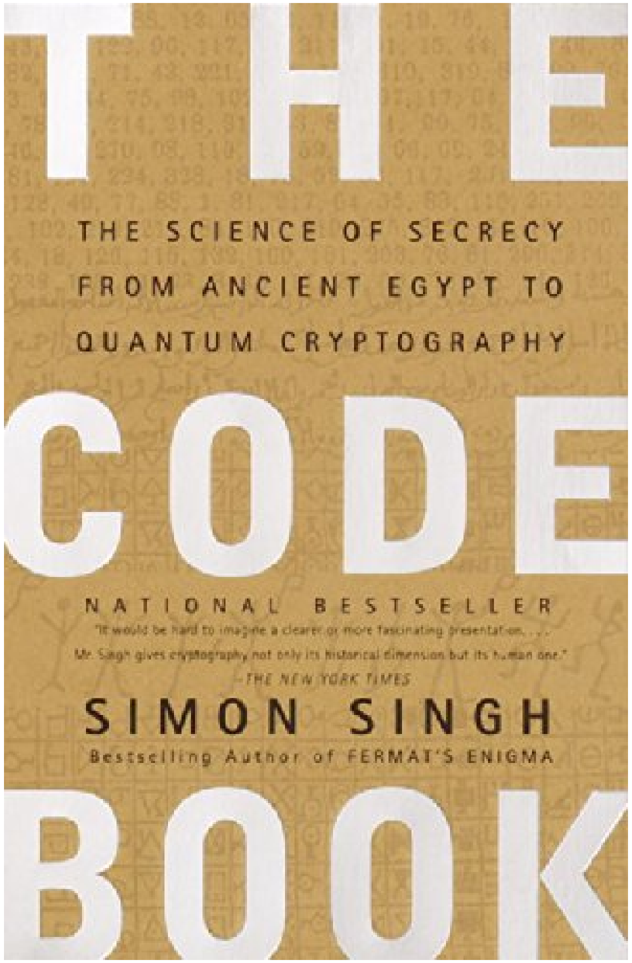
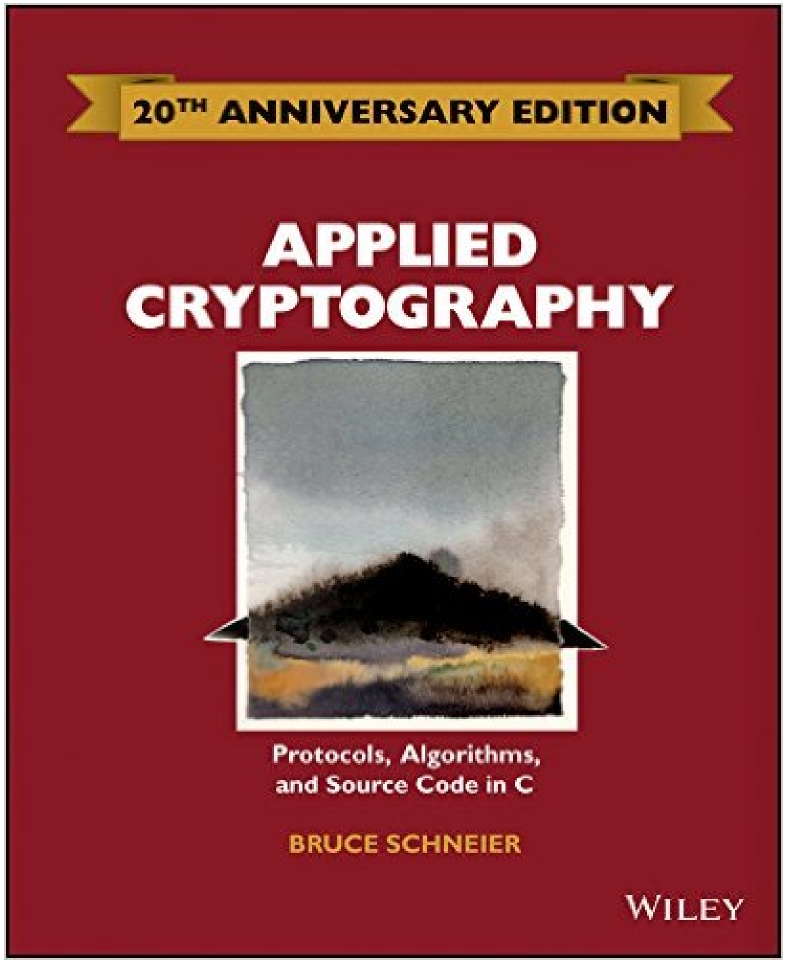
Data Security and Privacy

