FDM (frequency division multiplexing): send bits using multiple EM waves

→ e.g., A uses frequency 1 GHz, B uses 2 GHz

Simplified setting:

- n users employ n different EM waves with frequencies $f_1, f_2, \ldots, f_n$
- each user sends single bit

→ thus: transmit n in parallel over n EM waves

→ called carrier waves (or frequencies)

Consider previous example where cell tower transmits n bits in parallel to n customers (1 bit per customer).

Step 1: synthesis at sender (i.e., cell tower)

- to send n data bits $a_1, a_2, \ldots, a_n$ (+1 for 1, -1 for 0) in parallel using EM waves $\mathbf{v}^1, \mathbf{v}^2, \ldots, \mathbf{v}^n$, compute

  → $\mathbf{u} = a_1\mathbf{v}^1 + a_2\mathbf{v}^2 + \cdots + a_n\mathbf{v}^n$

  → same as in linear algebra

  → treat each wave as a vector

  → indexed by frequency

  → $\mathbf{u}$ is an EM wave produced by summing up n EM waves of different frequencies

- transmit $\mathbf{u}$

  → wired or wireless transmitter

Important: each carrier wave $\mathbf{v}^i$ is held constant (i.e., repeated) for an extended time period, say, τ.

Thus

→ $\mathbf{v}^i$ is repeated beyond its period $1/f_i$

→ redundant

→ view τ as slow clock at receiver (and sender)

Note: carrier frequency may be very fast (e.g., 60 GHz) but software that controls hardware runs at much slower clock rate (e.g., 2-5 GHz).

Step 2: receiver i hears EM wave $\mathbf{u}$; to decode its bit by finding a_i, it performs

- $\mathbf{u} \circ \mathbf{v}^i$

  → same as linear algebra

- $\mathbf{u} \circ \mathbf{v}^i = a_i(\mathbf{v}^i \circ \mathbf{v}^i) = a_i \times$ positive constant

  → by orthogonality

  → carrier waves $\mathbf{v}^1, \mathbf{v}^2, \ldots, \mathbf{v}^n$ are mutually orthogonal

Mathematically: two different carrier waves $\mathbf{v}^i$ and $\mathbf{v}^j$ (i.e., $f_i \neq f_j$) are mutually orthogonal over time interval $(-\infty, \infty)$.

→ hence each EM wave held constant for extended period τ

→ near orthogonality

For meaning of dot product of carrier waves refer to discussion in class.

Same goes for mathematical definition of carrier wave $\mathbf{v}^i$.

→ details are not necessary for logical understanding of FDM and OFDM

Note: compared to 4 steps in CDMA, only 2 steps in FDM.

→ direct from bits to EM wave

Communication problem solved?

