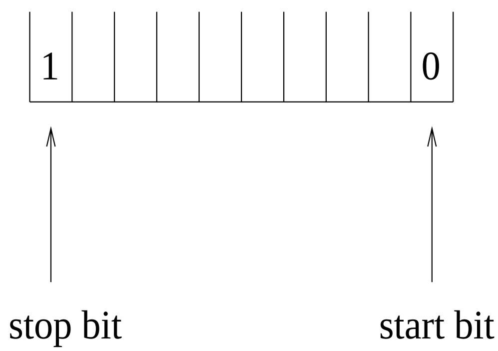
Some benefits:

- more secure (eavesdropping)
- resistant to jamming (esp. freq. hopping)
- noise resistant (esp. direct sequence)
- graceful multiplexing degradation

Synchronous vs. asynchronous transmission

$\longrightarrow$ framing problem

Asynchronous: e.g., ASCII character transmission between dumb terminal and host computer.



Each character is an independent unit; receiver needs to know bit duration.

Overhead problem; assuming 1 start bit, 1 stop bit, 8 data bits, only 80% efficiency.

$\longrightarrow$ inefficient for long messages

Synchronous: "Byte-oriented scheme"; e.g., BISYNC

SYN	SYN	SOH	Header	STX	Body	ETX	CRC

$\longrightarrow$ **`SYN, SOH, STX, ETX, DLE`**: sentinels

Two problems:

- How to maintain synchronization if |Body| is large?
- Control characters within Body of message.

$\longrightarrow$ inefficient for short messages

$\longrightarrow$ efficiency approaches 1 as $|\text{Body}| \to \infty$

"Bit-oriented scheme"; e.g., HDLC

Use fixed *preamble* and *postamble*; simply a bit pattern.

$\longrightarrow$ 01111110

How to avoid confusing 01111110 in the data part?

$\longrightarrow$ bit stuffing

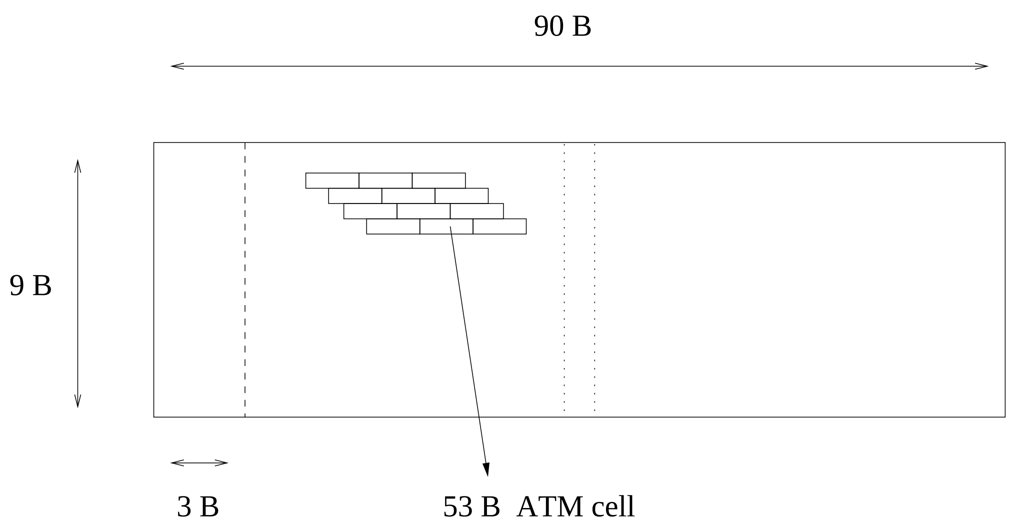
SONET (Synchronous Optical Network)

$\longrightarrow$ framing/transmission standard for optical fiber

Rates: STS-1 (51.84 Mbps), STS-3 (155.25 Mbps), STS-3c, STS-12c (622.08 Mbps), STS-24c (1.24416 Gbps), STS-48c, etc.

Common to use OC-n in place of STS-n.