Overview of Symmetric Cryptography

Books That Might Be Of Interest





Goals of Cryptography

- The most fundamental problem cryptography addresses: ensure security of communication over insecure medium
- What does secure communication mean?
  - confidentiality (secrecy)
    - only the intended recipient can see the communication
  - integrity (authenticity)
    - the communication is generated by the alleged sender
- What does insecure medium mean?
  - Two basic possibilities:
    - Passive attacker: the adversary can eavesdrop
    - Active attacker: the adversary has full control over the communication channel

Basic Terminology for Encryption

- Plaintext — original message
- Ciphertext — transformed message
- Key — secret used in transformation
- Encryption
- Decryption
- Cipher — algorithm for encryption/decryption

Mono-alphabetic Substitution Cipher

- The key space: all permutations of Σ = {A, B, C, …, Z}
- Encryption given a key π:
  - each letter X in the plaintext P is replaced with π(X)
- Decryption given a key π:
  - each letter Y in the cipherext P is replaced with π^{-1}(Y)

Example:

```
   A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
π= B A D C Z H W Y G O Q X S V T R N M L K J I P F E U
```

BECAUSE $\rightarrow$ AZDBJSZ

Cryptanalysis of Substitution Ciphers: Frequency Analysis

- Basic ideas:
  - Each language has certain features: frequency of letters, or of groups of two or more letters.
  - Substitution ciphers preserve the language features.
  - Substitution ciphers are vulnerable to frequency analysis attacks.

How to Defeat Frequency Analysis?

- Use different substitutions to get rid of frequency features.
  - Leads to polyalphabetical substituion ciphers, one-time pad, and to stream ciphers such as RC4

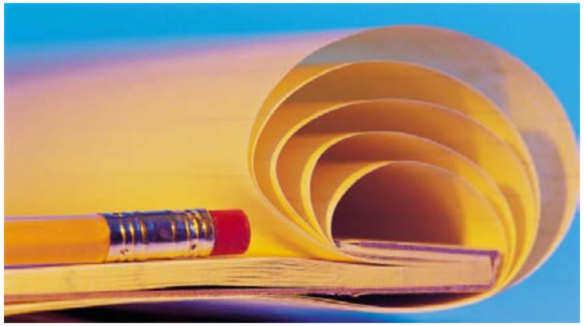
- Use larger blocks as the basis of substitution. Rather than substituting one letter at a time, substitute 64 bits at a time, or 128 bits.
  - Leads to block ciphers such as DES & AES.

One-Time Pad

- Key is a random string that is at least as long as the plaintext
- Encryption is similar to shift cipher
- Invented by Vernam in the 1920s

One-Time Pad

Let Z_m ={0,1,…,m-1} be the alphabet.



Plaintext space = Ciphtertext space = Key space = $(Z_m)^n$

The key is chosen uniformly randomly

Plaintext $X = (x_1\ x_2 \dots x_n)$

Key $K = (k_1\ k_2 \dots k_n)$

Ciphertext $Y = (y_1\ y_2 \dots y_n)$

$e_k(X) = (x_1+k_1\ \ x_2+k_2 \dots x_n+k_n) \bmod m$

$d_k(Y) = (y_1-k_1\ \ y_2-k_2 \dots y_n-k_n) \bmod m$

Shannon (Information-Theoretic) Security = Perfect Secrecy

Basic Idea: Ciphertext should reveal no “information” about Plaintext

Definition. An encryption over a message space $\mathcal{M}$ is perfectly secure if

$\forall$ probability distribution over $\mathcal{M}$

$\forall$ message m $\in \mathcal{M}$

$\forall$ ciphertext c $\in \mathcal{C}$ for which Pr[C=c] > 0

We have

$$\Pr[\mathbf{PT}=m \mid \mathbf{CT}=c] = \Pr[\mathbf{PT} = m].$$

Explanation of the Definition

- Pr [**PT** = m] is what the adversary believes the probability that the plaintext is m, before seeing the ciphertext
- Pr [**PT** = m | **CT**=c] is what the adversary believes after seeing that the ciphertext is c
- Pr [**PT**=m | **CT**=c] = Pr [**PT** = m] means that after knowing that the ciphertext is C_0, the adversary's belief does not change.

