Final step: OFDM

→ how to find n mutually orthogonal sinusoids over finite time interval τ

- what are the n carrier waves
- what is time interval τ

Method: choose harmonics of base frequency.

→ given base frequency f

→ its harmonics: integer multiples of f

→ i.e., f, $2f$, $3f$, ...

Set-up for OFDM:

Assume system is constrained to operate within frequency band f_a and f_b.

→ e.g., reliability for wired media

→ e.g., regulations for wireless media

Gap between f_a and f_b, $W = f_b - f_a$ (Hz), also called bandwidth

→ gap W will be important for system design

→ not absolute frequency f_a per se

For n users: choose n EM waves with frequencies

$$f_a + \frac{W}{n}, f_a + 2\frac{W}{n}, \cdots, f_a + n\frac{W}{n}$$

→ note: base frequency is bandwidth W

Example: for both [0.5 GHz, 0.7 GHz] and [5 GHz, 5.2 GHz], bandwidth W is

$\rightarrow$ the same: 0.2 GHz

Carrier waves: for $n = 20$ users $W/n = 10$ MHz

$\rightarrow$ carrier waves for [5 GHz, 5.2 GHz]

$\rightarrow$ 5.010 GHz, 5.020 GHz, $\ldots$, 5.190 GHz, 5.2 GHz

Note: ignore absolute frequency $f_a = 5$ GHz

$\rightarrow$ what matters: base frequency 0.010 GHz

What about time interval τ?

Time interval τ:

$$\tau = \frac{n}{W}$$

$\rightarrow$ called symbol period

$\rightarrow$ does not depend on f_a

For [5 GHz, 5.2 GHz] and $n = 20$:

$\rightarrow$ $\tau = 20$ / 10 MHz = 2 μs (microsecond)

$\rightarrow$ $n = 20$ carrier waves are mutually orthogonal over τ = 2μs

$\rightarrow$ [0, 2μs], [2μs, 4μs], [4μs, 6μs], . . .,[38μs, 40μs]

Per carrier wave: send one bit per symbol period

$\rightarrow$ same for slow and fast carrier waves

$\rightarrow$ 0.5 Mbps per carrier

$\rightarrow$ total for $n = 20$: 10 Mbps

Summary:

- every EM wave transmits 1 bit over a τ interval

  $\rightarrow$ assures orthogonality for all carrier waves

- for [5 GHz, 5.2 GHz] example, slower 5.010 GHz carrier wave and faster 5.2 GHz wave transmit bits at same speed (bps)

  $\rightarrow$ 1 bit per τ

  $\rightarrow$ 1 bit per 2μs

- thus bandwidth (in bps) per carrier wave is $1/\tau$ (bps)

  $\rightarrow$ assuming AM with two levels

  $\rightarrow$ what about four levels?

- total link bandwidth across all n users

  $\rightarrow$ n/τ (bps)

Implication:

Does increasing n (i.e., more parallel "lanes") increase speed (bps)?

$\rightarrow$ total link bandwidth (bps) $= n/\tau = n/(n/W) = W$

$\rightarrow$ does not depend on n; bandwidth (bps) only depends on W

$\rightarrow$ nor does it depend on absolute frequency f_a

Captured by Shannon's fundamental result (called 2nd theorem).

$\rightarrow$ bandwidth (bps) depends on bandwidth W (Hz)

$\rightarrow$ additionally: role of signal strength and noise

Conservation law at play:

→ increased parallelism n is offset by slow down of carrier waves due to τ

→ to satisfy orthogonality

→ no free lunch

Advantage of OFDM over FDM

- higher spectral efficiency: can transmit more bits in parallel
- clock used to compute and transmit bits (i.e., time interval τ) can be slow

  → i.e., determined by gap $W = f_b - f_a$, not absolute frequency f_a

  → final step of transmission: multiply carrier wave by f_a

  → shifts EM wave from $[W/n, W]$ to $[f_a + W/n, f_a + W]$