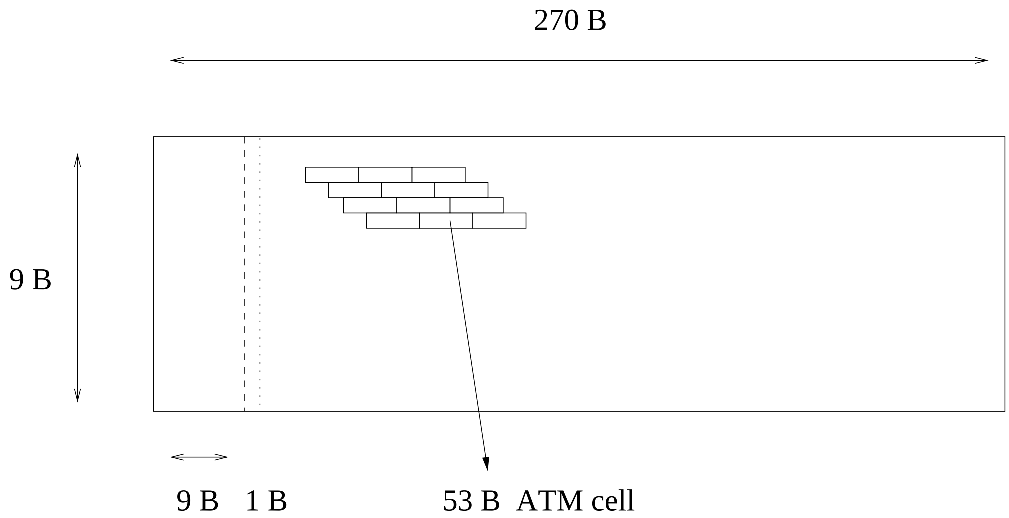
STS-1 frame:



Features:

- 125 μs frame duration (for all STS-n)
- 51.84 Mbps $= 810 \cdot 8 \cdot 8000$
- $3 + 1$ columns of overhead
- overhead includes synchronization, pointer fields
- overhead encoded using NRZ
- payload scrambled (XOR'ed) to achieve approximate self-clocking
- SONET also used for FDDI

STS-3c frame:



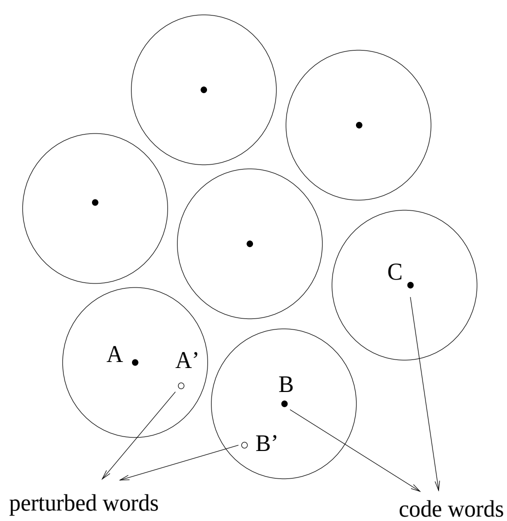
- contiguous payload area—SPE (synchronous payload envelop)
- STS-3c frame can carry about 44 ATM cells
- most relevant frame (with STS-12c) for ATM networking

Error-detection and correction

General theory: subject of *Information Theory*.

E.g., Hamming codes, Huffman codes, Shannon-Fano codes, Reed-Solomon, etc.

Intuitive idea: Want to transmit 8-bit words reliably; use, e.g., 12-bit code words.



In network protocol context: want practical error detection.

⟶ error-correction: use retransmission

⟶ two-level scheme

Parity: Odd or even parity; single bit error detection.

(*Internet*) *Checksum*: Group message into 16-bit words; calculate their sum (one's complement); append "checksum" to message.

Cyclic redundancy check (CRC): Polynomial arithmetic over finite field.

View n-bit string $a_{n-1}a_{n-2}\cdots a_0$ as a polynomial of degree $n-1$:

$$M(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0.$$

Fix some generator polynomial $G(x)$ of degree k. Choice of $G(x)$ is important.

Let $R(x)$ be the remainder of $x^k M(x)/G(x)$.

Let $T(x) = x^k M(x) - R(x)$.

$\longrightarrow$ $T(x)$ is the code word

Assume $T(x) + E(x)$ arrives at the receiver.

If $E(x) = 0$ then $T(X)/G(x) = 0$.

$\longrightarrow$ no errors

If $E(x) \neq 0$ then $T(X)/G(x) \neq 0$.

$\longrightarrow$ error has occured

Specific instances:

If $E(x) = x^i$, $0 \leq i \leq n + k - 1$ (i.e., a single error at position i), then assuming $G(x)$ contains at least two terms, $G(x)$ will fail to divide $E(x)$.

If $E(x) = x^i + x^j$, $i > j$, then first express $E(x) = x^j(x^{i-j} + 1)$.

Assuming x does not divide $G(x)$, to detect double errors it is sufficient that $G(x)$ not divide $x^{i-j} + 1$.

Fact: $G(x) = x^{15} + x^{14} + 1$ will not divide $x^r + 1$ for all $k < 32768$.

Some commonly used CRC generator polynomials:

- CRC-32: $x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$ (FDDI, Ethernet)
- CRC-CCITT: $x^{16} + x^{12} + x^5 + 1$ (HDLC)
- CRC-8: $x^8 + x^2 + x + 1$ (ATM)