An Equivalent Definition of Perfect Secrecy

Definition. An encryption scheme is perfectly secure if and only if **for any ciphertext c, and any two plaintext m1 and m2, the probability that m1 is encrypted to c is the same as the probability that m2 is encrypted to c.**

$\forall$ message m1 ,m2

$\forall$ ciphertext c

Pr [**CT**=c | **PT** = m1] = Pr [**CT** = c | PT = m2]

Perfect Secrecy

- Fact: When keys are uniformly chosen in a cipher, the cipher has perfect secrecy iff. the number of keys encrypting M to C is the same for any (M,C)
  - This implies that

    $\forall c \forall m_1 \forall m_2 \; \Pr[\mathbf{CT}=c \mid \mathbf{PT}=m_1] = \Pr[\mathbf{CT}=c \mid \mathbf{PT}=m_2]$

- One-time pad has perfect secrecy when limited to messages over the same length (Proof?)

Key Randomness in One-Time Pad

- One-Time Pad uses a very long key, what if the key is not chosen randomly, instead, texts from, e.g., a book are used as keys.
  - this is not One-Time Pad anymore
  - this does not have perfect secrecy
  - this can be broken
  - How?
- The key in One-Time Pad should never be reused.
  - If it is reused, it is Two-Time Pad, and is insecure!
  - Why?

Usage of One-Time Pad

- To use one-time pad, one must have keys as long as the messages.
- To send messages totaling certain size, sender and receiver must agree on a shared secret key of that size.
  - typically by sending the key over a secure channel
- Key agreement is difficult to do in practice.
- Can't one use the channel for sending the key to send the messages instead?
- Why is OTP still useful, even though difficult to use?

Usage of One-Time Pad

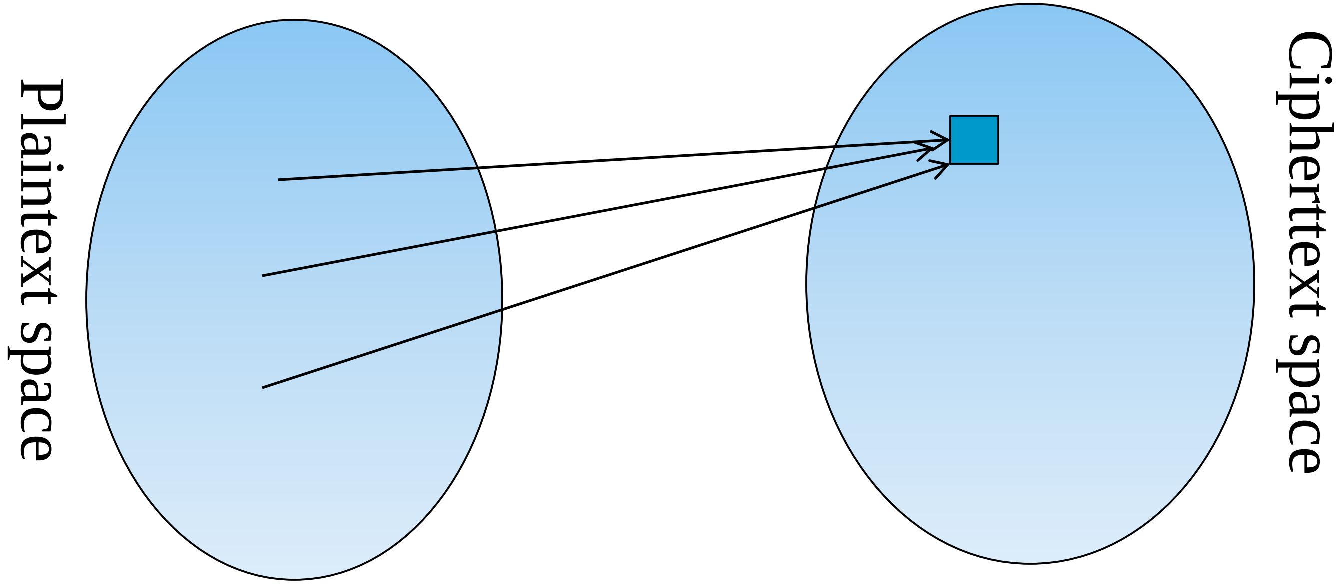
- The channel for distributing keys may exist at a different time from when one has messages to send.

