

## Digital vs. Analog Data

Digital data: bits.

- discrete signal
- both in time and amplitude

Analog “data”: audio/voice, video/image

- continuous signal
- both in time and amplitude

Both forms used in today’s network environment.

- burning CDs
- audio/video playback

In broadband networks:

- use analog signals to carry digital data

Important task: analog data is often digitized

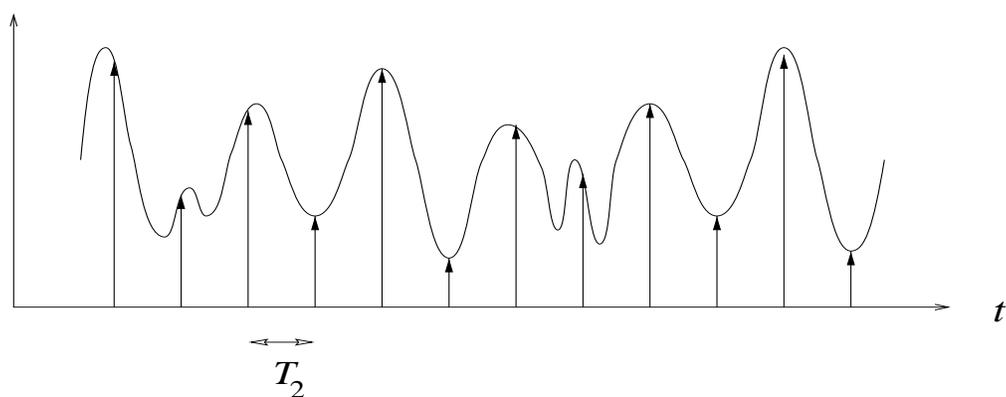
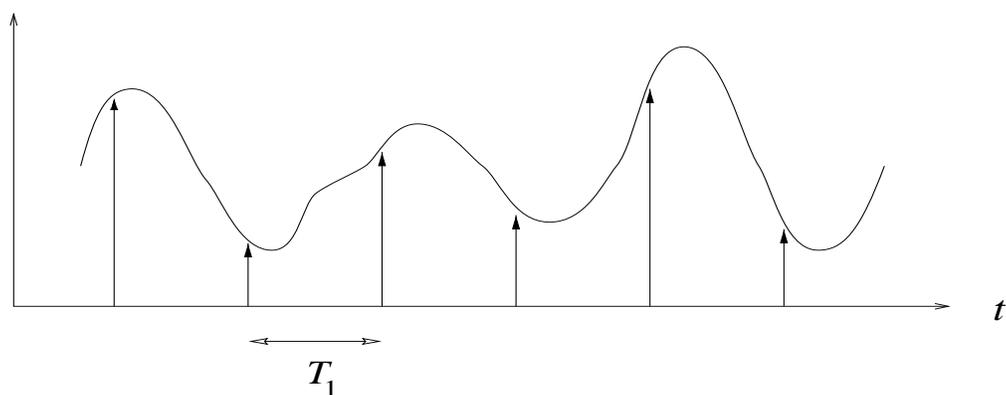
- useful: why?
- it's convenient
- use full power of digital computers
- simple form: digital signal processing
- analog computers are not as versatile/programmable
- cf. "Computer and the Brain," von Neumann (1958)

How to digitize such that digital representation is faithful?

- sampling
- interface between analog & digital world

Intuition behind sampling:

→ slowly vs. rapidly varying signal



If a signal varies quickly, need more samples to not miss details/changes.

$$\nu_1 = 1/T_1 < \nu_2 = 1/T_2$$

Sampling criterion for guaranteed faithfulness:

**Sampling Theorem (Nyquist):** Given continuous bandlimited signal  $s(t)$  with  $S(\omega) = 0$  for  $|\omega| > W$ ,  $s(t)$  can be reconstructed from its samples if

$$\nu > 2W$$

where  $\nu$  is the sampling rate.

