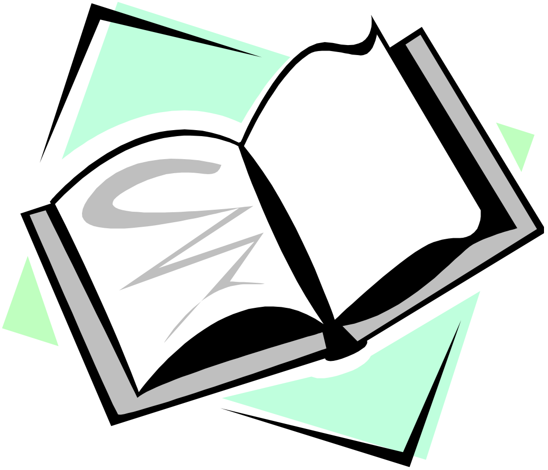
Cryptography
CS 555

Topic 21: Digital Schemes (1)

Outline and Readings

- Outline
  - Digital signature
  - RSA signatures
  - Hash and sign
- Readings:
  - Katz and Lindell: Chapter 12.1-12.4



Digital Signatures: The Problem

- Consider the real-life example where a person pays by credit card and signs a bill; the seller verifies that the signature on the bill is the same with the signature on the card
- Contracts are valid if they are signed.
- Signatures provide non-repudiation.
  - ensuring that a party in a dispute cannot repudiate, or refute the validity of a statement or contract.
- Can we have a similar service in the electronic world?
  - Does Message Authentication Code provide non-repudiation? Why?

Digital Signatures

- MAC: One party generates MAC, one party verifies integrity.
- Digital signatures: One party generates signature, many parties can verify.
- Digital Signature: a data string which associates a message with some originating entity.
- Digital Signature Scheme:
  - a signing algorithm: takes a message and a (private) signing key, outputs a signature
  - a verification algorithm: takes a (public) verification key, a message, and a signature
- Provides:
  - Authentication, Data integrity, Non-Repudiation

Digital Signature

- A signature scheme consists of the following three PPT algorithms
  - $(pk,sk) \leftarrow$ **Gen**(1^n) key generation
  - $\sigma \leftarrow$ **Sign**$_{sk}(m)$ signing
  - $b :=$ **Vrfy**$_{pk}(m,t)$ verification algorithm

    b=1 meaning valid, b=0 meaning invalid

  Must satisfy $\forall$ (pk,sk) $\forall m$ **Vrfy**$_{pk}(m,$ **Sign**$_{sk}(m))$ = 1

  Assume that receiver has an authentic copy of the sender's public key, then receiver can verify that a document is indeed sent by the sender.

Security of Signature Schemes

- The experiment **Sig-forge**$_{\mathbf{A},\Pi}$
  - $(pk,sk) \leftarrow$ **Gen**(1^n)
  - Adversary A is given pk and oracle access to **Sign**$_{sk}(\cdot)$
  - Adversary outputs (m, σ). Let Q denote the set of all queries that A asked to the oracle.
  - Adversary wins if **Vrfy**$_{pk}(m, t) = 1$ and $m \notin Q$

- A signature Π **is existential unforgeable under an adaptive chosen-message attack** (or just **secure**) if for all PPT A, there exists a negligible function negl such that

  Pr[**Mac-forge**$_{\mathbf{A},\Pi}$=1] $\leq$ negl(n)

“Textbook RSA” Signatures

Key generation (as in RSA encryption):

Public key: (e, n)	**used for verification**
Private key: d,	**used for generation**

Signing message m with private key

- Compute $\sigma = m^d \bmod n$

Verifying signature σ using public key (e, n)

- Check whether $\sigma^e \bmod n = m$

Insecurity of “Textbook RSA”

- A no-message attack
  - Choose arbitrary σ, compute m= σ^e mod n
  - (m,σ) is a valid pair
  - One cannot control what is m
- Forging signature on arbitrary message
  - To forge signature on message m, query signing oracle for m_1 (obtaining σ_1) and m_2=m/m_1 (mod n) (obtaining σ_2)
  - (m, σ_1 σ_2) is a valid pair

RSA Signatures with Hash

Use a hash function H: {0,1}*→ Z_n*

Signing message m with private key (n,d)

- Compute $\sigma = H(m)^d \bmod n$

Verifying signature σ using public key (e, n)

- Check whether $\sigma^e \bmod n = H(m)$

Can be proven secure assuming that H is random oracle. (This is not considered a valid proof of security, but means that no known attack exists.)

Hash and Sign Paradigm

- Enabling the signing of arbitrary long message.

- Given a secure signing scheme (for a fixed message space), and a collision-resistant hash function, first hash and then sign is also secure.
  - “Textbook RSA” is insecure, so this result does not apply to hash and sign with RSA
  - Any attack either finds a collision or breaks the security of the signing scheme.

Non-repudiation

- Nonrepudiation is the assurance that someone cannot deny something. Typically, nonrepudiation refers to the ability to ensure that a party to a contract or a communication cannot deny the authenticity of their signature on a document or the sending of a message that they originated.

- Can one deny a signature one has made?

- Does email provide non-repudiation?

Coming Attractions …

- Other Signature Schemes
- Reading: Katz & Lindell: Chapter 12.5,12.7