- The channel for distributing keys may have the property that keys can be leaked, but such leakage will be detected
  - Such as in Quantum cryptography

The “Bad News” Theorem for Perfect Secrecy

- Question: OTP requires key as long as messages, is this an inherent requirement for achieving perfect secrecy?
- Answer. Yes. Perfect secrecy implies that
  key-length ≥ msg-length

Proof:



- Implication: Perfect secrecy difficult to achieve in practice

Stream Ciphers

- In One-Time Pad, a key is a random string of length at least the same as the message
- Stream ciphers:
  - Idea: replace "rand" by "pseudo rand"
  - Use Pseudo Random Number Generator
  - PRNG: $\{0,1\}^s \rightarrow \{0,1\}^n$
    - expand a short (e.g., 128-bit) random seed into a long (e.g., 10^6 bit) string that "looks random"
  - Secret key is the seed
  - Basic encryption method: $E_{key}[M] = M \oplus PRNG(key)$

Pseudo Random Number Generator

- Useful for cryptography, simulation, randomized algorithm, etc.
  - Stream ciphers, generating session keys
- The same seed always gives the same output stream
  - Why is this necessary for stream ciphers?
- Simulation requires uniform distributed sequences
  - E.g., having a number of statistical properties
- **Cryptographically secure pseudo-random number generator** requires unpredictable sequences
  - satisfies the "next-bit test“: given consecutive sequence of bits output (but not seed), next bit must be hard to predict
- Some PRNG’s are weak: knowing output sequence of sufficient length, can recover key.
  - Do not use these for cryptographic purposes

Security Properties of Stream Ciphers

- Under known plaintext, chosen plaintext, or chosen ciphertext, the adversary knows the key stream (i.e., PRNG(key))
  - Security depends on PRNG
  - PRNG must be “unpredictable”
- Do stream ciphers have perfect secrecy?
- How to break a stream cipher in a brute-force way?
- If the same key stream is used twice, then easy to break.
  - This is a fundamental weakness of stream ciphers; it exists even if the PRNG used in the ciphers is strong

Using Stream Ciphers in Practice

- If the same key stream is used twice, then easy to break.
  - This is a fundamental weakness of stream ciphers; it exists even if the PRNG used in the ciphers is strong
- In practice, one key is used to encrypt many messages
  - Example: Wireless communication
  - Solution: Use Initial vectors (IV).
  - $E_{key}[M] = [IV, M \oplus PRNG(key || IV)]$
    - IV is sent in clear to receiver;
    - IV needs integrity protection, but not confidentiality protection
    - IV ensures that key streams do not repeat, but does not increase cost of brute-force attacks
    - Without key, knowing IV still cannot decrypt
  - Need to ensure that IV never repeats! How?

Towards Computational Security

- Perfect secrecy is too difficult to achieve.
- Computational security uses two relaxations:
  - Security is preserved only against **efficient** (computationally bounded) adversaries
    - Adversary can only run in feasible amount of time
  - Adversaries can potentially succeed with some **very small probability** (that we can ignore the case it actually happens)

Defining Security

- Desire “semantic security”, i.e., having access to the ciphertext does not help adversary to compute any function of the plaintext.
  – Difficult to use

- Equivalent notion: Adversary cannot distinguish between the ciphertexts of two plaintexts

Towards IND-CPA Security:

- Ciphertext Indistinguishability under a Chosen-Plaintext Attack: Define the following IND-CPA experiment :
  - Involving an Adversary and a Challenger
  - Instantiated with an Adversary algorithm *A*, and an encryption scheme Π = (Gen, Enc, Dec)

Challenger		Adversary
$k \leftarrow \text{Gen}()$	$\text{Enc}_k[\,]$ →	
	← m_0, m_1	chooses $m_0, m_1 \in \mathcal{M}$
$b \leftarrow_R \{0,1\}$	$C = \textbf{Enc}_k[m_b]$ →	
	← $b' \in \{0,1\}$	

Adversary wins if b=b'

Computational Security vs. Information Theoretic Security

- If a cipher has only computational security, then it can be broken by a brute force attack, e.g., enumerating all possible keys
  - Weak algorithms can be broken with much less time
- How to prove computational security?
  - Assume that some problems are hard (requires a lot of computational resources to solve), then show that breaking security means solving the problem
- Computational security is foundation of modern cryptography.