→  $\nu$ : samples per second

Remember simple rule: sample twice the bandwidth

Issue of digitizing amplitude/magnitude ignored

→ problem of quantization

→ possible source of information loss

→ exploit limitations of human perception

→ logarithmic scale

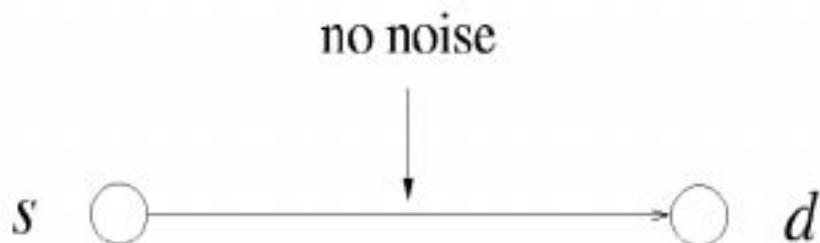
## Compression

Information transmission over noiseless medium

- medium or “channel”
- fancy name for copper wire, fiber, air/space

Sender wants to communicate information to receiver over noiseless channel.

- can receive exactly what is sent
- idealized scenario



Set-up:

- take a system perspective
- e.g., modem manufacturer

Need to specify two parts: property of data source—what are we supposed to send?—and how compression is done.

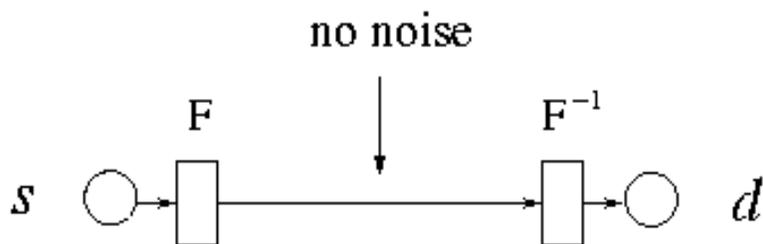
- need to know what we're dealing with
- if we want to do a good job compressing

Part I. What does the (data) source look like:

- source  $s$  emits symbols from finite alphabet set  $\Sigma$ 
  - e.g.,  $\Sigma = \{0, 1\}$ ;  $\Sigma =$  ASCII character set
- symbol  $a \in \Sigma$  is generated with probability  $p_a > 0$ 
  - e.g., books have known distribution for 'e', 'x' ...
  - let's play "Wheel of Fortune"

## Part II. Compression machinery:

- code book  $F$  assigns code word  $w_a = F(a)$  for each symbol  $a \in \Sigma$ 
  - $w_a$  is a binary string of length  $|w_a|$
  - $F$  could be just a table
- $F$  is invertible
  - receiver  $d$  can recover  $a$  from  $w_a$
  - $F^{-1}$  is the same table, different look-up



Ex.:  $\Sigma = \{A, C, G, T\}$ ; need at least two bits

- $F^1$ :  $w_A = 00$ ,  $w_C = 01$ ,  $w_G = 10$ ,  $w_T = 11$
- $F^2$ :  $w_A = 0$ ,  $w_C = 10$ ,  $w_G = 110$ ,  $w_T = 1110$

→ pros & cons?

Note: code book  $F$  is not unique

→ find a “good” code book

→ when is a code book good?

Performance (i.e., “goodness”) measure: average code length  $L$

$$L = \sum_{a \in \Sigma} p_a |w_a|$$

→ average number of bits consumed by given  $F$

Ex.: If DNA sequence is 10000 letters long, then require on average  $10000 \cdot L$  bits to be transmitted.

→ good to have code book with small  $L$

Optimization problem: Given source  $\langle \Sigma, \mathbf{p} \rangle$  where  $\mathbf{p}$  is a probability vector, find a code book  $F$  with least  $L$ .

→ practically super-important

→ shrink-and-send

→ lossless shrinkage

A fundamental result on what is achievable to attain small  $L$ .

→ kind of like speed-of-light

First, define entropy  $H$  of source  $\langle \Sigma, \mathbf{p} \rangle$

$$H = \sum_{a \in \Sigma} p_a \log \frac{1}{p_a}$$

Ex.:  $\Sigma = \{A, C, G, T\}$ ;  $H$  is maximum if  $p_A = p_C = p_G = p_T = 1/4$ .

→ when is it minimum?

**Source Coding Theorem (Shannon):** For all code books  $F$ ,

$$H \leq L_F$$

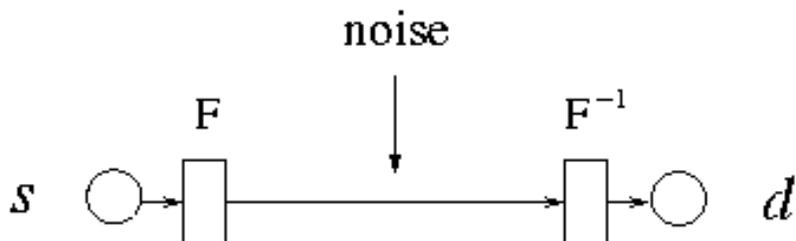
where  $L_F$  is the average code length under  $F$ .

