Introduction to Cryptography CS 355

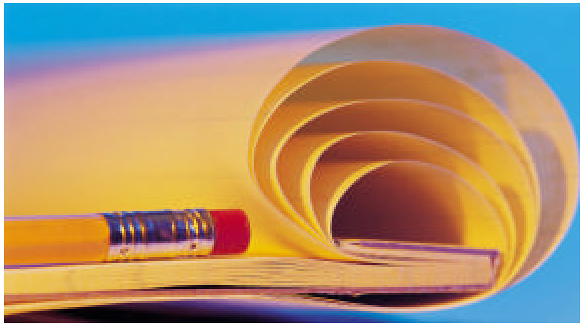
Lecture 9

One Time Pad

One-Time Pad

- Basic Idea: Extend Vigenère cipher so that the key is as long as the plaintext
  - No repeat, cannot be broken by finding key length + frequency analysis
- Key is a random string that is at least as long as the plaintext
- Encryption is similar to Vigenère

One-Time Pad



Plaintext space =

Ciphtertext space =

Keyspace = $(Z_m)^n$

Key is chosen randomly

Plaintext $X = (x_1\ x_2\ \ldots\ x_n)$

Key $K = (k_1\ k_2\ \ldots\ k_n)$

$Y = (y_1\ y_2\ \ldots\ y_n)$

$e_k(X) = (x_1+k_1\ \ x_2+k_2\ \ldots\ x_n+k_n) \bmod m$

$d_k(Y) = (y_1+k_1\ \ y_2+k_2\ \ldots\ y_n+k_n) \bmod m$

How Good is One-Time Pad?

- Intuitively, it is secure …
- The key is random, so the ciphertext is completely random

Shannon (Information-Theoretic) Security

- Basic Idea: Ciphertext should provide no "information" about Plaintext
  - Precise definition will be given towards the end of the course
- We also say such a scheme has perfect secrecy.
- One-time pad has perfect secrcy
  - E.g., suppose that the ciphertext is "Hello", can we say any plaintext is more likely than another plaintext?

What about the Ciphers We Have Studied

- Why Shift cipher does not have perfect secrecy?
- Why Vigenère cipher does not have perfect secrecy?

Key Randomness in One-Time Pad

- One-Time Pad uses a very long key, what if the key is not chosen randomly, instead, texts from, e.g., a book is used.
  – this is not One-Time Pad anymore
  – this does not have perfect secrecy
  – this can be broken easily

- The key in One-Time Pad should never be reused.
  – If it is reused, it is Two-Time Pad, and is insecure!

The “Bad News” Theorem for Perfect Secrecy

- Perfect secrecy $\Rightarrow$ key-length $\geq$ msg-length
- Difficult to use in practice

The Binary Version of One-Time Pad

Plaintext space = Ciphtertext space =

Keyspace = $\{0,1\}^n$

Key is chosen randomly

For example:

- Plaintext is 11011011
- Key is 01101001
- Then ciphertext is 10110010

Bit Operators

- Bit AND

  $0 \wedge 0 = 0$ $0 \wedge 1 = 0$ $1 \wedge 0 = 0$ $1 \wedge 1 = 1$

- Bit OR

  $0 \vee 0 = 0$ $0 \vee 1 = 1$ $1 \vee 0 = 1$ $1 \vee 1 = 1$

- Addition mod 2 (also known as Bit XOR)

  $0 \oplus 0 = 0$ $0 \oplus 1 = 1$ $1 \oplus 0 = 1$ $1 \oplus 1 = 0$

- Can we use operators other than Bit XOR for binary version of One-Time Pad?

Stream Ciphers

- In OTP, a key is described by a random bit string of length n
- Stream ciphers:
  - – Idea: replace “rand” by “pseudo rand”
  - – Use Pseudo Random Number Generator
  - – PRNG: $\{0,1\}^s \rightarrow \{0,1\}^n$
    - expand a short (e.g., 128-bit) random seed into a long (e.g., 10^6 bit) string that “looks random”
  - – Secret key is the seed
  - – $E_{seed}[M] = M \oplus PRNG(seed)$

Properties of Stream Ciphers

- Does not have perfect secrecy
  - security depends on PRNG
- PRNG must be “unpredictable”
  - given consecutive sequence of bits output (but not seed), next bit must be hard to predict
- Typical stream ciphers are very fast
- Used in many places, often incorrectly
  - SSL(RC4), DVD (LFSR), WEP (RC4), etc.

Fundamental Weaknesses of Stream Ciphers

- If the same stream is used twice ever, then easy to break.
- Highly malleable
  - easy to change ciphertext so that plaintext changes in predictable, e.g., flip bits
- Weaknesses exist even if the PRNG is strong

Coming Attractions …

- Linear Feedback Shift Register (LFSR)

- Recommended reading for next lecture:
  - Trappe & Washington: 2.10