Why Block Ciphers?

- One thread of defeating frequency analysis
  - Use different keys in different locations
  - Example: one-time pad, stream ciphers

- Another way to defeat frequency analysis
  - Make the unit of transformation larger, rather than encrypting letter by letter, encrypting block by block
  - Example: block cipher

Block Ciphers

- An n-bit plaintext is encrypted to an n-bit ciphertext
  - $\mathcal{P}$: $\{0,1\}^n$
  - $\mathcal{C}$: $\{0,1\}^n$
  - $\mathcal{K}$: $\{0,1\}^s$
  - **E**: $\mathcal{K} \times \mathcal{P} \rightarrow \mathcal{C}$: E_k: a permutation on $\{0,1\}^n$
  - **D**: $\mathcal{K} \times \mathcal{C} \rightarrow \mathcal{P}$: D_k is E_k^{-1}
  - Block size: n
  - Key size: s

Data Encryption Standard (DES)

- Designed by IBM, with modifications proposed by the National Security Agency
- US national standard from 1977 to 2001
- De facto standard
- Block size is 64 bits;
- Key size is 56 bits
- Has 16 rounds
- Designed mostly for hardware implementations
  - Software implementation is somewhat slow
- Considered insecure now
  - vulnerable to brute-force attacks
- Triple DES: $E_{k3}D_{k2}E_{K1}(M)$ has 112-bit strength, but slow

Advanced **E**ncryption **S**tandard

- In 1997, NIST made a formal call for algorithms stipulating that the AES would specify an unclassified, publicly disclosed encryption algorithm, available royalty-free, worldwide.
- Goal: replace DES for both government and private-sector encryption.
- The algorithm must implement symmetric key cryptography as a block cipher and (at a minimum) support block sizes of 128-bits and key sizes of 128-, 192-, and 256-bits.
- In 1998, NIST selected 15 AES candidate algorithms.
- On October 2, 2000, NIST selected **Rijndael** (invented by Joan Daemen and Vincent Rijmen) to as the AES.

Need for Encryption Modes

- A block cipher encrypts only one block
- Needs a way to extend it to encrypt an arbitrarily long message
- Want to ensure that if the block cipher is secure, then the encryption is secure
- Aims at providing Semantic Security (**IND-CPA)** assuming that the underlying block ciphers are strong

Block Cipher Encryption Modes: ECB

- Message is broken into independent blocks;
- Electronic Code Book (ECB): each block encrypted separately.
- **Encryption: $c_i = E_k(x_i)$**
- **Decrytion: $x_i = D_k(c_i)$**

Properties of ECB

- Deterministic:
  - the same data block gets encrypted the same way,
    - reveals patterns of data when a data block repeats
  - when the same key is used, the same message is encrypted the same way
- Usage: not recommended to encrypt more than one block of data

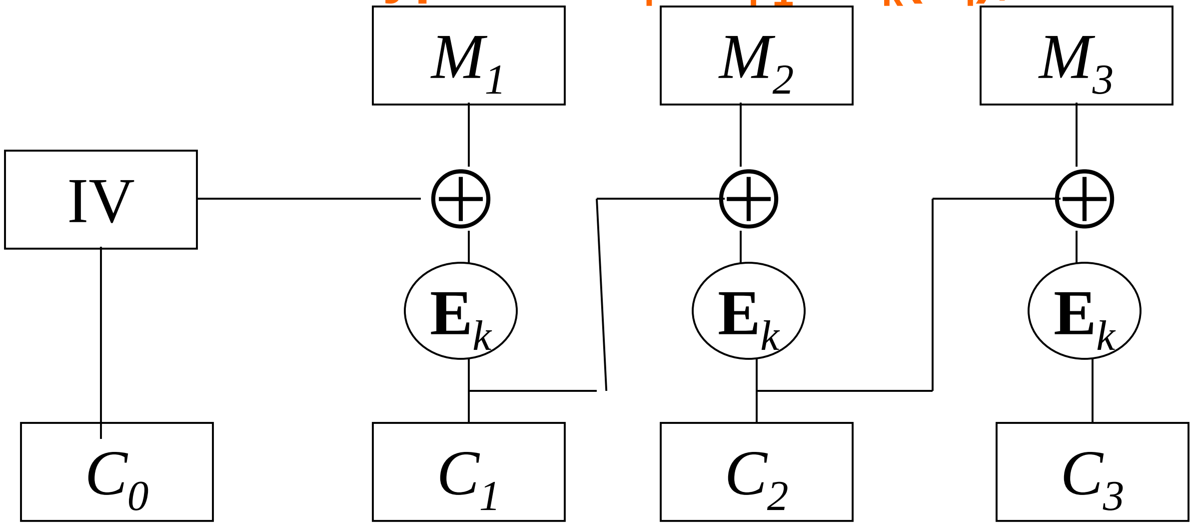
- How to break the semantic security (IND-CPA) of a block cipher with ECB?