Furthermore,  $L_F$  can be made to approach  $H$  by selecting better and better  $F$ .

Remark:

- to approach minimum  $H$  use blocks of  $k$  symbols
  - e.g., treat “THE” as one unit (not 3 separate letters)
  - called extension code
- entropy is innate property of data source  $s$
- limitation of ensemble viewpoint
  - e.g., sending number  $\pi = 3.1415927\dots$
  - better way?

## Information Transmission under Noise



Uncertainty introduced by noise:

- encoding/decoding:  $a \mapsto w_a \mapsto w \mapsto [?]$
- $w_a$  gets corrupted, i.e., becomes  $w$
- if  $w = w_b$ , incorrectly conclude  $b$  as symbol
- detect  $w$  is corrupted: error detection
- correct  $w$  to  $w_a$ : error correction

Would like: if received code word  $w = w_c$  for some symbol  $c \in \Sigma$ , then probability that actual symbol sent is indeed  $c$  is high

→  $\Pr\{\text{actual symbol sent} = c \mid w = w_c\} \approx 1$

→ noiseless channel: special case (prob = 1)

In practice,  $w$  may not match any legal code word:

→ for all  $c \in \Sigma$ ,  $w \neq w_c$

→ good or bad?

→ what's next?

Shannon showed that there is a fundamental limitation to reliable data transmission.

→ the noisier the channel, the smaller the reliable throughput

→ overhead spent dealing with bit flips

Definition of channel capacity  $C$ : maximum achievable reliable data transmission rate (bps) over a noisy channel (dB) with bandwidth  $W$  (Hz).

**Channel Coding Theorem (Shannon):** Given bandwidth  $W$ , signal power  $P_S$ , noise power  $P_N$ , channel subject to white noise,

$$C = W \log \left( 1 + \frac{P_S}{P_N} \right) \text{ bps.}$$

$P_S/P_N$ : signal-to-noise ratio (SNR)

→ upper bound achieved by using longer codes

→ detailed set-up/conditions omitted

Increasingly important for modern day networking:

- Power control (e.g., pocket PCs)
  - trade-off w.r.t. battery power
  - trade-off w.r.t. multi-user interference
  - signal-to-interference ratio (SIR)
- Recent trend: software radio
  - hardware-to-software migration
  - kind of like cordless phones (e.g., 2.4 GHz)
  - configurable: make it programmable

Signal-to-noise ratio (SNR) is expressed as

$$\text{dB} = 10 \log_{10}(P_S/P_N).$$

**Example:** Assuming a decibel level of 30, what is the channel capacity of a telephone line?

*Answer:* First,  $W = 3000$  Hz,  $P_S/P_N = 1000$ . Using Channel Coding Theorem,

$$C = 3000 \log 1001 \approx 30 \text{ kbps.}$$

- compare against 28.8 kbps modems
- what about 56 kbps modems?
- DSL lines?

## Digital vs. Analog Transmission

Two forms of *transmission*:

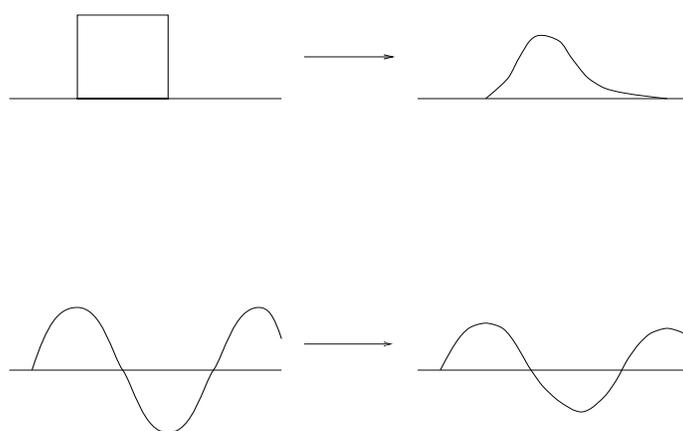
- digital transmission: data transmission using square waves
- analog transmission: data transmission using all other waves

Four possibilities to consider:

- analog data via analog transmission  
→ “as is” (e.g., radio)
- analog data via digital transmission  
→ sampling (e.g., voice, audio, video)
- digital data via analog transmission  
→ broadband & wireless (“high-speed networks”)
- digital data via digital transmission  
→ baseband (e.g., Ethernet)

Why consider digital transmission?