→ use many carrier frequencies

→ use AM to transmit bit stream over each carrier wave

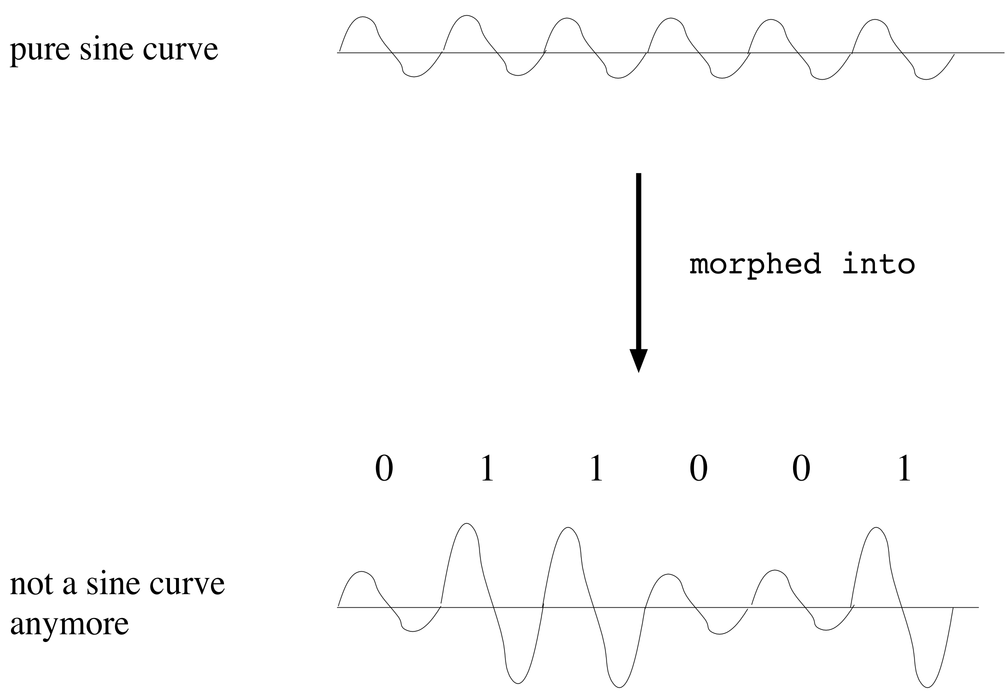
Unfortunately, not.

→ FDM harbors significant limitation

Crux of problem: transmit not single bit but multiple bits over same carrier wave.

→ bit stream per carrier wave

Previous set-up of single user using AM to send multiple bits over single carrier wave.



→ e.g., carrier frequency is $f = 1$ GHz

→ what is the issue?

Performing AM to sequentially to send 011001 corrupts pure EM wave.

→ amplitude not constant across different periods

→ yields a new function $s(t)$: called signal

→ still kind of resembles sine curve

→ but not pure sinunoid anymore

What is $s(t)$?

By better understanding what $s(t)$ is we can understand the limitations of FDM.

→ which leads us to OFDM

Utilize concepts from linear algebra again.

In 3-D, three mutually orthogonal vectors $\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3$ form a basis if:

→ any other 3-D vector $\mathbf{z}$ is their weighted sum

→ i.e., $\mathbf{z} = a_1\mathbf{x}^1 + a_2\mathbf{x}^2 + a_3\mathbf{x}^3$

→ basis vectors span the space

Weights a_1, a_2, a_3 are called spectrum of $\mathbf{z}$.

→ quantify how important the building blocks $\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3$ are

→ signature or "DNA" of $\mathbf{z}$

Same goes for n-dimensional space.

Analogous for signal $s(t)$ obtained from AM modulation of single EM wave.

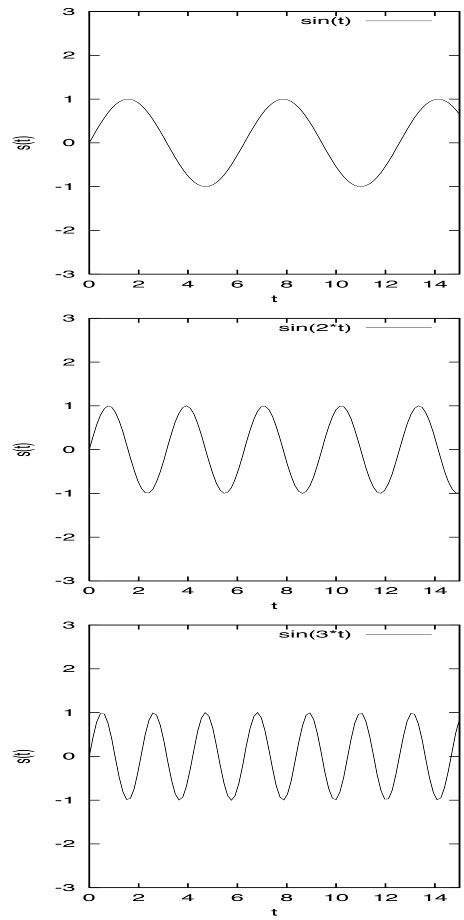
→ since time t is continuous $s(t)$ an infinite sum of weighted pure EM waves