DES Encryption Modes: CBC

- Cipher Block Chaining (CBC):
  - Uses a random Initial Vector (IV)
  - Next input depends upon previous output

Encryption: $C_i = E_k(M_i \oplus C_{i-1})$, **with** $C_0 = IV$

Decryption: $M_i = C_{i-1} \oplus D_k(C_i)$, **with** $C_0 = IV$

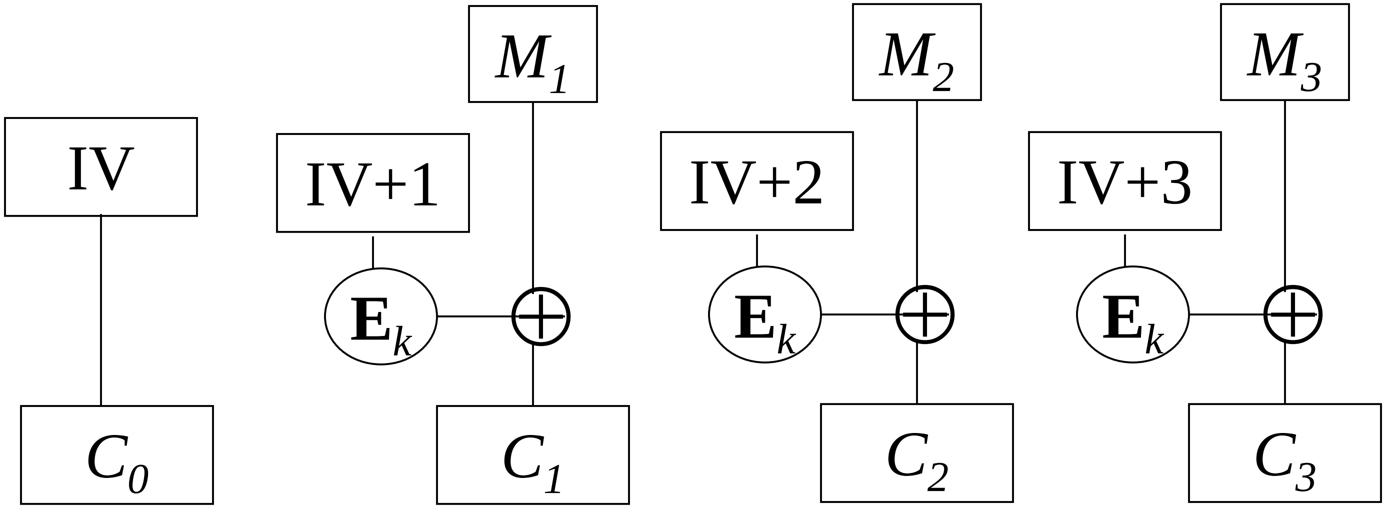


Properties of CBC

- Randomized encryption: repeated text gets mapped to different encrypted data.
  - can be proven to provide IND-CPA assuming that the block cipher is secure (i.e., it is a Pseudo Random Permutation (PRP)) and that IV's are randomly chosen and the IV space is large enough (at least 64 bits)
- Each ciphertext block depends on all preceding plaintext blocks.
- Usage: chooses **random** IV and protects the **integrity** of IV
  - The IV is not secret (it is part of ciphertext)
  - The adversary cannot control the IV