Common to both: problem of *attenuation*.



- decrease in signal strength as a function of distance
- increase in attenuation as a function of frequency

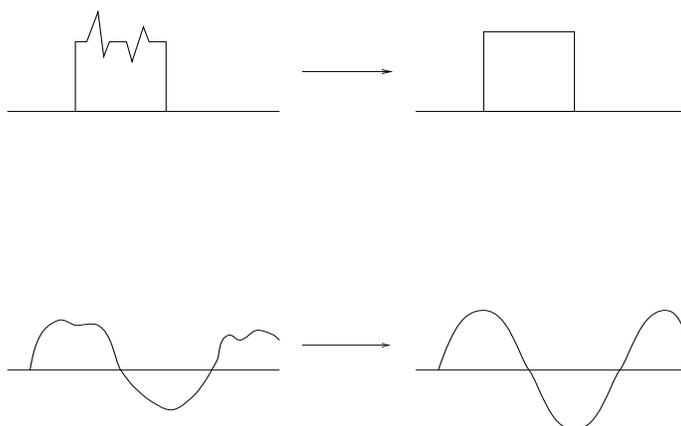
Rejuvenation of signal via amplifiers (analog) and repeaters (digital).

Delay distortion: different frequency components travel at different speeds.

Most problematic: effect of noise

→ thermal, interference, ...

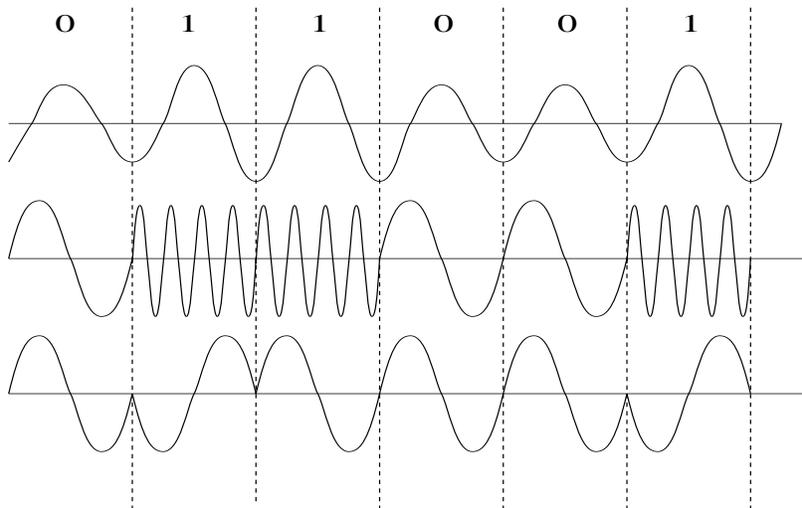
- Analog: Amplification also amplifies noise—filtering out just noise, in general, is a complex problem.
- Digital: Repeater just generates a new square wave; more resilient against ambiguity.



## Analog Transmission of Digital Data

Three pieces of information to manipulate: amplitude, frequency, phase.

- Amplitude modulation (AM): encode bits using amplitude levels.
- Frequency modulation (FM): encode bits using frequency differences.
- Phase modulation (PM): encode bits using phase shifts.



FM radio uses ... FM!

AM radio uses ... AM!

iPod & radio experiment uses ... ?

Why is FM radio clearer (“high fidelity”) than AM radio?

Broadband uses ... ?

## Baud Rate vs. Bit Rate

*Baud rate*: Unit of time within which carrier wave can be altered for AM, FM, or PM.