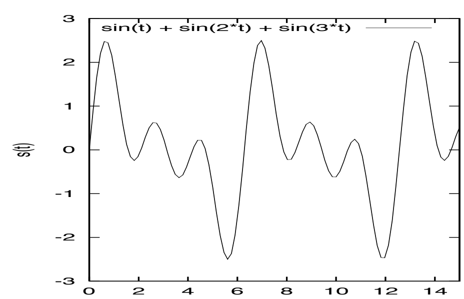
Weight a_f of each EM wave in the sum (i.e., integral) is identified by its frequency f

→ weights a_f across all frequencies called spectrum of $s(t)$

→ quantifies importance of EM wave with frequency f

Spectrum of superposition of three sine waves example?

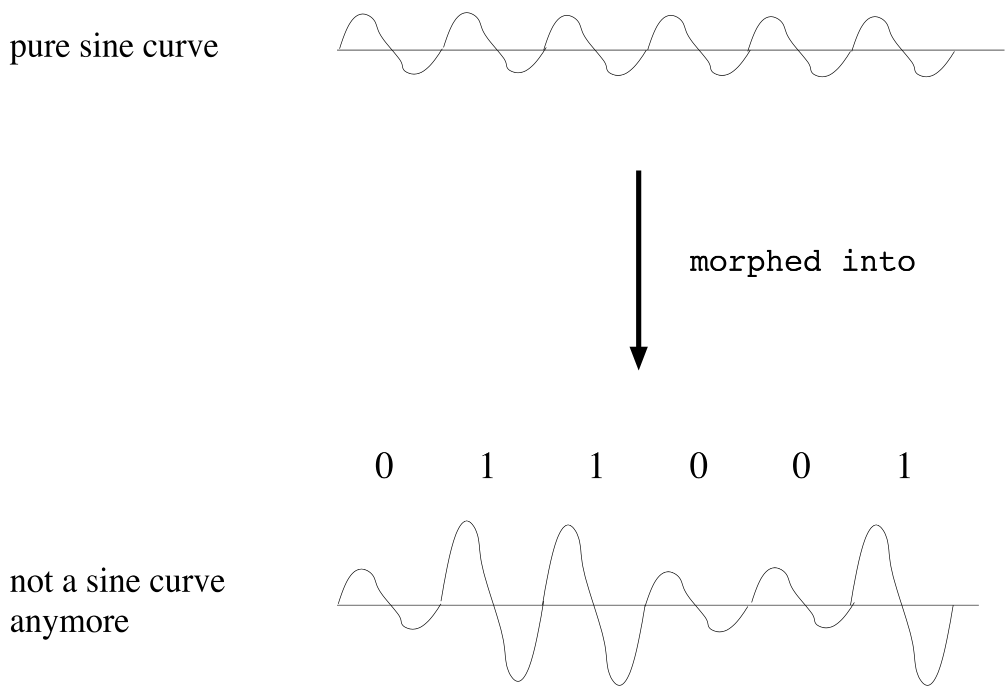




Example: square wave

→ demo link at course web site

Back to AM modulation example:

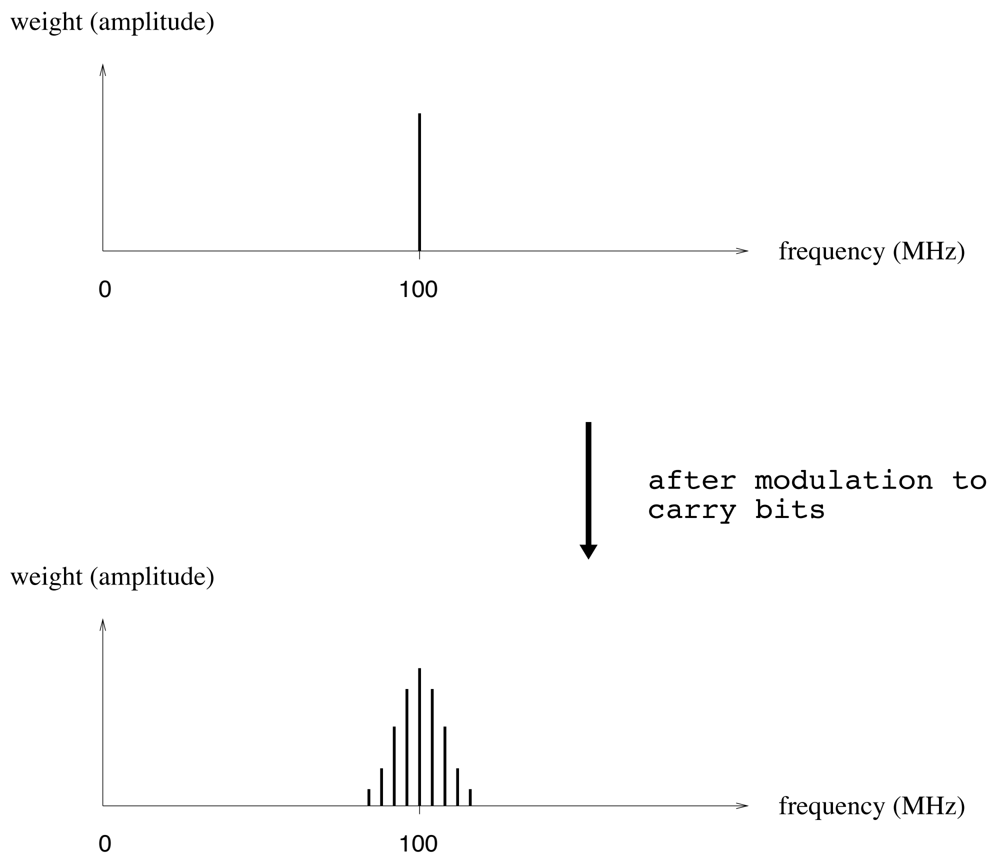


→ not pure sinusoid anymore

→ but bears resemblance

Qualitative picture of what happens: e.g., $f = 100$ MHz

$\rightarrow$ spectrum: before and after performing AM



$\rightarrow$ footprint spreads

$\rightarrow$ concentrated near 100 MHz

Why useful for understanding limitation of FDM?

→ can answer what happens when multiple carrier frequencies are used

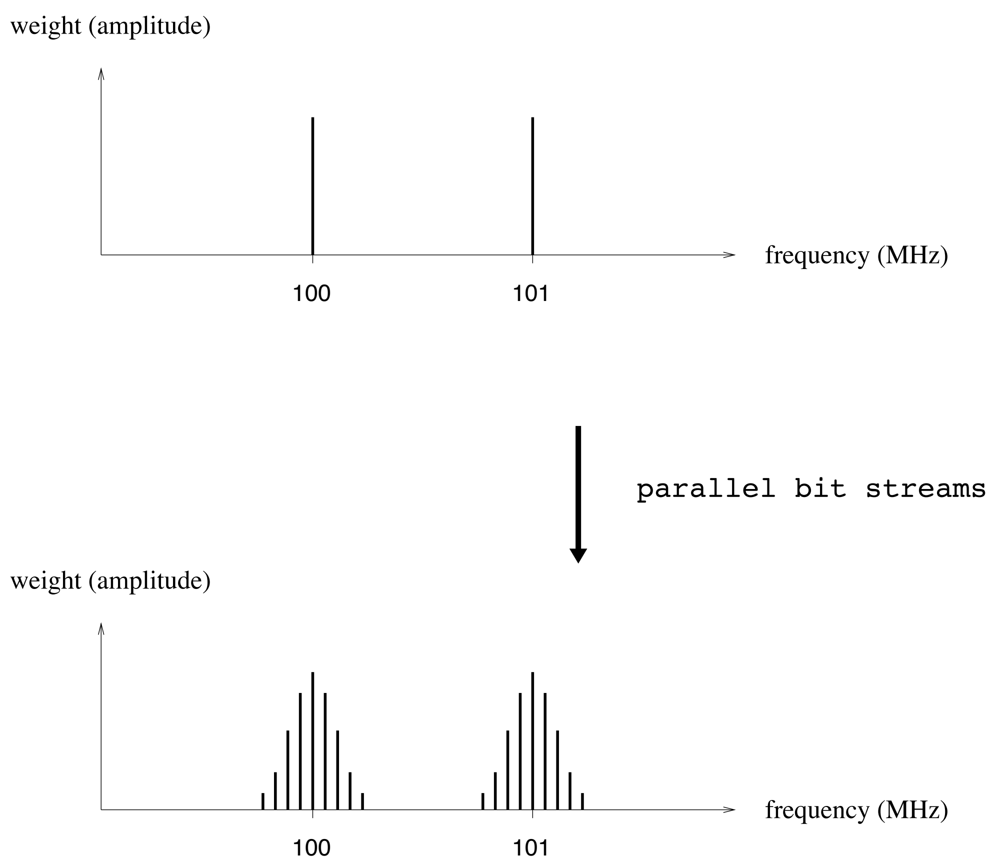
→ feasible or not?