Encryption Modes: CTR

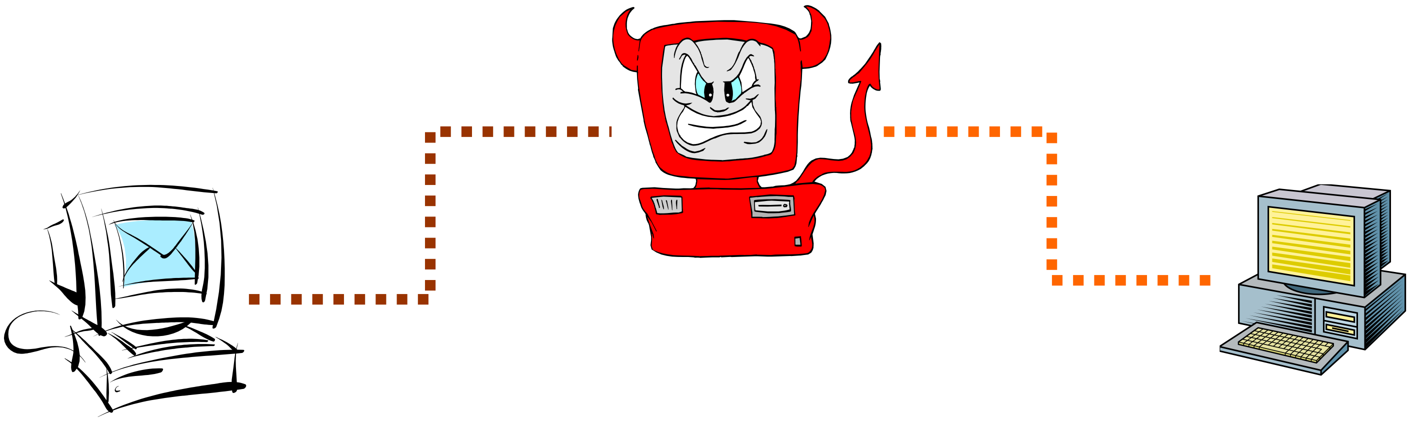
- Counter Mode (CTR): Defines a stream cipher using a block cipher
  - Uses a random IV, known as the counter
  - Encryption: C_0=IV, $C_i = M_i \oplus E_k[IV+i]$
  - Decryption: IV=C_0, $M_i = C_i \oplus E_k[IV+i]$



Properties of CTR

- Gives a stream cipher from a block cipher
- Randomized encryption:
  - when starting counter is chosen randomly
- Random Access: encryption and decryption of a block can be done in random order, very useful for hard-disk encryption.
  - E.g., when one block changes, re-encryption only needs to encrypt that block. In CBC, all later blocks also need to change

Data Integrity and Source Authentication



- Encryption does not protect data from modification by another party.
- Most encryption schemes are **malleable**:
  - Modifying ciphertext result in (somewhat) predictable change in plaintext
- Need a way to ensure that data arrives at destination in its original form as sent by the sender.

Hash Functions

- A hash function maps a message of an arbitrary length to a m-bit output
  - output known as the fingerprint or the message digest
- What is an example of hash functions?
  - Give a hash function that maps Strings to integers in [0,2^{32}-1]
- Cryptographic hash functions are hash functions with additional security requirements

Security Requirements for Cryptographic Hash Functions

Given a function $h:X \rightarrow Y$, then we say that h is:

- preimage resistant (one-way):

  if given $y \in Y$ it is computationally infeasible to find a value $x \in X$ s.t. $h(x) = y$

- 2-nd preimage resistant (weak collision resistant):

  if given $x \in X$ it is computationally infeasible to find a value $x' \in X$, s.t. $x' \neq x$ and $h(x') = h(x)$

- collision resistant (strong collision resistant):

  if it is computationally infeasible to find two distinct values $x', x \in X$, s.t. $h(x') = h(x)$

Usages of Cryptographic Hash Functions

- Software integrity
  - E.g., tripwire
- Timestamping (cryptographic commitment)
  - How to prove that you have discovered a secret on an earlier date without disclosing the context of a secret?
- Other applications
  - Message authentication
  - One-time passwords
  - Digital signature