→ signalling rate

→ e.g., clock

Not synonymous with bit rate: e.g., AM with 8 levels, PM with 8 phases

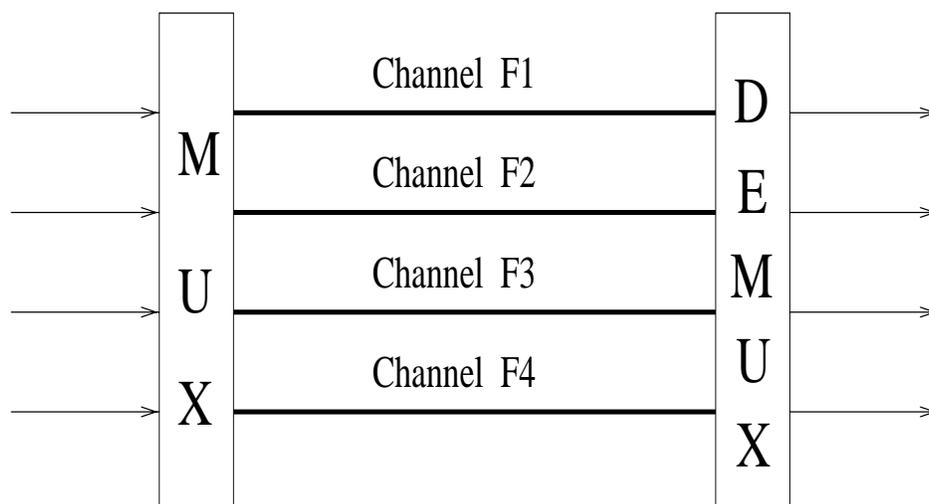
→ bit rate (bps) = 3 × baud rate

... less than one bit per baud?

## Broadband vs. Baseband

Presence or absence of carrier wave: allows many channels to co-exist at the same time

→ frequency division multiplexing (FDM)



Ex.: AM radio (535 kHz–1705 kHz)

→ tuning to specific frequency: Fourier transform

→ coefficient (magnitude) carries bit information

Ex.: FM radio

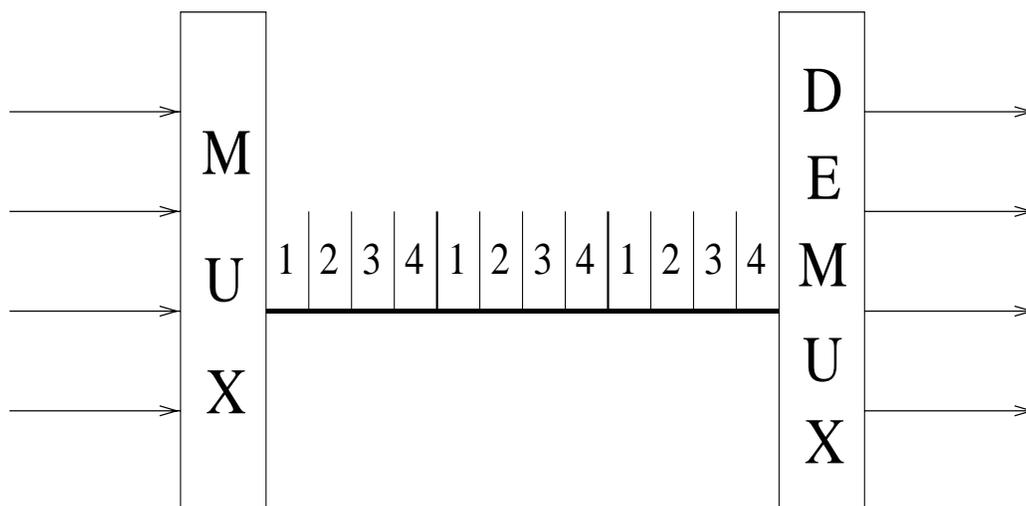
- 88 MHz–108 MHz
- 200 kHz slices
- how does it work?
- better or worse than AM?

Ex.: Digital radio

- digital audio radio service
- GEO satellites (a.k.a. satellite radio)
- uses 2.3 GHz spectrum (a.k.a. S-band)
- e.g., XM, Sirius

