Issues with just increasing frequency:

Increasing frequency requires increase in processing speed  $\rightarrow \cos t$ 

Wireless: above 10 GHz requires line-of-sight (LOS)

For a given frequency band (say 2.4–2.5 GHz) want to pack as many bits as possible

- $\rightarrow$  utilize the band as much as possible
- $\rightarrow$  also called "bandwidth"
- $\rightarrow$  multiple lanes, i.e., broadband
- $\rightarrow$  if one lane per user: multi-user communication

Wireless: simultaneous uplink (to base station) transmission by multiple clients (from mobiles)

- $\rightarrow$  problem of multi-user communication
- $\rightarrow$  also referred to as multiplexing

Simple solution to multi-user communication:

- $\rightarrow$  share a single lane by time reservation
- $\rightarrow$  time division multiplexing (TDM)
- Ex.: 4 users sharing a single lane, i.e., frequency
- $\rightarrow$  divide time into blocks
- $\rightarrow$  reserve blocks to 4 users: 1, 2, 3, 4, 1, 2, 3, 4, ...



 $\rightarrow$  each block can carry multiple bits: block size  $\rightarrow 1, 2, 3, 4$ : frame or packet

## Real-world example: T1 carrier (1.544 Mbps)



- 24 simultaneous users
- 8-bit block size
- squeeze 8000 frames into 1 second
  - $\rightarrow$  frame duration: 125  $\mu$ sec
- bandwidth:  $8000 \times 193 = 1.544$  Mbps
- drawbacks of using TDM for multi-user communication?

TDM allows sharing of single lane—called carrier frequency by multiple users

- $\rightarrow$  baseband communication
- $\rightarrow$  users alternate in time: not truly simultaneous
- $\rightarrow$  what we want is broadband: multiple lanes
- $\rightarrow$  increase the "size of the pie"
- $\rightarrow$  truly simultaneous

Key problem of broadband or high-speed networks:

Given a frequency band (wired or wireless), how to create as many parallel lanes as possible

- $\rightarrow$  frequency band or "bandwidth" (Hz): scarce resource
- $\rightarrow$  especially wireless
- $\rightarrow$  utilize multiple frequencies for parallel transmission
- $\rightarrow$  frequency division multiplexing (FDM)
- $\rightarrow$  how many lanes are possible?

State-of-the-art: OFDM (orthogonal FDM)

- $\rightarrow$  ubiquitous in wireless networks
- $\rightarrow$  IEEE 802.11g/n WLANs (not 802.11b)
- $\rightarrow$  WiMAX, cellular, etc.

- three wireless hosts are sending bits to base station
- $\bullet$  each host uses its own carrier frequency

 $\rightarrow f_1, f_2, f_3$ 

 $\rightarrow$  e.g., 2.42 GHz, 2.44 GHz, 2.46 GHz

• base station receives



 $\rightarrow$  what bits did the 3 hosts send?



- $\rightarrow$  to recover the bits sent requires recovering the shape of the individual carrier waves
- $\rightarrow$  with hundreds, thousands of carrier frequencies, how to do that?

Note: same problem applies to wireline broadband communication

Ex.: point-to-point link from A to B

- $\rightarrow A$  transmits bits to B over 3 parallel lanes
- $\rightarrow$  faster file exchange

Root of FDM solution: Joseph Fourier

- $\rightarrow$  18th century idea ("old technology")
- $\rightarrow$  Fourier analysis
- $\rightarrow$  engineering bread and butter
- $\rightarrow$  worth knowing for its own sake (great idea)

Fourier's key insight:

A complicated looking signal s(t) whose shape (i.e., strength) varies over time

is just the sum of very simple building blocks  $\rightarrow$  sinusoids

## Thus:



- $\rightarrow$  may require adding many sinusoids of different frequencies
- $\rightarrow$  key caveat: before adding sinusoid with frequency f, multiply its magnitude by a weight  $\alpha$
- $\rightarrow$  therefore complicated looking s(t) is just the weighted sum of sinusoids

Some conceptual similarity to periodic table and matter

- $\rightarrow$  elements of periodic table: building blocks (sinusoids)
- $\rightarrow$  matter: complicated looking signal
- $\rightarrow$  H<sub>2</sub>O: 2 parts H and 1 part O
- $\rightarrow C_8 H_{10} N_4 O_2$
- $\rightarrow$  of course, matter has additional structure: not simple weighted sum

- The value of the weight  $\alpha_f$  of sinusoid f indicates how important sinusoid f is
- For example, if  $\alpha_f = 0$  then sinusoid of frequency f is not needed at all for creating s(t)
- Since there are an infinite number of frequencies (from 0 to ∞) the weighted sum may entail an infinite number of sinusoids

 $\rightarrow$  not relevant for FDM: why?

Fourier's key insight is accompanied by a key technical contribution:

If given some complicated looking signal s(t), then for any sinusoid f Fourier provides a simple formula for finding its weight  $\alpha_f$ 

 $\rightarrow$  a way to decompose into building blocks

Let's make use of Fourier's insight for enabling broadband communication: point-to-point link from A to B

A wishes to send 3 bits to B in parallel using three carrier frequencies  $f_1$ ,  $f_2$ , and  $f_3$ 

- $\rightarrow$  say 3 bits: 1, 0, 1
- $\rightarrow$  carrier frequencies: 1 Hz, 2 Hz, 3 Hz
- $\rightarrow$  how to do it?

Fourier's conceptual and technical contribution in more precise language (aka math):

1. Complicated looking signal s(t) is weighted sum of sinusoids

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \alpha_f e^{ift} df$$

- $\rightarrow$  called Fourier expansion
- $\rightarrow$  integral " $\int$ " is just continuous sum
- $\rightarrow$  recall:  $e^{ift} = \cos ft + i \sin ft$
- $\rightarrow$  Euler's formula
- $\rightarrow$  why complex sinusoids involving  $i = \sqrt{-1}$ ?

2. Given s(t) and frequency f, how to find weight  $\alpha_f$ :

$$\alpha_f = \int_{-\infty}^{\infty} s(t) e^{-ift} dt$$

- $\rightarrow$  another weighted sum
- $\rightarrow$  called Fourier transform
- $\rightarrow$  algorithm to compute Fourier transform quickly: fast Fourier transform (FFT)
- $\rightarrow$  see algorithms book