Example: $f_1 = 100$ MHz, $f_2 = 101$ MHz

→ two parallel bit streams

→ each undergoing AM

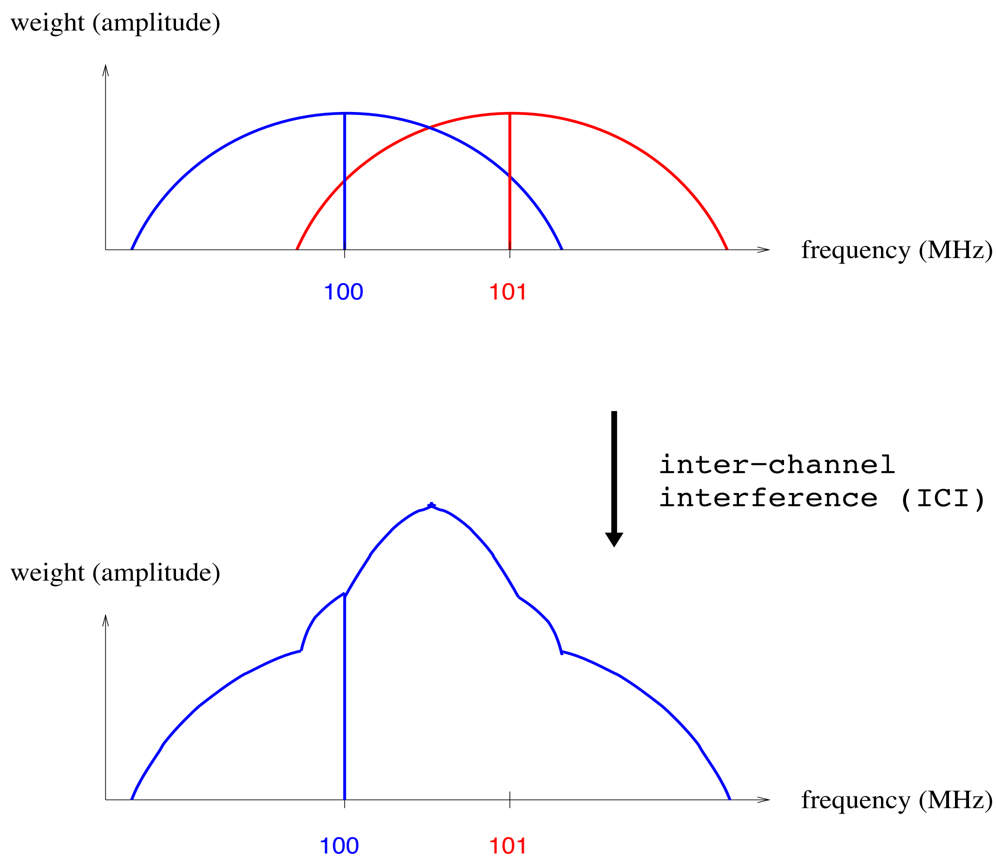
Good case: spreading due to performing AM does not overlap