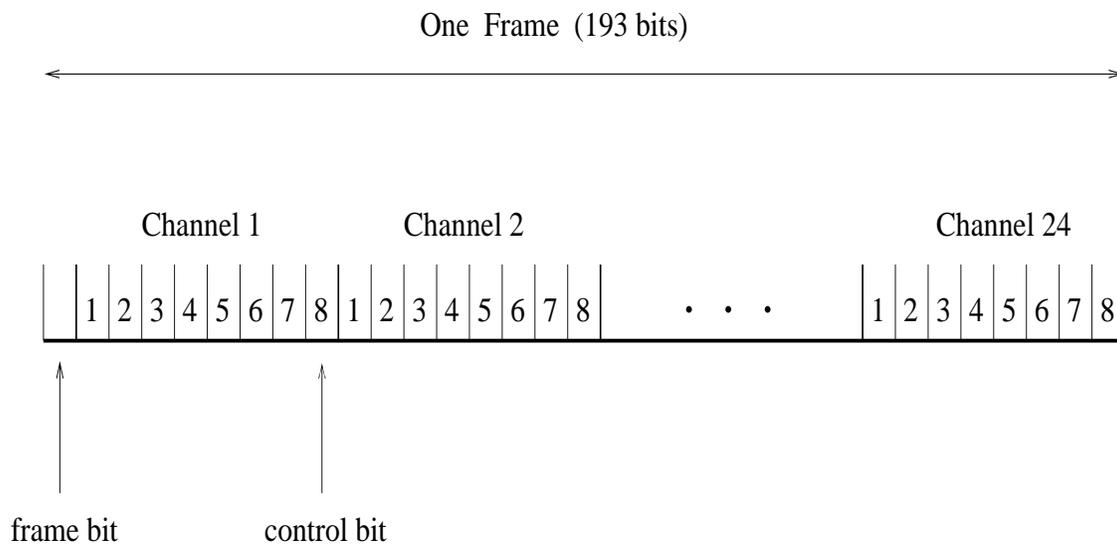
In the absence of carrier wave, can still use multiplexing:

→ time-division multiplexing (TDM)



- digital transmission of analog data
  - first digitize
  - PCM (e.g., PC sound cards), modem
- digital transmission of digital data
  - e.g., telephony backbone network

## Example: T1 carrier (1.544 Mbps)



- 24 simultaneous users
- 7 bit quantization

Assuming 4 kHz telephone channel bandwidth, Sampling Theorem dictates 8000 samples per second.

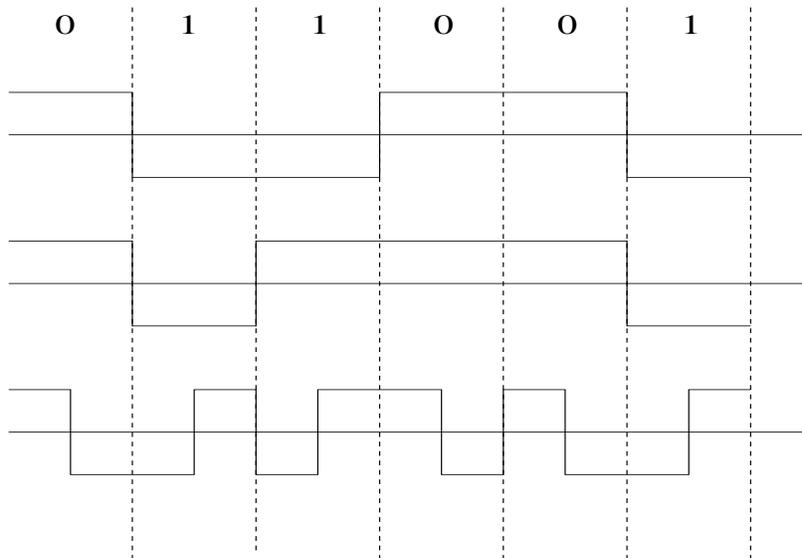
→ 125  $\mu$ sec inter-sample interval

Bandwidth =  $8000 \times 193 = 1.544$  Mbps

## Digital Transmission of Digital Data

Direct encoding of square waves using voltage differentials; e.g.,  $-15\text{V}$ – $+15\text{V}$  for RS-232-C.

- NRZ-L (non-return to zero, level)
- NRZI (NRZ invert on ones)
- Manchester (biphase or self-clocking codes)



→ baud rate vs. bit rate of Manchester?

Trade-offs:

- NRZ codes—long sequences of 0's (or 1's) causes synchronization problem; need extra control line (clock) or sensitive signalling equipment.
- Manchester codes—synchronization achieved through self-clocking; however, achieves only 50% efficiency vis-à-vis NRZ codes.

4B/5B code

Encode 4 bits of data using 5 bit code where the code word has at most one leading 0 and two trailing 0's.

0000  $\leftrightarrow$  11110, 0001  $\leftrightarrow$  01001, etc.

→ at most three consecutive 0's

→ efficiency: 80%

Multiplexing techniques:

- TDM
- FDM
- mixture (FDM + TDM); e.g., TDMA
- CDMA (code division multiple access) or spread spectrum
  - wireless communication
  - competing scheme with TDMA