Choosing Parameters

- The level of security (for collision resistance) of a hash function that outputs n bits, is about n/2 bits
  - i.e., it takes $2^{n/2}$ time to bruteforce it
  - Assuming that no better way of attacking the hash function is known
- Longer outputs often means more computation time and more communication overhead
- The level of security for encryption function using k-bit key is about k bits

Choosing the length of Hash outputs

- The Weakest Link Principle:
  - A system is only as secure as its weakest link.
  - Hence all links in a system should have similar levels of security.
- Because of the birthday attack, the length of hash outputs in general should double the key length of block ciphers
  - SHA-224 matches the 112-bit strength of triple-DES (encryption 3 times using DES)
  - SHA-256, SHA-384, SHA-512 match the new key lengths (128,192,256) in AES
  - If small output size is highly important, and one is sure that collision-resistance is not needed (only one-wayness is needed), then same size should be okay.

Well Known Hash Functions

- MD5
  - output 128 bits
  - collision resistance completely broken by researchers in China in 2004
- SHA1
  - output 160 bits
  - In Feb 2017, collision found with $\approx 10^{19} \approx 2^{63}$ SHA-1 evaluations (costing around $100K using cloud)
  - one-wayness still holds
- SHA2 (SHA-224, SHA-256, SHA-384, SHA-512)
  - outputs 224, 256, 384, and 512 bits, respectively
  - No practical security concerns yet
- SHA3 (224, 256, 384, 512)

Using Hash Functions for Message Integrity

- Method 1: Uses a Hash Function h, assuming an authentic (adversary cannot modify) channel for short messages
  - Transmit a message M over the normal (insecure) channel
  - Transmit the message digest h(M) over the **authentic** channel
  - When receiver receives both M' and h, how does the receiver check to make sure the message has not been modified?

- This is insecure. How to attack it?
- A hash function is a many-to-one function, so collisions can happen.

Hash Family

- A hash family is a four-tuple (X,Y,K,H), where
  - X is a set of possible messages
  - Y is a finite set of possible message digests
  - K is the keyspace
  - For each $K \in K$, there is a hash function $h_K \in H$. Each $h_K: X \rightarrow Y$
- Alternatively, one can think of H as a function $K \times X \rightarrow Y$

Message Authentication Code (MAC)

- A MAC scheme is a hash family, used for message authentication
- $MAC(K,M) = H_K(M)$
- The sender and the receiver share secret K
- The sender sends $(M, H_k(M))$
- The receiver receives (X,Y) and verifies that $H_K(X)=Y$, if so, then accepts the message as from the sender
- To be secure, an adversary shouldn't be able to come up with (X',Y') such that $H_K(X')=Y'$.

MAC: Using a shared secret (or a limit-bandwidth confidential channel) to achieve authenticity/integrity.

Security Requirements for MAC

- Secure against the "Existential Forgery under Chosen Plaintext Attack"
  - Challenger chooses a random key K
  - Adversary chooses a number of messages M_1, M_2, .., M_n, and obtains t_j=MAC(K,M_j) for $1 \leq j \leq n$
  - Adversary outputs M' and t'
  - Adversary wins if $\forall$j M'$\neq M_j$, and t'=MAC(K,M')

- Basically, adversary cannot create the MAC value for a message for which it hasn't seen an MAC

Constructing MAC from Hash Functions

- Let h be a one-way hash function

- MAC(K,M) = h(K || M), where || denote concatenation
  - Insecure as MAC with a hash function that uses the Merkle-Damgard construction:
  - given M and t=h(K || M), adversary can compute M’=M||Pad(M)||X and t’, such that h(K||M’) = t’

HMAC: Constructing MAC from Cryptographic Hash Functions

$$HMAC_K[M] = Hash[(K^+ \oplus opad) \,||\, Hash[(K^+ \oplus ipad)||M)]]$$

- K^+ is the key padded (with 0) to B bytes, the input block size of the hash function
- ipad = the byte 0x36 repeated B times
- opad = the byte 0x5C repeated B times.

At high level, $HMAC_K[M] = H(K \,||\, H(K \,||\, M))$

Hash function is used twice, in nested fashion.

HMAC Security

- If used with a secure hash functions (e.g., SHA-256) and according to the specification (key size, and use correct output), no known practical attacks against HMAC

Next Lecture

- Overview of Public-Key Cryptography