$\rightarrow$ to decode bits carried by 100 MHz EM wave: ignore values around 101 MHz

$\rightarrow$ filtering

Bad case: spreading overlaps



→ receiver hears superposition of overlapping spectra

→ overlap called inter-channel interference (ICI)

→ cannot reconstruct original signal carried by 100 MHz EM wave; same for 101 MHz

To prevent ICI from overlap must sufficiently separate neighboring carrier frequencies

→ gap called guardband

→ traditional FDM approach: still used today

Limits how many carrier frequencies can be squeezed in a given frequency range.

Example (wired): if copper wire of given grade (i.e., quality) can carry bits reliably over specific distance for EM waves in 100 MHz to 500 MHz range, can only squeeze in so many carrier waves.

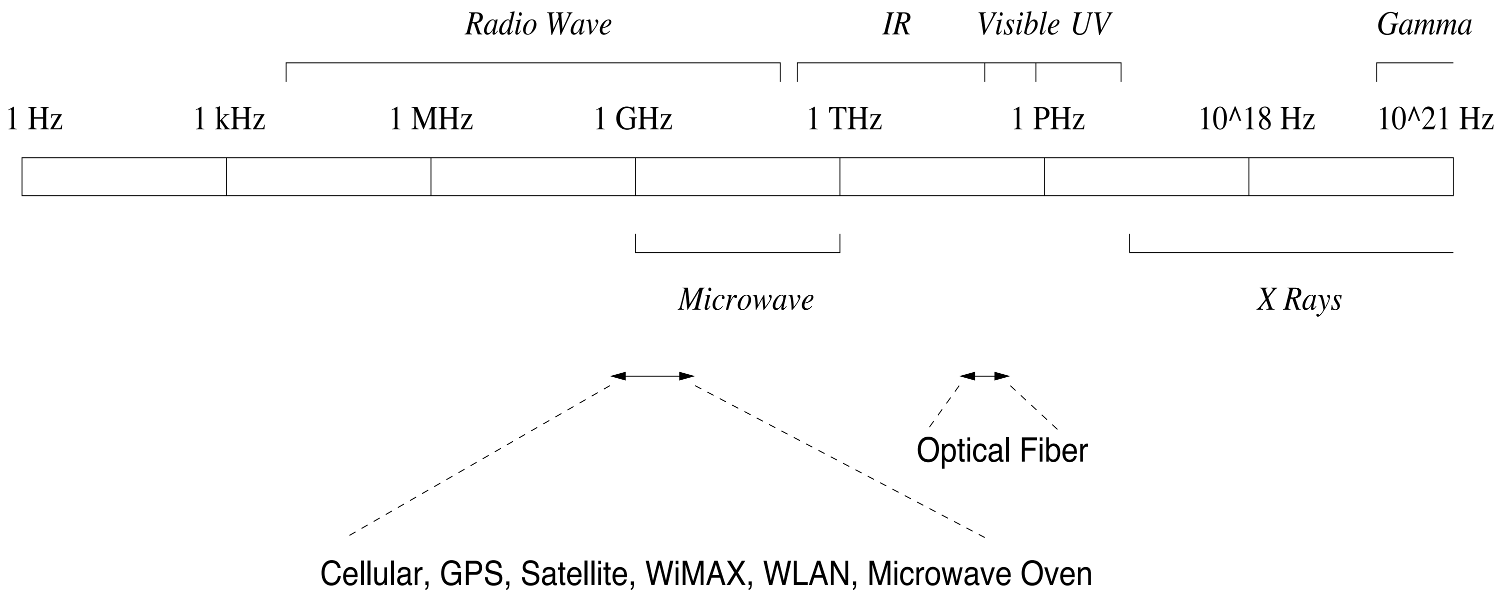
→ low spectral efficiency

→ limits bps

Example (wireless): by regulation/law only allowed to transmit EM waves in 2.4 GHz to 2.5 GHz range. can be feasibly communicated.

→ same issue

Wireless electromagnetic spectrum:



→ logarithmic scale

→ wireless spectrum: crowded

→ bandwidth scarce resource

Modern approach that mitigates limitation of FDM:

→ orthogonal FDM (OFDM)

Key idea: using AM transmit bits over multiple EM waves

→ use EM waves that are mutually orthogonal over finite time interval τ

With n parallel EM waves:

- each EM wave carries first bit (of its bit stream) in first time interval $(0, \tau)$

  → second bit in second time interval $(\tau, 2\tau)$

  → third bit in third time interval $(2\tau, 3\tau)$

  → etc.

Since EM waves are chosen so that they are mutually orthogonal over $(0, \tau)$, can decode first bit of first EM wave using orthogonality.

$\rightarrow$ same for first bit of second EM wave, etc.

$\rightarrow$ hence: n parallel first bits

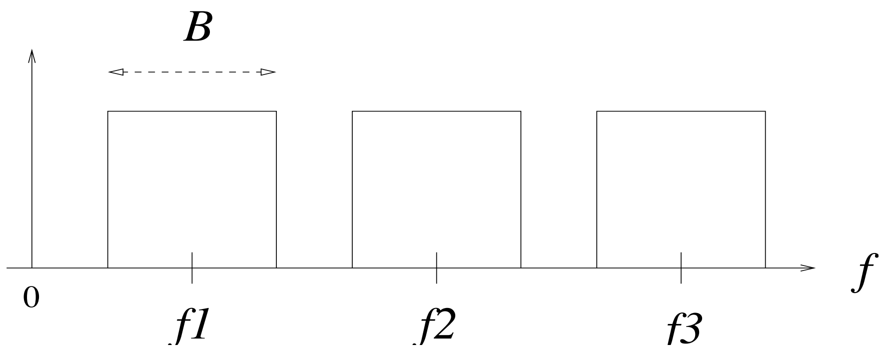
Same holds for second time interval $(\tau, 2\tau)$, third time interval $(2\tau, 3\tau)$, etc.

$\rightarrow$ overlap of neighboring spectra due to AM spreading does not hinder decoding

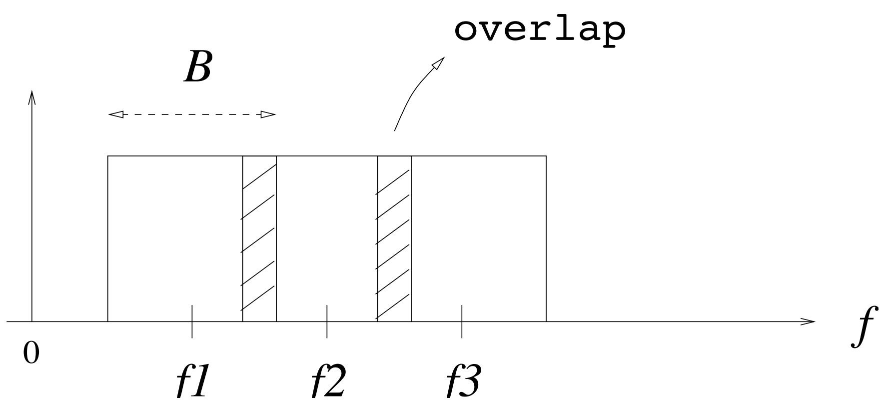
Orthogonality over finite time interval "neutralizes" ICI.

OFDM's advantage over FDM:

FDM:



OFDM:



→ overlapping spectra does not impede decoding

→ higher spectral efficiency and bps